SECONDARY
JHS

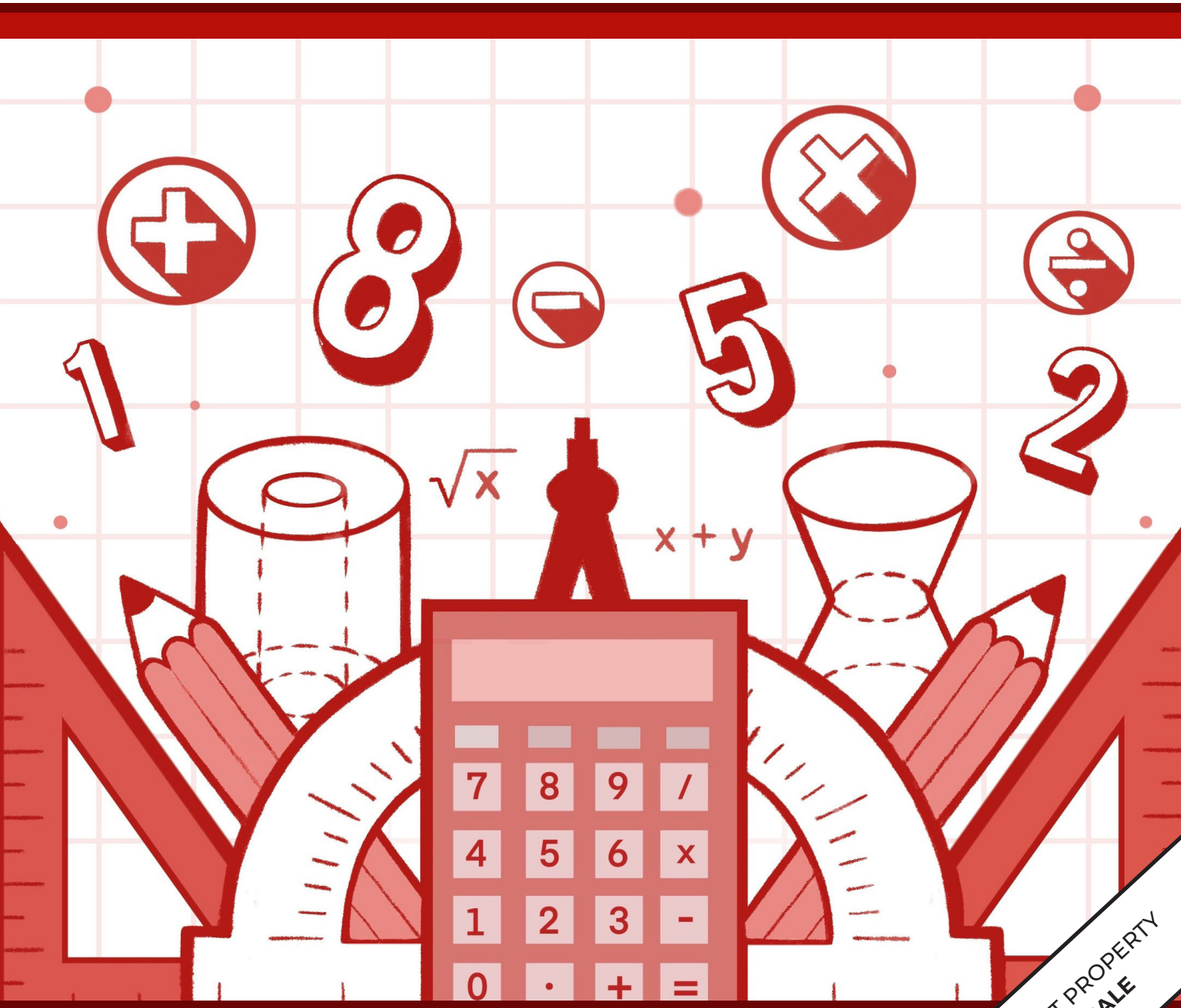
DepED DEPARTMENT OF EDUCATION

ALS Alternative Learning System LIFELONG LEARNING

LEARNING STRAND 3
MATHEMATICAL & PROBLEM-SOLVING SKILLS

MODULE 5: HOW MUCH WILL IT GROW?

ALS Accreditation and Equivalency Program: Junior High School



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LEARNING STRAND 3

HOW MUCH WILL IT GROW?

MATHEMATICAL AND PROBLEM-SOLVING SKILLS
MODULE 5

ALS Accreditation and Equivalency Program: Junior High School
Learning Strand 3: Mathematical and Problem-Solving Skills
Module 5: How Much Will It Grow?

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User's Guide

For the ALS Learner:

Welcome to this Module entitled How Much Will It Grow? under Learning Strand 3 Mathematical and Problem-Solving Skills of the ALS K to 12 Basic Education (BEC).

This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace and time. You will be enabled to process the contents of the learning resource while being an active learner.

This module has the following parts and corresponding icons:

 *Let's Get to Know*	This will give you an idea of the skills or competencies you are expected to learn in the module.
 *Pre-assessment*	This part includes an activity that aims to check what you already know about the lesson. If you get all the answers correct (100%), you may decide to skip this module.
 *Setting the Path*	This section provides a brief discussion of the lesson. This aims to help you discover and understand new concepts and skills.
 *Trying This Out*	This comprises activities for independent practice to solidify your understanding and skills of the topic. You may check the answers to the exercises using the Answer Key at the end of the module.
 *Understanding What You Did*	This includes questions that process what you learned from the lesson.
 *Sharpening Your Skills*	This section provides an activity that will help you transfer your new knowledge or skill in real-life situations or concerns.
 *Treading the Road to Mastery*	This is a task which aims to evaluate your level of mastery in achieving the given learning competency.
 *Don't Forget*	This part serves as a summary of the lesson in the module.
 *Explore More*	In this portion, another activity will be given to you to enrich your knowledge or skill of the lesson learned. This also tends retention of learned concepts.
 *Reach the Top*	This part will assess your level of mastery in achieving the learning competencies in each lesson in the module.
Answer Key	This contains answers to all activities in the module.
Glossary	This portion gives information about the meanings of the specialized words used in the module.

At the end of this module you will also find:

References	This is a list of all sources used in developing this module.

The following are some reminders in using this module:

1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
2. Don't forget to answer the Pre-assessment before moving on to the other activities included in the module.
3. Read the instruction carefully before doing each task.
4. Observe honesty and integrity in doing the tasks and checking your answers.
5. Finish the task at hand before proceeding to the next.
6. Return this module to your ALS Teacher/Instructional Manager/Learning Facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your ALS Teacher/Instructional Manager/Learning Facilitator. Always bear in mind that you are not alone.

We hope that through this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!

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MODULE 5

LET'S GET TO KNOW

Abdul and Sahaya are a young couple that is planning to start a family soon. Both, however, want to be financially stable before they decide to marry. They are exploring ways to save money so they can better provide for their future family.

MODULE 5
PRE-ASSESSMENT

Choose the letter of the correct answer. Write your answers on a separate sheet of paper.

1. Given an arithmetic sequence of 3, 1, −1, −3, −5, −7,… Find the 8th term of the given sequence.

   A. 11 **B.** −11 **C.** 9 **D.** −9

2. What is the common difference in the arithmetic sequence given in item no. 1?

   A. 3 **B.** −3 **C.** −2 **D.** 2

3. Find the next two terms in the geometric sequence 5, −10, 20, −40 ___, ___.

   A. 80 and −160 **B.** −80 and 160 **C.** −80 and −160 **D.** 80 and 160

4. Find the common difference in the arithmetic sequence 2, $\frac{5}{2}$, 3, $\frac{7}{2}$,…

   A. $\frac{3}{4}$ **B.** $\frac{1}{4}$ **C.** $\frac{1}{2}$ **D.** $-\frac{1}{2}$

5. Find the sum of all the even integers between 3 and 33?

   A. 274 **B.** 272 **C.** 270 **D.** 268

6. What is the 9th term of an arithmetic sequence if the common difference is 3 and the 6th term is 16?

   A. 22 **B.** 25 **C.** 27 **D.** 31

7. Which shape continues the given sequence below?

   A.  **B.**  **C.**  **D.** 

8. What is the sum of all the multiples of 5 from 3 to 78?

A. 595 **B.** 600 **C.** 605 **D.** 610

9. If two geometric means are inserted between 3 and 375, find the second geometric mean.

A. 60 **B.** 65 **C.** 70 **D.** 75

10. Which term of the geometric sequence 1, −2, 4, −8,… is −128 ?

A. 9th term **B.** 8th term **C.** 7th term **D.** 6th term

11. What is the 7th term in the geometric sequence −2, 6, −18, 54, −162,….

A. −1457 **B.** 1457 **C.** −1458 **D.** 1458

12. Find the common ratio of the geometric sequence $2, \frac{5}{2}, \frac{25}{8}, \frac{125}{32}, \ldots$

A. $\frac{5}{4}$ **B.** $\frac{5}{3}$ **C.** $\frac{5}{2}$ **D.** 5

13. If the first term of a geometric sequence is 1 and the second term is 4, what is the sum of the first five terms?

A. 339 **B.** 340 **C.** 341 **D.** 342

14. What are the last two terms of the geometric sequence given in item 13?

A. 16 and 64 **B.** 64 and 256 **C.** 256 and 1024 **D.** 16 and 256

15. During the community lockdown, Christian decided to do push up sessions every day. On the first day of the session, he accomplished 20 push ups. He made 25 push ups on the second day and 30 push ups on the third day. How many push ups will he have accomplished during the 7th day session?

A. 60 **B.** 55 **C.** 50 **D.** 45

LESSON 1

SETTING THE PATH

SUMMING IT UP

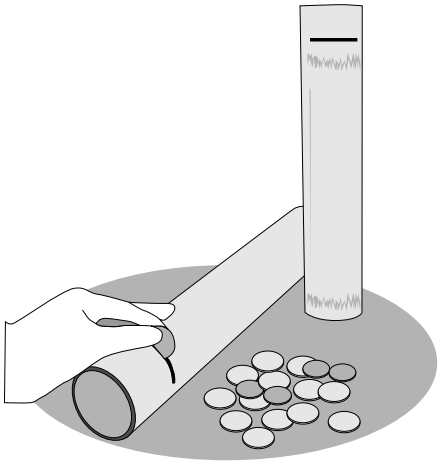
At the end of this lesson, you will be able to:

- illustrate an arithmetic sequence (LS3MP-PA-PSE-JHS-74);
- find the unknown term/s of arithmetic sequence (LS3MP-PA-PSE-JHS-77);
- find the arithmetic series and other related unknown values (LS3MP-PA-PSE-JHS-78); and
- find the arithmetic mean/s (LS3MP-PA-PSE-JHS-79).

LESSON 1
TRYING THIS OUT

Abdul and Sahaya would like to try the *Kuripot* Challenge in which a person starts saving a fixed amount of money and continuously increases it over time. For example, one person starts saving ₱20.00 for the first day and decides to increase it by ₱10.00 per day, as displayed in the table below:

TIME	AMOUNT SAVED
DAY 1	₱20.00
DAY 2	₱30.00
DAY 3	₱40.00
DAY 4	₱50.00
and so on...	₱...



Abdul decided to start saving ₱150.00 and continued to save every month from January to December. The amount he saves increases by ₱50.00 per month.

Sahaya decided to start saving ₱500.00 quarterly or every three months (March, June, September, and December). The amount increases by ₱500.00 per quarter.

On a separate sheet of paper, create a list for both Abdul and Sahaya's *Kuripot* Challenge amount similar to the given table above. Then, answer the questions below.

1. Who has the bigger amount saved at the end of the year? By how much?
2. How much is the total amount of money will they save together at the end of the year?

LESSON 1

UNDERSTANDING WHAT YOU DID

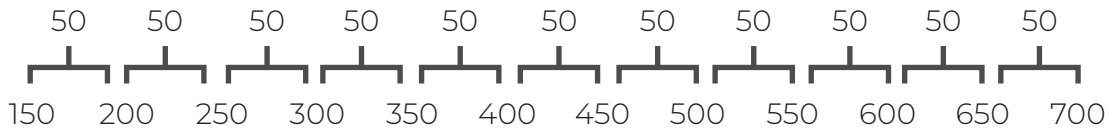
To be able to compare Abdul's and Sahaya's *Kuripot* Challenge, it may be presented as:

ABDUL'S *KURIPOT* CHALLENGE	
MONTH	AMOUNT
JANUARY	150
FEBRUARY	200
MARCH	250
APRIL	300
MAY	350
JUNE	400
JULY	450
AUGUST	500
SEPTEMBER	550
OCTOBER	600
NOVEMBER	650
DECEMBER	700

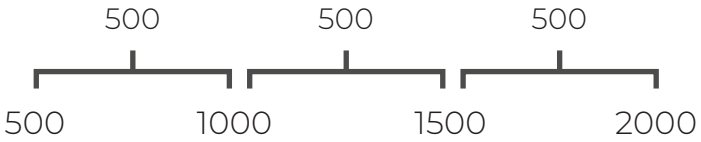
SAHAYA'S *KURIPOT* CHALLENGE	
MONTH	AMOUNT
MARCH	500
JUNE	1000
SEPTEMBER	1500
DECEMBER	2000

Both list of numbers in Abdul and Sahaya's *Kuripot* Challenge follows a pattern where any two consecutive numbers in the group has the same difference.

ABDUL



SAHAYA



The set of numbers in both *Kuripot* Challenges are examples of a **sequence.**

SEQUENCE OF NUMBERS

A sequence of numbers is a list of numbers that usually follow a pattern. Each number in a sequence is called a **term.**

Examples: 1, 3, 5, 7, 9,...
3, 6, 9, 12, 15,...
1, 2, 4, 8, 16, 32,...

In Abdul's *Kuripot* Challenge, each amount is ₱50.00 greater than the previous amount. While in Sahaya's *Kuripot* Challenge, each amount is ₱500 greater than the previous amount. The type of sequence portrayed by Abdul and Sahaya's *Kuripot* Challenge is called an **arithmetic sequence.**

ARITHMETIC SEQUENCE

An **arithmetic sequence** is a sequence of numbers where every two consecutive terms differ by a constant called the **common difference (d).**

To get the common difference, *subtract* the first term from the second term, *subtract* the second term from the third term, and so on ($a_2 - a_1$; $a_3 - a_2$; and so on).

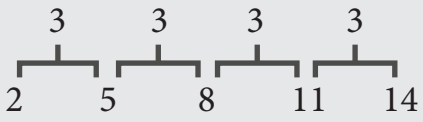
In the activity, the common difference in Abdul's saving is 50 ($d = 50$), while Sahaya has a common difference of 500 ($d = 500$).

To find the next term in an arithmetic sequence, *add* the common difference to the previous term.

Examples: For each arithmetic sequence, find the common difference and the next term.

a. 2, 5, 8, 11, 14,…
b. $\frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2},\ldots$
c. 4, −1, −6, −11, −16,…

a. **2, 5, 8, 11, 14,...**



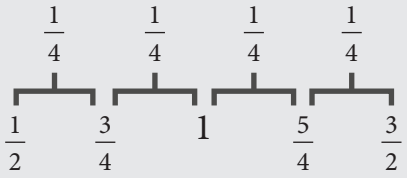
second term − first term = 5 − 2 = 3
third term − second term = 8 − 5 = 3
fourth term − third term = 11 − 8 = 3…

Therefore, the common difference is $d = 3$.

To get the next term, add the common difference $d = 3$ to the last term, 14. Thus, the next term of the sequence is

$$14 + 3 = 17.$$

b. $\frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2},\ldots$



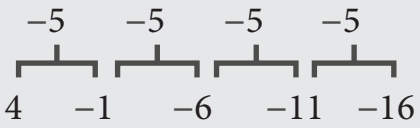
second term − first term = $\frac{3}{4} - \frac{1}{2} = \frac{1}{4}$
third term − second term = $1 - \frac{3}{4} = \frac{1}{4}$
fourth term − third term = $\frac{5}{4} - 1 = \frac{1}{4}$…

Therefore, the common difference is $d = \frac{1}{4}$.

To get the next term, add the common difference $d = \frac{1}{4}$ to the last term, $\frac{3}{2}$. Thus, the next term of the sequence is

$$\frac{3}{2} + \frac{1}{4} = \frac{7}{4}.$$

c. **4, −1, −6, −11, −16,…**



second term − first term = −1 − 4 = −5
third term − second term = −6 − (−1) = −5
fourth term − third term = −11 − (−6) = −5

Therefore, the common difference is $d = -5$.

To get the next term, add the common difference $d = -5$ to the last term, −16. Thus, the next term of the sequence is

$$-16 + (-5) = -21.$$

FINDING MISSING TERMS IN AN ARITHMETIC SEQUENCE

Going back to the activity, what happens if Abdul would like to continue saving up beyond the 12 months? What amount does he need to save on the 30th month?

Listing down all the amount he will save per month is tiring and prone to mistakes. Instead, let us look for a pattern to find the 30th term in the sequence.

To do that, let us first define some variables that can help us.

$\boldsymbol{a_1}$ = first term of the arithmetic sequence;
$\boldsymbol{n}$ = number of the term in the sequence;
$\boldsymbol{a_n}$ = nth term of the sequence; and
$\boldsymbol{d}$ = the common difference

Using Abdul's *Kuripot* Challenge as an example, we have

$$a_1 = 150 \qquad d = 50.$$

We want to find a_{30}. Listing down the terms,

TERM	ORDER	VALUE	PROCESS
a_1	1	150	
a_2	2	200	= 150 + 50
a_3	3	250	= 150 + 50 + 50
a_4	4	300	= 150 + 50 + 50 + 50

For the first four terms in the arithmetic sequence, what is the relationship between the **order of the term** and the number of times the **common difference** is added?

$a_1 \rightarrow$ order 1 = 0 times added
$a_2 \rightarrow$ order 2 = 1 time added
$a_3 \rightarrow$ order 3 = 2 times added
$a_4 \rightarrow$ order 4 = 3 times added

The number of times the common difference is added is given by **subtracting 1** from the order of the term. So, when we look for the $\boldsymbol{n^{th}}$ term in the sequence, add the common difference a number $(n - 1)$ times. That is,

$a_n \rightarrow$ Order n = $(n - 1)$ times added

For Abdul's savings on the 30th month, we can write the process as:

TERM	ORDER	PROCESS
a_n	n	150 + 50 + 50 + 50 + ... + 50 $(n-1)$ *times* $a_n = 150 + (n-1)50$
a_{30}	30	$a_{30} = 150 + (30-1)50$ $= 150 + (29)50$ $= 150 + 1450$ $= 1600$

This means that Abdul will save an amount of ₱1,600.00 on the 30th month.

In general, to find the n^{th} **term** of an arithmetic sequence, we can use the formula given below:

$$a_n = a_1 + (n-1)d$$

where a_n = n^{th} term
a_1 = first term
d = common difference
n = number of terms

Example 1. Find the 12th term of the arithmetic sequence 6, 13, 20, 27,…

Given $a_1 = 6$ $n = 12$
$d = 7$ $a_{12} =$ unknown

Solution

$a_n = a_1 + (n-1)d$	*formula*
$a_{12} = 6 + (12-1)7$	*substitute the given*
$= 6 + (11)7$	*multiply 11 by 7*
$= 6 + 77$	*add*
$a_{12} = 83$	*answer*

Therefore, the 12th term of the sequence is 83.

Example 2. What is the first term in the arithmetic sequence with a common difference of 4, if the 7th term is 21?

Given $d = 4$ $a_7 = 21$
$n = 7$ $a_1 =$ unknown

Solution

$a_n = a_1 + (n-1)d$	*formula*
$a_7 = a_1 + (7-1)4$	*substitute the given*
$21 = a_1 + (6)4$	*simplify*
$21 = a_1 + 24$	*combine constant numbers*
$21 - 24 = a_1$	*simplify*
$-3 = a_1$	*answer*

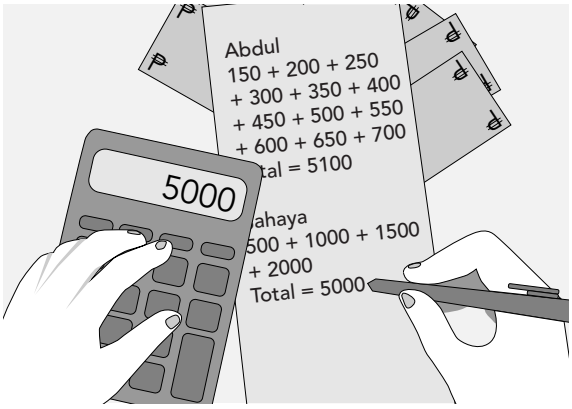
Therefore, the 1st term of the sequence is -3.

ARITHMETIC SERIES

From the activity,

Abdul's savings are computed by:
= 150 + 200 + 250 + 300 + 350 + 400 + 450 + 500 + 550 + 600 + 650 + 700
= 5,100

Sahaya's savings are computed by:
= 500 + 1000 + 1500 + 2000
= 5,000



Therefore, Abdul will have more money saved in a year by ₱100.00 than Sahaya. And they will have 5,100 + 5,000 = 10,100 in total.

The sum of the terms of a sequence is called **series.** The sum of the terms of an arithmetic sequence is called an **arithmetic series.** Here are some more examples of arithmetic series:

ARITHMETIC SEQUENCE	ARITHMETIC SERIES
2, 5, 8, 11, 14	$2 + 5 + 8 + 11 + 14 = 40$
$\frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2}$	$\frac{1}{2} + \frac{3}{4} + 1 + \frac{5}{4} + \frac{3}{2} = 5$

The sum of the first n terms of a series is denoted by $\boldsymbol{S_n}$. For instance, S_5 stands for the sum of the first 5 terms of a series.

But arithmetic series may be hard to find when there are many terms in the sequence.

To develop a formula for an arithmetic series, let us first use Sahaya's savings. Observe that we can write Sahaya's total savings as

$$S_4 = 500 + 1000 + 1500 + 2000$$

We use the process of writing S_4 into two different orders and add them:

$$\begin{aligned} S_4 &= 500 + 1000 + 1500 + 2000 \\ + \; S_4 &= 2000 + 1500 + 1000 + 500 \\ \hline 2S_4 &= 2500 + 2500 + 2500 + 2500 \\ 2S_4 &= 4(2500) \\ S_4 &= \frac{4}{2}(2500) \\ S_4 &= 5000 \end{aligned}$$

starts at a_1, ends at a_4
starts at a_4, ends at a_1

since the sum has 4 terms
2500 comes from the sum of the first up to the last terms
eliminate 2 using the Multiplication Property of Equality
We got the same answer.

We now use Abdul's savings. Observe that we can write Abdul's total savings as

$$S_{12} = 150 + 200 + 250 + 300 + 350 + 400 + 450 + 500 + 550 + 600 + 650 + 700$$

We use the process of writing S_{12} into two different orders and add them:

$$\begin{aligned} S_{12} &= 150 + 200 + 250 + 300 + 350 + 400 + 450 + 500 + 550 + 600 + 650 + 700 \\ + \; S_{12} &= 700 + 650 + 600 + 550 + 500 + 450 + 400 + 350 + 300 + 250 + 200 + 150 \\ \hline 2S_{12} &= 850 + 850 + 850 + 850 + 850 + 850 + 850 + 850 + 850 + 850 + 850 + 850 \\ 2S_{12} &= 12(850) \\ S_{12} &= \frac{12}{2}(850) \\ S_{12} &= 5100 \end{aligned}$$

since the sum has 12 terms
850 comes from the sum of the first up to the last terms
eliminate 2 using the Multiplication Property of Equality
We got the same answer.

In general, to find the **sum of the first n terms** of an arithmetic sequence, we can use the formula given below:

$$S_n = \frac{n}{2}(a_1 + a_n)$$

where S_n = sum of the first n terms
a_1 = first term
a_n = last term
n = number of terms

However, we can replace a_n with $a_1 + (n-1)d$. Substituting this to S_n, we get

$$S_n = (a_1 + a_n) = \frac{n}{2}[a_1 + a_1 + (n-1)d]$$

or

$$S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

The second formula can be used if the last term a_n is not given.

Example 3. Find the sum of the first 20 terms in the arithmetic sequence 4, 8, 12,…

Given $a_1 = 4$ $n = 20$ $d = 4$

Solution

$S_n = \frac{n}{2}[2a_1 + (n-1)d]$

$S_{20} = \frac{20}{2}[2(4) + (20-1)4]$ *substitute the given to the formula*

$= 10[8 + (19)4]$ *simplify using PEMDAS*

$= 10(8 + 76)$ *simplify*

$= 10(84)$ *simplify*

$= 840$ *final answer*

Therefore, the sum of the first 20 terms of the given arithmetic sequence is 840.

Example 4. The arithmetic series of a sequence is 253 with the first term equal to 8. Find the common difference if there are 11 terms.

Given $S_{11} = 253$ $n = 11$ $a_1 = 8$

Solution

$S_n = \frac{n}{2}[2a_1 + (n-1)d]$

$253 = \frac{11}{2}[2(8) + (11-1)d]$ *substitute the given*

$253 = \frac{11}{2}[16 + (10)d]$ *simplify the left side of the equation*

$253 = \frac{11}{2}[16 + 10d]$ *Distributive Property*

$253 = \frac{176}{2} + \frac{110}{2}d$ *simplify*

$253 = 88 + 55d$ *Addition Property of Equality*

$253 - 88 = 55d$

$165 = 55d$ *Multiplication Property of Equality*

$\frac{165}{55} = d$ *simplify*

$3 = d$ *answer*

Therefore, the common difference is 3.

ARITHMETIC MEANS

There are times that we are given two terms of a sequence, but they are non-consecutive terms. **Arithmetic means** refer to terms to be placed between any two nonconsecutive terms in an arithmetic sequence.

For instance, given the sequence 9, 12, 15, 18, 21, 24, 27,…, there are two arithmetic means between 15 and 24 and these are 18 and 21.

9, 12, 15, **18, 21,** 24, 27,…

2 arithmetic means
between 15 and 24

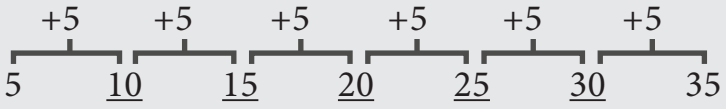
Example 5. Insert 5 arithmetic means between 5 and 35.

Given $a_1 = 5$ $a_7 = 35$

5, _____, _____, _____, _____, _____, 35

Since we are finding for terms to be inserted, we first solve for d:

$$\begin{aligned} a_n &= a_1 + (n-1)d \\ a_7 &= 5 + (7-1)d \\ 35 &= 5 + 6d \\ 35 - 5 &= 6d \\ 30 &= 6d \\ 5 &= d \end{aligned}$$



Therefore, the 5 arithmetic means are 10, 15, 20, 25, and 30.

LESSON 1

SHARPENING YOUR SKILLS

Write your answers on a separate sheet of paper.

I. Solve for the n^{th} term in the arithmetic sequence using the given values.

1.	$a_1 = -5$	$d = 3$	$n = 8$
2.	$a_1 = 7$	$d = -4$	$n = 15$
3.	$a_1 = 11$	$d = 8$	$n = 33$
4.	$a_1 = -6$	$d = -10$	$n = 12$
5.	$a_1 = 13$	$d = 9$	$n = 50$

II. Determine the arithmetic series of a sequence that satisfies the stated conditions.

1.	$a_1 = 21$	$a_n = 63$	$n = 11$
2.	$a_1 = -12$	$a_n = 46$	$n = 9$
3.	$a_1 = -8$	$a_n = -30$	$n = 15$
4.	$a_1 = 25$	$n = 16$	$d = -3$
5.	$a_1 = 13$	$n = 5$	$d = 7$

LESSON 1

TREADING THE ROAD TO MASTERY

Write your answers on a separate sheet of paper.

Solve the problem.

1. Find three arithmetic means between 16 and 76.
2. Find three arithmetic means between 55 and 115.
3. Find two arithmetic means between 10 and −8.
4. Find four arithmetic means between −8 and 7.
5. Find five arithmetic means between 3 and 27.

LESSON 2

SETTING THE PATH

MULTIPLY IT CONTINUOUSLY!

At the end of this lesson, you will be able to:

- illustrate a geometric sequence (LS3MP-PA-PSE-JHS-80);
- find the unknown terms of geometric sequence (LS3MP-PA-PSE-JHS-83); and
- find the geometric series and other unknown values (LS3MP-PA-PSE-JHS-84).

LESSON 2
TRYING THIS OUT

To prepare for their future, Sahaya and Abdul are looking at putting their money on investments. They want to choose an investment that will allow their money to grow more. Sahaya found a stocks investment offered by a bank while Abdul found a mutual trust fund offered by an insurance company.

	STOCKS	**MUTUAL FUND**
PRINCIPAL	INVEST ₱80	INVEST ₱100
TERMS	Total investment *multiplies by 3* every year after year 1	Total investment *multiplies by 2* every year after year 1
MATURITY	5 years	7 years
AMOUNT SCHEDULE		
YEAR 1	80	100
YEAR 2	80(3) = 240	100(2) = 200
YEAR 3	240(3) = 720	200(2) = 400
YEAR 4		
YEAR 5		
YEAR 6	INVESTMENT MATURITY	
YEAR 7	INVESTMENT MATURITY	

1. Fill in the amount for the remaining years for both investments.
2. Which of the two is the better choice for investment at their maturity?

LESSON 1

UNDERSTANDING WHAT YOU DID

The increase in the total investment for both options should look like this:

AMOUNT SCHEDULE	STOCKS	MUTUAL FUND
YEAR 1	80	100
YEAR 2	80(3) = 240	100(2) = 200
YEAR 3	240(3) = 720	200(2) = 400
YEAR 4	720(3) = 2160	400(2) = 800
YEAR 5	2160(3) = 6480	800(2) = 1600
YEAR 6		1600(2) = 3200
YEAR 7		3200(2) = 6400

Investing on stocks results in a larger amount at its maturity making it a better choice.

STOCKS	240, 720, 2160, 6480
MUTUAL FUND	200, 400, 800, 1600, 3200, 6400

In stocks, each amount is being multiplied by 3 to get the next amount. While in mutual fund, each amount is being multiplied by 2 to get the next amount. The type of sequence portrayed by these investments is called a **geometric sequence.**

GEOMETRIC SEQUENCE

A **geometric sequence** is a set of numbers where each term after the first is found by multiplying the previous term by a constant called the **common ratio** (r).

To get the common ratio, *divide* the second term by the first term, *divide* the third term by the second term, *divide* the fourth term by the third term, and so on.

In the activity, the common ratio in stocks is 3 ($r = 3$), while in mutual fund, the common ratio is 2 ($r = 2$).

To find the next term in a geometric sequence, *multiply* the common ratio to the previous term.

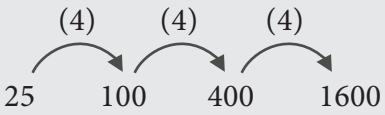
Examples: For each geometric sequence, find the common ratio and the next term.

a. 25, 100, 400, 1600,…

b. 3, −6, 12, −24,…

c. $\frac{1}{2}, \frac{1}{6}, \frac{1}{18}, \frac{1}{54}, \ldots$

a. **25, 100, 400, 1600,…**



second term ÷ first term = 100 ÷ 25 = 4
third term ÷ second term = 400 ÷ 100 = 4
fourth term ÷ third term = 1600 ÷ 400 = 4

To get the common ratio, *divide* the second term by the first term, *divide* the third term by the second term, *divide* the fourth term by the third term.

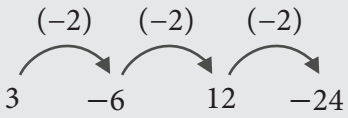
Therefore, the common ratio is $r = 4$.

To get the next term of the given sequence, multiply the common ratio ($r = 4$) to the last term (1600) of the sequence.

$$1600(4) = 6400$$

Thus, the next term of the given sequence is 6400.

b. **3, –6, 12, –24,…**



second term ÷ first term = –6 ÷ 3 = –2
third term ÷ second term = 12÷ (–6) = –2
fourth term ÷ third term = –24 ÷ 12 = –2

To get the common ratio, *divide* the second term by the first term, *divide* the third term by the second term, *divide* the fourth term by the third term.

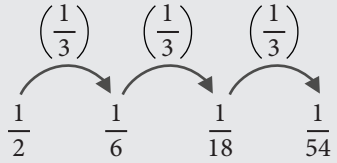
Therefore, the common ratio is $r = -2$.

To get the next term of the given sequence, multiply the common ratio ($r = -2$) to the last term (–24) of the sequence.

$$(-24)(-2) = 48$$

Thus, the next term of the given sequence is 48.

c. $\frac{1}{2}, \frac{1}{6}, \frac{1}{18}, \frac{1}{54}, \ldots$



second term ÷ first term = $\frac{1}{6} \div \frac{1}{2} = \frac{1}{3}$
third term ÷ second term = $\frac{1}{18} \div \frac{1}{2} = \frac{1}{3}$
fourth term ÷ third term = $\frac{1}{54} \div \frac{1}{18} = \frac{1}{3}$

Therefore, the common ratio is $r = \frac{1}{3}$.

To get the next term of the given sequence, multiply the common ratio $\left(r = \frac{1}{3}\right)$ to the last term $\left(\frac{1}{54}\right)$ of the sequence. Thus, the next term of the sequence is

$$\left(\frac{1}{54}\right)\left(\frac{1}{3}\right) = \frac{1}{162}$$

Thus, the next term of the sequence is $\frac{1}{162}$.

FINDING MISSING TERMS IN A GEOMETRIC SEQUENCE

Going back to the activity, what if the investment for the stocks, which is the better choice for investment, continues beyond 5 years of maturity? How do we find the amount of money that will result after 10 years?

Let us investigate using the first few terms in the geometric sequence of Sahaya's stocks.

Let us first define some variables that can help us:

$\boldsymbol{a_1}$ = first term of the geometric sequence;
$\boldsymbol{n}$ = order of the term in the sequence;
$\boldsymbol{a_n}$ = n^{th} term of the sequence; and
$\boldsymbol{r}$ = the common ratio

Using Sahaya's stocks investment, we have

$$a_1 = 80 \qquad r = 3$$

We want to find a_{10}. Let us list down the terms on the next page.

TERM	ORDER	VALUE	PROCESS
a_1	1	80	
a_2	2	240	$= 80 \cdot 3$
a_3	3	720	$= 80 \cdot 3 \cdot 3$
a_4	4	2160	$= 80 \cdot 3 \cdot 3 \cdot 3$

For the first four terms in the geometric sequence, what is the relationship between the **order of the term** and the number of times the **common ratio** is multiplied?

$a_1 \rightarrow$ order 1 $=$ 0 times multiplied
$a_2 \rightarrow$ order 2 $=$ 1 time multiplied
$a_3 \rightarrow$ order 3 $=$ 2 times multiplied
$a_4 \rightarrow$ order 4 $=$ 3 times multiplied

The number of times the common ratio is multiplied is given by **subtracting 1** from the order of the term. So, when we look for the $\boldsymbol{n}^{\text{th}}$ term in the sequence, multiply the common ratio a number $(n - 1)$ times. That is,

$$a_n \quad \rightarrow \quad \text{order } n \; = (n - 1) \text{ times multiplied}$$

For Sahaya's stock investment on the 10th year, we can write the process as:

TERM	ORDER	PROCESS
a_n	n	$80 \cdot 3 \cdot 3 \cdot 3 \cdot \cdots \cdot 3$ $\underbrace{\qquad\qquad}_{(n-1) \text{ times}}$ $\downarrow$ $a_n = 80 \cdot 3^{n-1}$

TERM	ORDER	PROCESS
a_{10}	10	$a_{10} = 80 \cdot 3^{(10-1)}$ $= 80 \cdot 3^9$ $= 80 \cdot 19{,}683$ $= 1{,}574{,}640$

This means that Sahaya's stocks investment after 10 years is ₱1,574,640.

In general, to find the n^{th} term of a geometric sequence, we can use the formula given below:

$$a_n = a_1 r^{(n-1)}$$

where a_n = n^{th} term
a_1 = first term
r = common ratio
n = number of terms

Example 1. Use the formula to find the 7th term in the sequence of 3, 6, 12,….

Given $a_1 = 3$ $n = 7$
$r = 2$ a_7 = unknown

Solution

$a_n = a_1 r^{(n-1)}$
$a_7 = 3(2)^{7-1}$ *substitute the given*
$= 3(2)^6$ *simplify*
$= 3(64)$
$a_7 = 192$ *answer*

Therefore, the 7th term of the sequence is 192.

Example 2. The fourth term in a geometric sequence is 320. Find the common ratio if the first term is 5.

Given $a_1 = 5$ $a_4 = 320$
$n = 4$ r = unknown

Solution

$$a_n = a_1 r^{(n-1)}$$
$$a_4 = 5(r)^{4-1} \quad \textit{substitute}$$
$$320 = 5r^3 \quad \textit{simplify}$$
$$\frac{320}{5} = r^3 \quad \textit{Multiplication Property of Equality}$$
$$64 = r^3 \quad \textit{simplify}$$
$$\sqrt[3]{64} = r \quad \textit{extracting roots}$$
$$4 = r \quad \textit{answer}$$

Therefore, the common ratio of the sequence is 4.

GEOMETRIC SERIES

From the activity,

Abdul's mutual fund's sum (for 7 years) is computed by:
$= 100 + 200 + 400 + 800 + 1600 + 3200 + 6400$
$=$ **12,700**

Sahaya's stock's sum (for 5 years) is computed by:
$= 80 + 240 + 720 + 2160 + 6480$
$=$ **9,680**

A **geometric series** is the sum of all the terms in a geometric sequence. Here are some examples of geometric series:

GEOMETRIC SEQUENCE	GEOMETRIC SERIES
25, 100, 400, 1600	$25 + 100 + 400 + 1600 = 2125$
$\frac{1}{2}, \frac{1}{6}, \frac{1}{18}, \frac{1}{54}$	$\frac{1}{2} + \frac{1}{6} + \frac{1}{18} + \frac{1}{54} = \frac{20}{27}$

To investigate a formula for a geometric series, we consider Sahaya's stock investment. Observe that we can write Sahaya's total investment as

$$S_5 = 80 + 240 + 720 + 2160 + 6480$$

Use this in the process below:

$$S_5 = 80 + 240 + 720 + 2160 + 6480$$

multiply S_5 by the common ratio 3, then subtract

$$(-)3S_5 = \quad 240 + 720 + 2160 + 6480 + 19440$$

$$(1-3)S_5 = 80 + 0 + 0 + 0 + 0 - 19440$$

$$(1-3)S_5 = 80 - 19440$$

19440 comes from the last term of the series multiplied by the common ratio 3

$$S_5 = \frac{80 - 19440}{1 - 3}$$

$$S_5 = 9680$$

We got the same answer.

We now use Abdul's mutual fund. Observe that we can write Abdul's total investment as

$$S_7 = 100 + 200 + 400 + 800 + 1600 + 3200 + 6400$$

Use this in the process below:

$$S_7 = 100 + 200 + 400 + 800 + 1600 + 3200 + 6400$$

multiply S_7 by the common ratio 2, then subtract

$$(-)2S_7 = \quad 200 + 400 + 800 + 1600 + 3200 + 6400 + 12800$$

$$(1-2)S_7 = 100 + 0 + 0 + 0 + 0 + 0 + 0 - 12800$$

$$(1-2)S_7 = 100 - 12800$$

12800 comes from the last term multiplied by common ratio 2

$$S_7 = \frac{100 - 12800}{1 - 2}$$

$$S_7 = 12700$$

We got the same answer.

In general, to find **the sum of the first n terms** of a geometric sequence, we can use the formula given below:

$$S_n = \frac{a_1 - a_1 r^n}{1 - r} = \frac{a_1(1 - r^n)}{1 - r} = a_1\left(\frac{1 - r^n}{1 - r}\right)$$

where S_n = sum of the first n terms
a_1 = first term
r = common ratio
n = number of terms

Example 3. Find the geometric series with five terms if the first term is 4 with a common ratio of 2.

Given $a_1 = 4$ $\quad r = 2$
$n = 5$ $\quad S_n =$ unknown

Solution

$$S_n = a_1\left(\frac{1 - r^n}{1 - r}\right)$$

$$S_5 = 4\left(\frac{1 - 2^5}{1 - 2}\right)$$

$$= 4\left(\frac{1 - 32}{1 - 2}\right)$$

$$= 4\left(\frac{-31}{-1}\right)$$

$$= 4(31)$$

$$S_5 = 124$$

Therefore, the sum of the first 5 terms of the given geometric sequence is 124.

Example 4. Find the geometric series with four terms if the first term is 12 with common ratio −3.

Given $a_1 = 12$
$n = 4$
$r = -3$
$S_n =$ unknown

Solution

$$S_n = a_1\left(\frac{1-r^n}{1-r}\right)$$

$$S_4 = 12\left[\frac{1-(-3)^4}{1-(-3)}\right]$$

$$= 12\left(\frac{1-81}{1+3}\right)$$

$$= 12\left(\frac{-80}{4}\right)$$

$$= 12\,(-20)$$

$$S_4 = -240$$

Therefore, the sum of the first 4 terms of the given geometric sequence is −240.

LESSON 2

SHARPENING YOUR SKILLS

Write your answers on a separate sheet of paper.

I. Find the common ratio for the given geometric sequence.

1. $-1, 6, -36, 216,$

2. $\frac{3}{5}, \frac{1}{5}, \frac{1}{15}, \frac{1}{45}, \ldots$

3. $2, 1, 0.5, 0.25, \ldots$

4. $3, -3, 3, -3, \ldots$

5. $16, -4, 1, -\frac{1}{45}, \ldots$

II. Determine the n^{th} term of the geometric sequence.

1. $25, 100, 400, \ldots$

A. 8th term **B.** 12th term

2. $3, -6, 12, -24, \ldots$

A. 9th term **B.** 14th term

3. $2, 1, \frac{1}{2}, \frac{1}{4}, \ldots$

A. 8th term **B.** 11th term

LESSON 2

TREADING THE ROAD TO MASTERY

Write your answers on a separate sheet of paper.

Answer the following items.

1. Determine the common ratio of the geometric sequence with $a_1 = 14$ and $a_6 = -3{,}402$.
2. Determine the common ratio of the geometric sequence with $a_1 = 3$ and $a_8 = 384$.
3. Find the sum of the first 10 terms of −4, 12, −36, 108…
4. Find the sum of the first 8 terms of 8, 4, 2, 1,…

LESSON 3

SETTING THE PATH

APPLYING SEQUENCES AND SERIES

At the end of the lesson, you will be able to:

- solve problems that involve arithmetic and geometric sequence/series (LS3MP-PA-PSE-JHS-86); and
- express satisfaction in mastery of new ways of thinking through application of mathematics (LS3MP-NS-PSE-BL/LE/AE/JHS-3).

LESSON 3

TRYING THIS OUT

Write whether the given word problem involves arithmetic sequence or geometric sequence. Then, write all the given values.

1. A networking business targets to recruit 5 new members every week. If they started with 11 members, how many members would they have after 8 weeks?

2. A coastal clean-up campaign is able to decrease the area of garbage pollution in a day by 4 square kilometers. If the total amount of land being cleaned is 52 square kilometers, how long will it take the coastal clean-up to clean the given land area?

3. A cell in a body divides into two in one minute. If there are initially 1,000 cells, how many cells are there after 6 minutes?

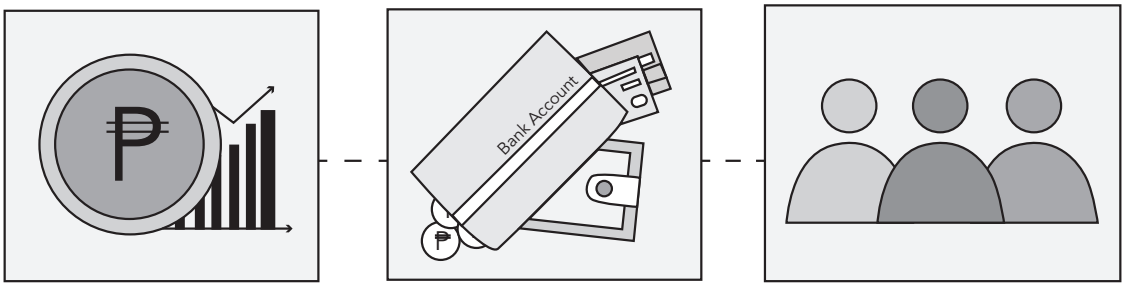
4. You deposited Php 300.00 in a bank that pays interest at 20% in one year. How much interest will be given after 3 years?

5. A stack of boxes has 8 layers. If the first layer has 20 boxes and the last layer has 13 boxes, how many boxes are in the stack?

LESSON 3

UNDERSTANDING WHAT YOU DID

Arithmetic and geometric series are applicable when solving problems in every day life especially in finance and the sciences.



Word Problems Involving Arithmetic Sequence

Let us apply the concepts of arithmetic sequence and series into word problems.

Example 1. A networking business targets to recruit 5 new members every week. If they started with 11 members, how many members would they have after 8 weeks?

We want to know how many members the business will have after 8 weeks so the missing value is a_8.

Given $a_1 = 11$ $d = 5$
$n = 8$ $a_8 =$ unknown

Solution

$$a_n = a_1 + (n - 1)d$$
$$a_8 = 11 + (8 - 1)5$$
$$= 11 + (7)5$$
$$= 11 + 35$$
$$a_8 = 46$$

Therefore, the networking business will have 46 members after 8 weeks.

Example 2. A coastal clean-up campaign is able to decrease the area of garbage pollution in a day by 4 square kilometers. If the total amount of land being cleaned is 52 square kilometers, how long will it take the coastal clean-up to clean the given land area?

We want to find the total number of days it will take to finish cleaning the total area of the garbage pollution, so the missing value is n.

Given $a_1 = 52$ $d = 4$
$a_n = 0$ $n =$ unknown

Solution

$$a_n = a_1 + (n - 1)d$$

$$
\begin{aligned}
0 &= 52 + (n - 1)(-4) \\
&= 52 + (-4n + 4) \\
&= 56 - 4n \\
4n &= 56 \\
n &= \frac{56}{4} \\
n &= 14
\end{aligned}
$$

Therefore, the coastal clean-up will be able to clean the whole area in 14 days.

Example 3. A stack of boxes has 8 layers. If the first layer has 20 boxes and the last layer has 13 boxes, how many boxes are in the stack?

We want to know the sum of all the boxes in the stack, so the missing value is S_8.

Given $a_1 = 20$ $\quad n = 8$
$a_8 = 13$ $\quad S_8 =$ unknown

Solution

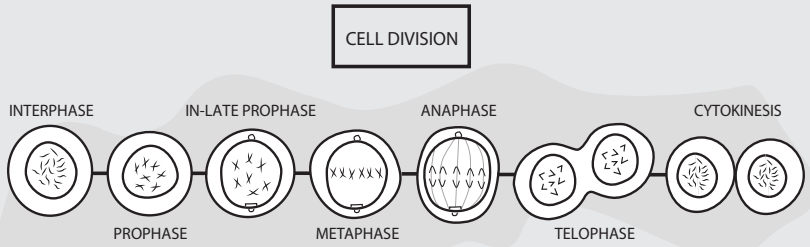
$$
\begin{aligned}
S_n &= \frac{n}{2}(a_1 + a_n) \\
S_8 &= \frac{8}{2}(20 + 13) \\
&= 4\,(33) \\
a_8 &= 132
\end{aligned}
$$

Therefore, there are 132 boxes in the stack.

Word Problems Involving Geometric Sequence

Let us apply the concepts of geometric sequence and series into word problems.

Example 4. A cell in a body divides into two in one minute. If there are initially 1,000 cells, how many cells are there after 6 minutes?



We want to know the number of cells after 6 minutes, so the missing value is a_6.

Given

$a_1 = 1000$
$n = 6$
$r = 2$
$a_6 =$ unknown

Solution

$$a_n = a_1 r^{n-1}$$
$$a_6 = 1000(2)^{6-1}$$
$$= 1000(2)^5$$
$$= 1000(32)$$
$$a_6 = 32{,}000$$

Therefore, there are 32,000 cells in the body after 6 minutes.

Example 5. You deposited ₱300.00 in a bank that pays interest at 20% in one year. How much interest will be given after 3 years?

We want to know the interest after 3 years, so the missing value is a_3.

Given

$a_1 = 300$
$r = 0.2$
$n = 3$
$a_3 =$ unknown

Solution

$a_n = a_1 r^{n-1}$
$a_3 = 300(0.2)^{3-1}$
$= 300(0.2)^2$
$= 300(0.04)$
$a_3 = 12$

Therefore, the interest after 3 years is ₱12.00.

LESSON 3

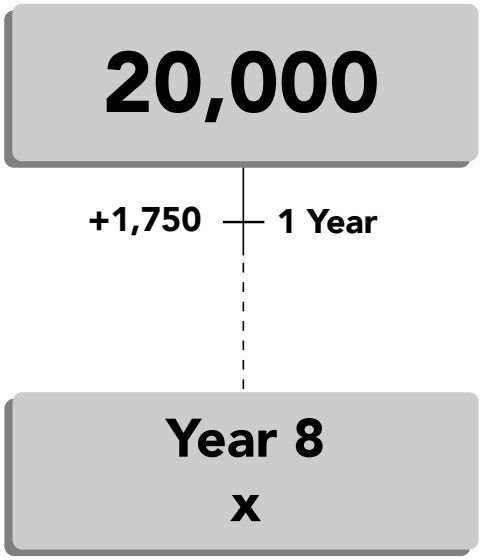
SHARPENING YOUR SKILLS

Write your answers on a separate sheet of paper.

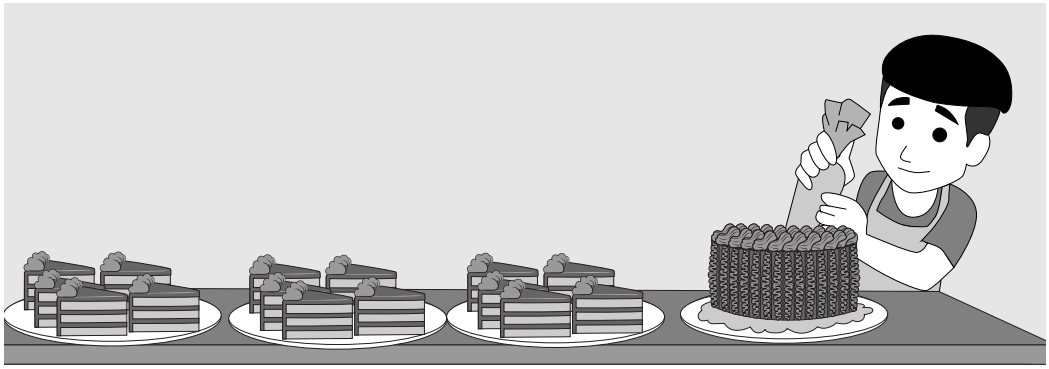
I. Answer the following word problems involving arithmetic sequence and series.

1. Find the sum of the first 100 positive odd integers.

2. Jenny deposited ₱20,000 on an investment that will give ₱1,750 for every year that the money stays in the account. How much money will she have in her account by the end of year 8?



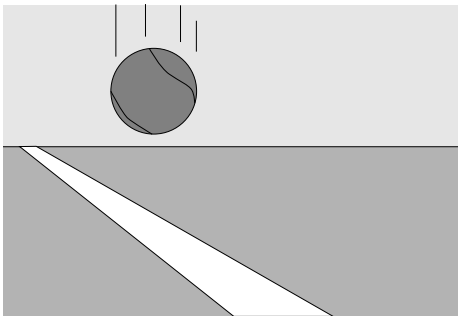
3. Christian decided to start baking chocolate cakes on his spare time during community lockdown because of COVID-19 for his extra income. On his first day, he has five (5) customers who want to avail his chocolate cakes. In continuous rate of orders, his first customer orders 15 chocolate cakes, his second customer orders 18 chocolate cakes, his third customer orders 21 chocolate cakes and so on. If one chocolate cake is being sold at ₱150.00, how much is his total sale for his first day?



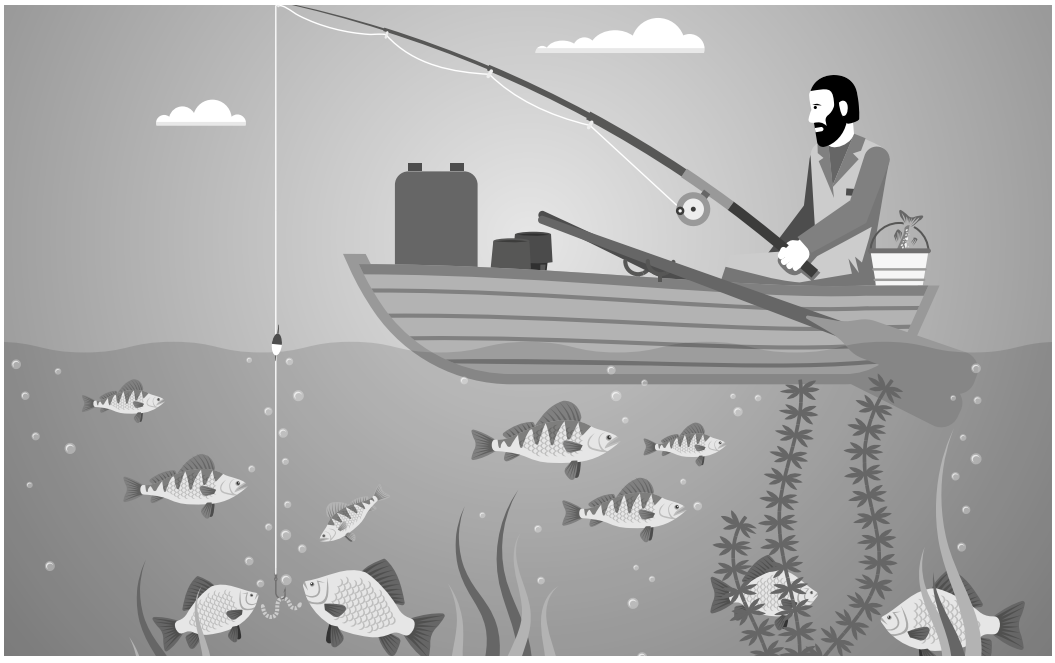
II. Answer the following word problems involving geometric sequence and series.

1. Sam has purchased a ₱30,000-car for his business. The car depreciates 30% every year. Depreciation means the value of an object goes down by a certain percentage each year (the value becomes 70% of the previous value). What will be the value of the car after the 5th year?

2. Suppose you drop a tennis ball from a height of 15 feet. After the ball hits the floor, it rebounds to 0.85 of its previous height. How high will the ball rebound after its third bounce? Round to the nearest tenth.



3. Hardy started a fishery where the fishes grow twice as big in population every month. If the initial population of his fingerlings was 100, what will be the population of the fishes in 10 months?

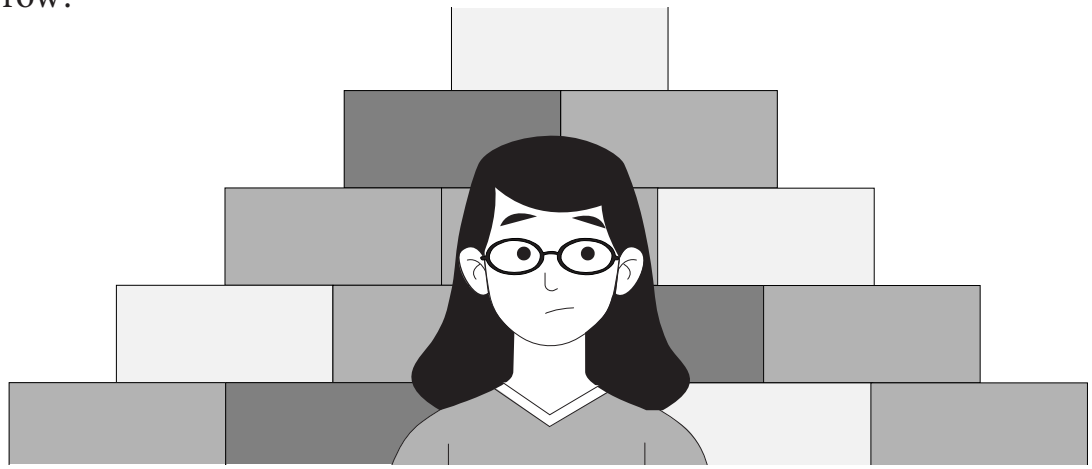


LESSON 3

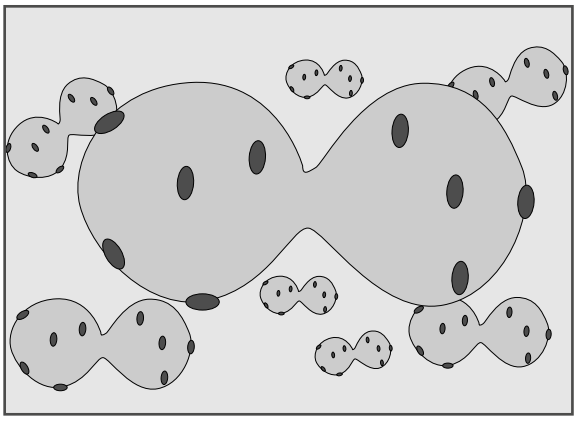
TREADING THE ROAD TO MASTERY

Directions: Indicate if the word problem involves arithmetic or geometric sequence and series. Then, answer the word problem. Write your answers on a separate sheet of paper.

1. Sonia has 55 hollow blocks. She decides to stack up all the hollow blocks so that each row has one less block than the row below it. She wants to end up with just 1 block on top. How many should she put in the bottom row?

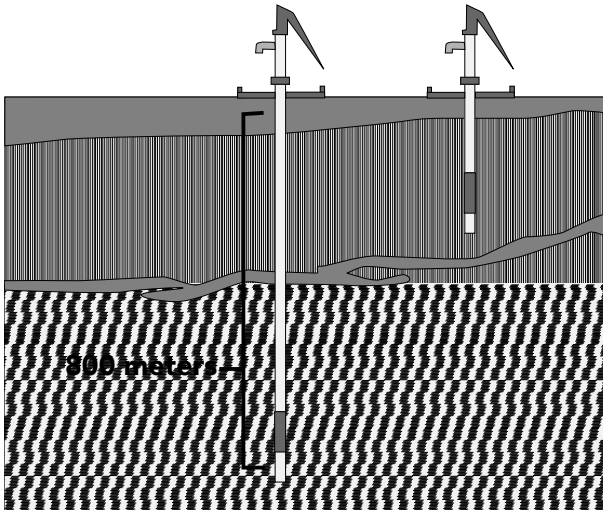


2. A virus reproduces by dividing into two, and after a certain growth period, it divides into two again. As the virus continues to reproduce, it will continue to divide in two. How many viruses will be in a system starting with a single virus after 10 divisions?



3. Money value depreciates by 0.10 (becomes 0.90 of the value) every year. What would be the value of a 10,000-peso deposit after 5 years?

4. A tube well is buried 800 meters deep. The 1st meter costs ₱250 and the cost per meter increases by ₱50 for every subsequent meter. Find the cost of drilling the 750th meter and the total cost incurred for the entire job.



MODULE 5

DON'T FORGET

- A sequence of numbers is a list of numbers that usually follow a pattern. Each number in a sequence is called a **term.**
- An **arithmetic sequence** is a sequence of numbers where every two consecutive terms differ by a constant called the **common difference** (d).
- To get the common difference, *subtract* the first term from the second term, *subtract* the second term from the third term, and so on ($a_2 - a_1$; $a_3 - a_2$; and so on).
- The n^{th} **term** of an arithmetic sequence can be computed using the formula:

$$a_n = a_1 + (n-1)d$$

where a_n = nth term
a_1 = first term
d = common difference
n = number of terms

- The sum of the terms of a sequence is called **series.**
- The sum of the terms of an arithmetic sequence is called an **arithmetic series.**
- The sum of the first n **terms** of an arithmetic sequence can be computed using the formula:

$$S_n = \frac{n}{2}(a_1 + a_n) \quad \text{or} \quad S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

where S_n = sum of the first n terms
a_1 = first term
a_n = last term
n = number of terms

- **Arithmetic means** refers to terms to be placed between any two non-consecutive terms in an arithmetic sequence.
- A **geometric sequence** is a set of numbers where each term after the first is found by multiplying the previous term by a constant called the **common ratio (r).**
- To get the common ratio, *divide* the second term by the first term, *divide* the third term by the second term, *divide* the fourth term by the third term, and so on.
- The **n^{th} term** of a geometric sequence can be computed using the formula:

$$a_n = a_1 r(n-1)$$

where $a_n = n^{\text{th}}$ term
a_1 = first term
r = common ratio
n = number of terms

- A **geometric series** is the sum of all the terms in a geometric sequence.

- The sum of the first ***n* terms** of a geometric sequence can be computed using the formula:

$$S_n = \frac{a_1 - a_1 r^n}{1 - r} = \frac{a_1 (1 - r^n)}{1 - r} = a_1 \left(\frac{1 - r^n}{1 - r} \right)$$

where S_n = sum of the first n terms
a_1 = first term
r = common ratio
n = number of terms

MODULE 5

EXPLORE MORE

If you wish to study further, here are some additional materials you can refer to:

"Arithmetic Sequences Introduction"
https://www.youtube.com/watch?v=GijrTfz0tIU

"Arithmetic Sequences"
https://www.transum.org/Maths/Exercise/Sequences/Arithmetic.asp

"Arithmetic Progression: Online Calculator"
https://planetcalc.com/177/

"Arithmetic Mean"
https://www.mathgoodies.com/lessons/vol8/mean

"Geometric Sequences Introduction"
https://www.youtube.com/watch?v=nwazO55Bkzo

MODULE 5
REACH THE TOP

Choose the letter of the correct answer by writing it on a separate sheet of paper.

1. What the missing term in the arithmetic sequence:
$\frac{1}{8}$, 2, ____, $\frac{23}{4}$, $\frac{61}{8}$,…

   a. $\frac{11}{8}$ **b.** $\frac{21}{8}$ **c.** $\frac{31}{8}$ **d.** $\frac{41}{8}$

2. Find the common difference in the given arithmetic sequence 9, 2, −5, −12, −19,…?

   a. −1 **b.** −3 **c.** −5 **d.** −7

3. What is the next two terms in the geometric sequence 4, −12, 36,… ?

   a. −108 and 324 **b.** 108 and −324 **c.** −72 and 216 **d.** 72 and −216

4. Find the common difference in the arithmetic sequence 3, $\frac{10}{3}$, $\frac{11}{3}$, 4,…

   a. $\frac{19}{3}$ **b.** $\frac{1}{3}$ **c.** $\frac{5}{2}$ **d.** 2

5. What is the sum of all the odd integers between 4 and 32?

   a. 251 **b.** 252 **c.** 253 **d.** 254

6. What is the 10th term in the given arithmetic sequence in item no. 2?

   a. −40 **b.** −47 **c.** −54 **d.** −61

7. Find the arithmetic mean between 7 and 127.

   a. 69 **b.** 67 **c.** 65 **d.** 63

8. Which of the following is the sum of all the multiples of 2 from 1 to 30 ?

   a. 210 **b.** 220 **c.** 230 **d.** 240

9. If three geometric means are inserted between 5 and 3125, find the third geometric mean.

a. 75 **b.** 125 **c.** 375 **d.** 625

10. Which term of the arithmetic sequence 2, 5, 8, 11,… is 38 ?

a. 14th term **b.** 13th term **c.** 12th term **d.** 11th term

11. Find the common ratio in the geometric sequence 3, 15, 75, 375, 1875,…

a. 3 **b.** 4 **c.** 5 **d.** 6

12. What is the 5th term of the geometric sequence 2, −12, 72,… ?

a. −2250 **b.** 2250 **c.** −2592 **d.** 2592

13. The first term of an arithmetic sequence is 2 while the 14th term is 93. Find the common difference of the sequence.

a. 7 **b.** 6 **c.** 5 **d.** 4

14. What is the 7th term of the geometric sequence 3, 9, 27 … ?

a. 2187 **b.** 1295 **c.** 2189 **d.** 1267

15. One day, Christian saw a photo on Instagram. At 1pm, he shared the photo link to 5 distinct people. Then at 2pm, each of his friends shared it to 5 distinct people. Then at 3pm, each of their friends shared it to another 5 distinct people. If this pattern kept going, how many distinct people would have received the photo link by 6pm?

a. 15,625 **b.** 3,125 **c.** 3,145 **d.** 965

ANSWER KEY

PRE-ASSESSMENT

PAGE 2

1. b
2. c
3. a
4. c
5. c
6. b
7. b
8. b
9. d
10. b
11. c
12. a
13. c
14. b
15. c

LESSON 1: SUMMING IT UP!

SHARPENING YOUR SKILLS

PAGE 19

ACTIVITY I

1. $a_8 = 16$
2. $a_{15} = -49$
3. $a_{33} = 267$
4. $a_{12} = -116$
5. $a_{50} = 454$

ACTIVITY II

1. $S_{11} = 462$
2. $S_9 = 153$
3. $S_{15} = -285$
4. $S_{16} = 40$
5. $S_5 = 135$

TREADING THE ROAD TO MASTERY

PAGE 20

1. 31, 46, 61
2. 70, 85, 100
3. 4, −2
4. −5, −2, 1, 4
5. 7, 11, 15, 19, 23

ANSWER KEY

LESSON 2: MULTIPLY IT CONTINUOUSLY!

SHARPENING YOUR SKILLS

PAGE 34

ACTIVITY I

1. -6
2. $\frac{1}{3}$
3. $\frac{1}{2}$
4. -1
5. $-\frac{1}{4}$

ACTIVITY II

1. **A.** $a_8 = 409{,}600$
   B. $a_{12} = 104{,}857{,}600$
2. **A.** $a_9 = 768$
   B. $a_{14} = -24{,}576$
3. **A.** $a_8 = \frac{1}{64}$
   B. $a_{11} = \frac{1}{512}$

TREADING THE ROAD TO MASTERY

PAGE 35

1. $r = -3$
2. $r = 2$
3. $S_{10} = 59{,}048$
4. $S_8 = \frac{255}{16}$

ANSWER KEY

LESSON 3: APPLYING SEQUENCES AND SERIES

SHARPENING YOUR SKILLS

PAGE 44

ACTIVITY I

1. $S_{100} = 10{,}000$
2. $a_8 = 32{,}350$
3. $S_5 = 105$
   Price = ₱15,750,000

ACTIVITY II

1. $a_5 = 7{,}203$
2. $a_3 = 10.84$ ft
3. $a_{10} = 51{,}200$

TREADING THE ROAD TO MASTERY

PAGE 46

1. Arithmetic, $a_{10} = 10$ blocks
2. Geometric, $a_{10} = 512$ virus
3. Geometric, $a_5 = 6{,}561$
4. Arithmetic, $S_{750} = 14{,}231{,}250$

REACH THE TOP

PAGE 52

1. c	**6.** c	**11.** c
2. d	**7.** b	**12.** c
3. a	**8.** d	**13.** a
4. b	**9.** d	**14.** a
5. b	**10.** c	**15.** a

GLOSSARY

Arithmetic Means	Terms to be placed between any two non-consecutive terms in an arithmetic sequence.
Arithmetic Sequence	A sequence of numbers where every two consecutive terms differ by a constant called the common difference (d).
Arithmetic Series	The sum of the terms of an arithmetic sequence.
Common Difference	The difference of two consecutive terms in an arithmetic sequence.
Common Ratio	The quotient of two consecutive terms in a geometric sequence.
Geometric Sequence	A sequence of numbers where each term after the first is found by multiplying the previous term by a constant.
Geometric Series	The sum of all the terms in a geometric sequence.
Sequence of Numbers	A list of numbers that follow a pattern.
Series	The sum of the terms of a sequence.
Term	Each number in a sequence.

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