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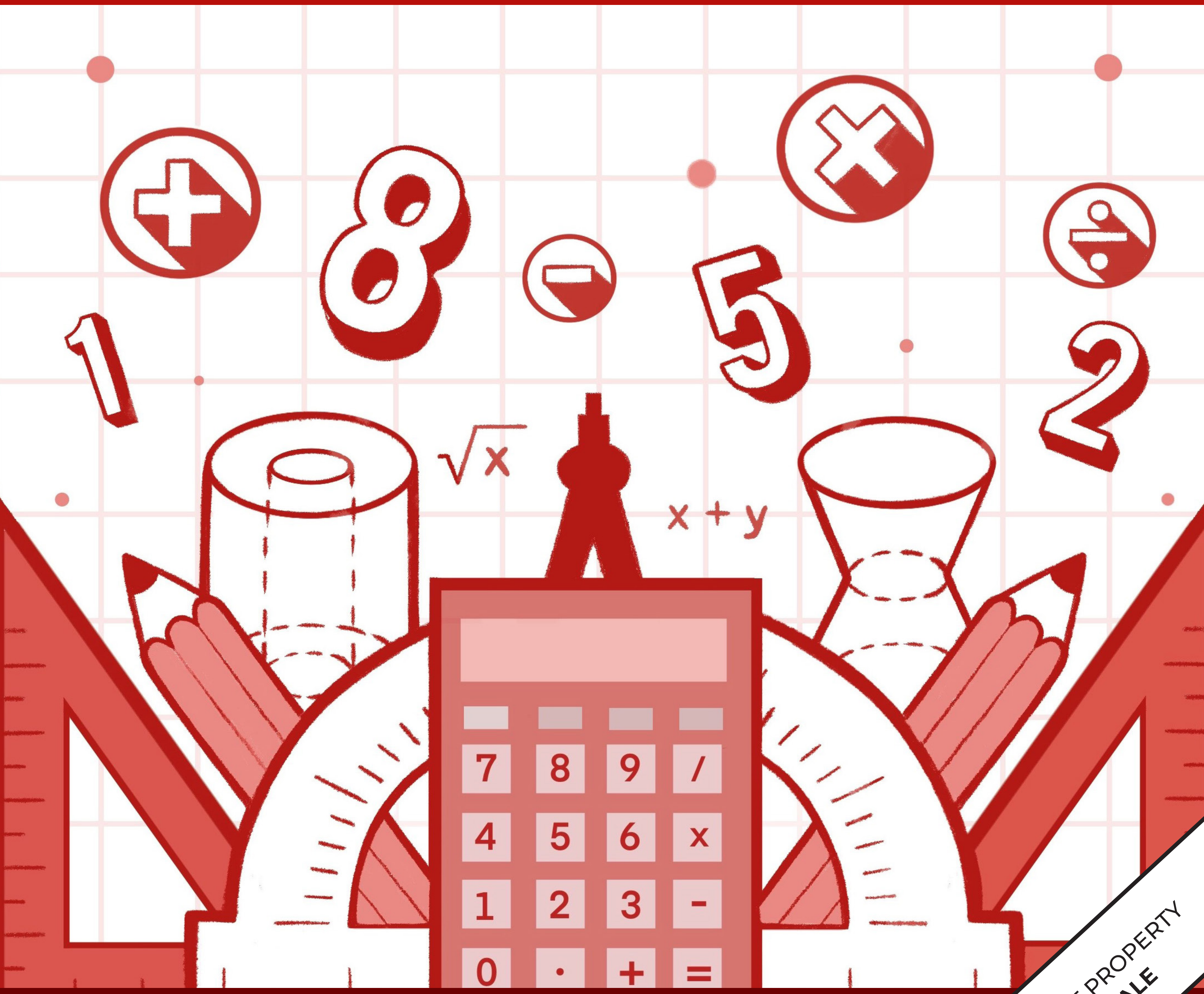
ALS
Alternative Learning System
LIFELONG LEARNING

LEARNING STRAND 3
MATHEMATICAL & PROBLEM-SOLVING SKILLS

MODULE 2: PLAYING WITH MISSING X's

ALS Accreditation and Equivalency Program: Junior High School



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PLAYING WITH MISSING X'S

MATHEMATICAL AND PROBLEM-SOLVING SKILLS
MODULE 2

ALS Accreditation and Equivalency Program: Junior High School
Learning Strand 3: Mathematical and Problem-Solving Skills
Module 2: Playing with Missing X's

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User's Guide

For the ALS Learner:

Welcome to this Module entitled Playing with Missing X's under Learning Strand 3 Mathematical and Problem-solving Skills of the ALS K to 12 Basic Education (BEC).

This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace and time. You will be enabled to process the contents of the learning resource while being an active learner.

This module has the following parts and corresponding icons:

Let's Get to Know	This will give you an idea of the skills or competencies you are expected to learn in the module.
Pre-assessment	This part includes an activity that aims to check what you already know about the lesson. If you get all the answers correct (100%), you may decide to skip this module.
Setting the Path	This section provides a brief discussion of the lesson. This aims to help you discover and understand new concepts and skills.
Trying This Out	This comprises activities for independent practice to solidify your understanding and skills of the topic. You may check the answers to the exercises using the Answer Key at the end of the module.
Understanding What You Did	This includes questions that process what you learned from the lesson.
Sharpening Your Skills	This section provides an activity that will help you transfer your new knowledge or skill in real-life situations or concerns.
Treading the Road to Mastery	This is a task which aims to evaluate your level of mastery in achieving the given learning competency.
Don't Forget	This part serves as a summary of the lessons in the module.
Explore More	In this portion, another activity will be given to you to enrich your knowledge or skill of the lesson learned. This also tends retention of learned concepts.
Reach the Top	This part will assess your level of mastery in achieving the learning competencies in each lesson in the module.
Answer Key	This contains answers to all activities in the module.
Glossary	This portion gives information about the meanings of the specialized words used in the module.

At the end of this module you will also find:

References	This is a list of all sources used in developing this module.

The following are some reminders in using this module:

1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
2. Don't forget to answer the Pre-assessment before moving on to the other activities included in the module.
3. Read the instruction carefully before doing each task.
4. Observe honesty and integrity in doing the tasks and checking your answers.
5. Finish the task at hand before proceeding to the next.
6. Return this module to your ALS Teacher/Instructional Manager/Learning Facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your ALS Teacher/Instructional Manager/Learning Facilitator. Always bear in mind that you are not alone.

We hope that through this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can do it!

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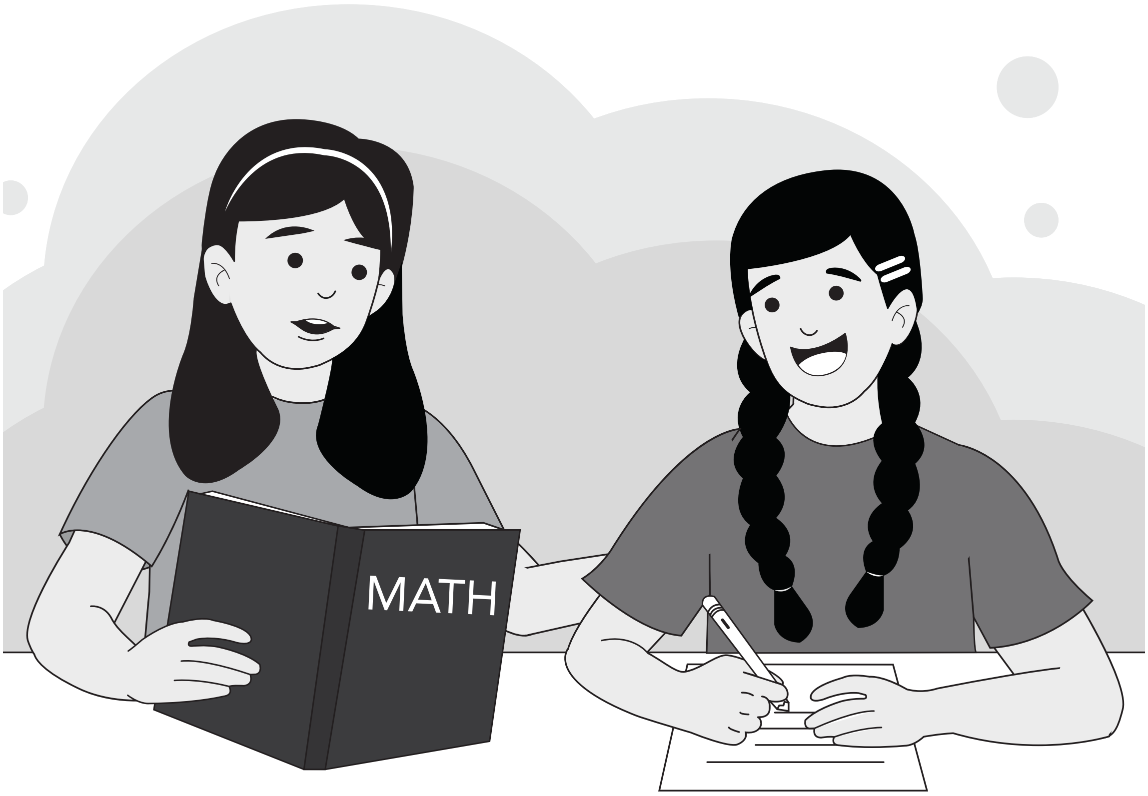
MODULE 2

LET'S GET TO KNOW



Your little sister told you that her math teacher said, "We use mathematical variables almost every day even without realizing it." Because she wanted to understand how these mathematical variables are used, she asked for your help to show her examples. Let's check out some examples here.

MODULE 2

PRE-ASSESSMENT

Instructions: Choose the letter of the correct answer by writing it on a separate sheet of paper.

1. Which of the following is a polynomial?

   a. $5x + 1$ **b.** $3x^{-2} + 1$ **c.** $2x - 1$ **d.** $4^{\sqrt{x}} - 10$

2. Which of the following is a binomial?

   a. $2x^2 + x$ **b.** $3x^2 + x + 1$ **c.** $x^3 + x - 2$ **d.** $4x^3$

3. How many terms does $4x^3 - 3x^2 + x + 1$ have?

   a. $2x^2 + x$ **b.** $3x^2 + x + 1$ **c.** $x^3 + x - 2$ **d.** $4x^3$

4. What do you call a polynomial with exactly three terms?

   a. trinomial **b.** binomial **c.** monomial **d.** multinomial

5. What do you call the highest sum of the exponents of the variables in a polynomial?

   a. degree **b.** term **c.** variable **d.** exponent

6. What is the degree of $x^2 y^3 + x^4 - 2$?

   a. 2 **b.** 3 **c.** 4 **d.** 5

7. What is the sum of $(7x^2 - 4x + 3) + (x^2 + 3x - 2)$?

   a. $8x^2 + 7x + 1$ **b.** $x^2 - x + 1$ **c.** $x^2 + 7x + 1$ **d.** $8x^2 - x + 1$

8. Find the difference of $(5x - 3) - (2x + 6)$.

a. $3x + 3$ b. $3x - 9$ c. $7x - 9$ d. $7x + 3$

9. What is the product of $x^2 (x + 3)$?

a. $x^3 - 3x^2$ b. $3x^2 - x$ c. $x^3 + 3x^2$ d. $x + 3x^2$

10. Multiply $(x - 3)$ by $(x + 2)$.

a. $x^2 + x + 6$ b. $x^2 - x + 5$ c. $x^2 - x - 6$ d. $x^2 + x - 6$

11. What is the quotient of $(10x - 12) \div 2$?

a. $20x - 24$ b. $20x + 24$ c. $5x - 6$ d. $5x + 6$

12. Divide $(x^2 + 7x + 12)$ by $(x + 3)$.

a. $x + 3$ b. $x + 4$ c. $x + 5$ d. $x + 6$

13. Determine the sum of $\frac{x+3}{2} + \frac{2x-3}{2}$.

a. $\frac{3x-6}{2}$ b. $\frac{x}{2}$ c. $\frac{x-6}{2}$ d. $\frac{3x}{2}$

14. What is the difference of $\frac{3x+14}{3} - \frac{x+10}{3}$?

a. $\frac{2x+24}{3}$ b. $\frac{4x+4}{3}$ c. $\frac{2x+4}{3}$ d. $\frac{4x+24}{3}$

15. What the result if $\frac{3x}{5}$ is added to $\frac{x-5}{2x}$?

a. $\frac{2x-5}{10x}$ b. $\frac{6x^2+5x-25}{10x}$ c. $\frac{6x^2-5x-25}{10x}$ d. $\frac{4x-5}{10x}$

LESSON 1

SETTING THE PATH

TO COMBINE OR NOT TO COMBINE

At the end of this lesson, you will be able to:

- define polynomials (LS3MP-PA-PSE-JHS-18);
- classify algebraic expressions which are polynomials according to degree and number of terms (LS3MP-PA-PSE-JHS-22); and
- add and subtract polynomials (LS3MP-PA-PSE-AE-23).

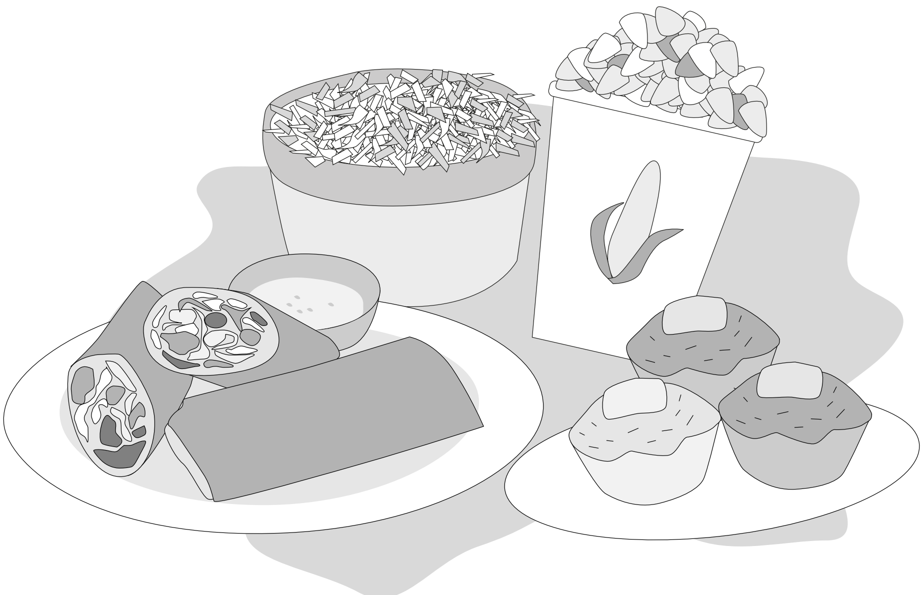
LESSON 1

TRYING THIS OUT

You and your friends Ivy, Fidel, and Charmaine work as canteen food crew members. One day, your manager told each of you to record the items that your branch has consumed in a month. You decided to use this as an example for your sister.

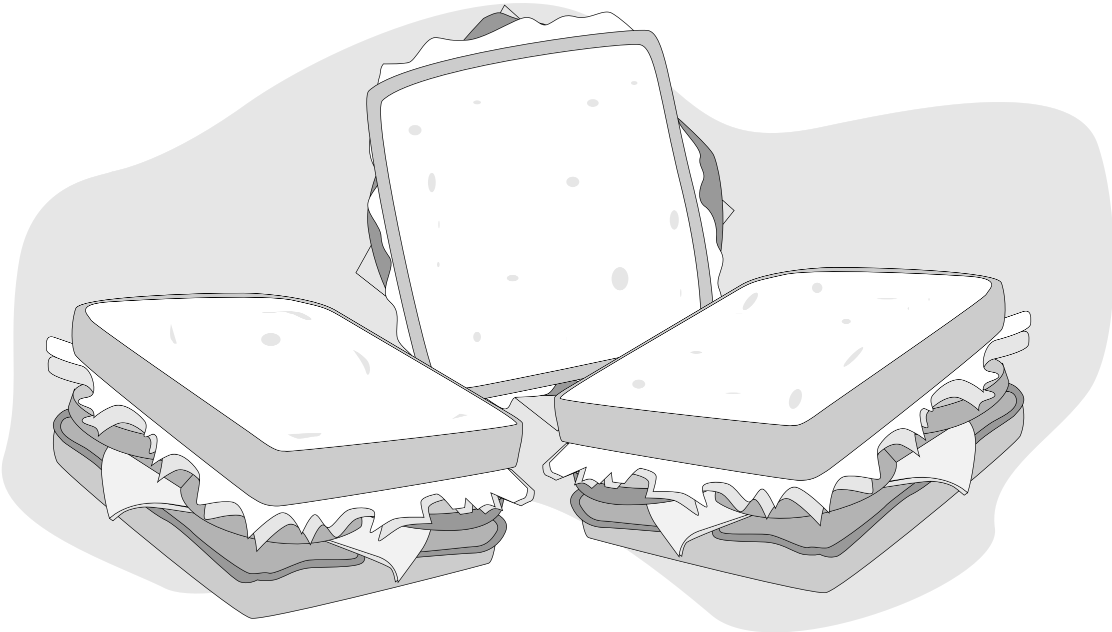
Organize the items and prepare a simple presentation to explain to your sister the process you used to determine the total of each item.

Me: Side Dishes ______________



Week 1	Week 2	Week 3	Week 4
9 Lumpiang Toge	**3** Coleslaw	**8** Puto	**4** Coleslaw
4 Coleslaw	**7** Corn	**11** Lumpiang Toge	**8** Puto
5 Corn	**11** Lumpiang Toge		**7** Lumpiang Toge
	2 Puto		

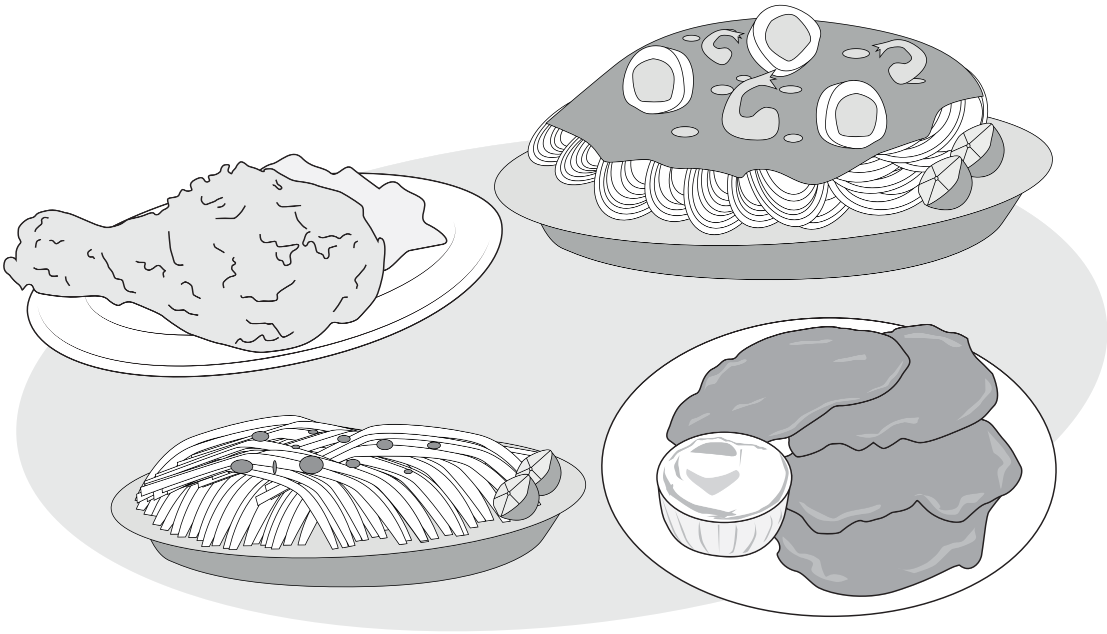
Ivy: Sandwiches ____________



Week 1	Week 2	Week 3	We4k 1
4 Egg Sandwich	**3** Tuna Sandwich	**4** Egg-Cheese Sandwich	**5** Egg Sandwich
5 Tuna Sandwich	**1** Egg-Cheese Sandwich	**2** Tuna Sandwich	**1** Tuna-Cheese Sandwich
2 Egg-Cheese Sandwich	**2** Tuna-Cheese Sandwich	**1** Egg Sandwich	
		2 Tuna-Cheese Sandwich	

MODULE 2

Fidel: Meals ______________



Week 1	Week 2	Week 3	Week 1
4 Fried Chicken	**1** Palabok	**1** Boiled Egg	**4** Pancit
2 Palabok	**3** Pancit	**6** Fried Chicken	**1** Tortang Isda
5 Pancit	**2** Boiled Egg	**2** Tortang Isda	**4** Boiled Egg
	7 Tortang Isda	**3** Palabok	**3** Fried Chicken
		2 Fried Chicken	**2** Palabok

Charmaine: Drinks ____________

Week 1	Week 2	Week 3	We4k 1
5 Gulaman	**7** Soya Milk	**6** Juice	**5** Water
3 Juice	**8** Gulaman	**11** Soya Milk	**6** Gulaman
4 Soya Milk	**3** Water		**2** Soya Milk
			13 Juice

LESSON 1

UNDERSTANDING WHAT YOU DID

To determine the number of each item on the list given to you, you must first **organize** the items accordingly.

	Lumpiang toge consumed in each week	Coleslaw consumed in each week	Corn consumed in each week	Puto consumed in each week
Week 1	9 Lumpiang Toge	4 Coleslaw	5 Corn	
Week 2	11 Lumpiang Toge	3 Coleslaw	7 Corn	2 Puto
Week 3	12 Lumpiang Toge			8 Puto
Week 4	7 Lumpiang Toge	4 Coleslaw		8 Puto
Total:	39 Lumpiang Toge	11 Coleslaw	12 Corn	18 Puto

Table 1.1 *Number of Side Dishes Consumed*

In mathematics, we can use variables to represent values which will help in simplifying calculations. A **variable** is a letter that represents a number whose value may vary or change.

We can replace each item in Table 1.1 with variables using different letters. We let

x = price of one lumpiang toge
y = price of one coleslaw
w = price of one corn
z = price of one puto

Week 1	$9x$	$+4y$	$+5w$		This represents the expense for Week 1
Week 2	$11x$	$+3y$	$+7w$	$+2z$	This represents the expense for Week 2
Week 3	$12x$			$+8z$	This represents the expense for Week 3
Week 4	$7x$	$+4y$		$+8z$	This represents the expense for Week 4
Total:	$39x$	$+11y$	$+12w$	$+18z$	This represents the expense for the month

Table 1.2 *The use of variables to represent the name of the sides*

We use the "+" symbol to indicate that we are adding amount to each side dish.

Each representation of expenses for each week in Table 1.2 using variables is called a **polynomial.**

$$\left.\begin{array}{l}\textbf{Week 1 expenses: } 9x + 4y + 5w \\ \textbf{Week 2 expenses: } 11x + 3y + 7w + 2z \\ \textbf{Week 3 expenses: } 12x + 8z \\ \textbf{Week 4 expenses : } 7x + 4y + 8z\end{array}\right\} \textbf{Polynomial}$$

For the sandwiches, the same items for each week must be identified to determine their total as shown below:

	Egg Sandwich consumed in each week	Tuna Sandwich consumed in each week	Tuna-Cheese Sandwich consumed in each week	Egg-Cheese Sandwich consumed in each week
Week 1	4 Egg Sandwiches	5 Tuna Sandwiches		2 Egg-Cheese Sandwiches
Week 2		3 Tuna Sandwiches	2 Tuna-Cheese sandwiches	1 Egg-Cheese Sandwich
Week 3	1 Egg Sandwich	2 Tuna Sandwiches	2 Tuna-Cheese Sandwiches	4 Egg-Cheese Sandwiches
Week 4	5 Egg Sandwiches		1 Tuna-Cheese Sandwich	
Total:	10 Egg Sandwiches	10 Tuna Sandwiches	5 Tuna-cheese Sandwiches	7 Egg-Cheese Sandwiches

Table 1.3 *Number of sandwiches consumed*

Let us again use variables to represent the different items.

Represent egg sandwich and egg-cheese sandwich using:

e = amount of each egg sandwich
e^2 = amount of each egg-cheese sandwich

Represent tuna sandwich and tuna-cheese sandwich using:

t = amount of each tuna sandwich
t^2 = amount of each tuna-cheese sandwich

Week 1	$4e$	$+5t$		$2e^2$	This represents the expense for Week 1
Week 2		$+3t$	$+2t^2$	$1e^2$	This represents the expense for Week 2
Week 3	e	$+2t$	$+2t^2$	$4e^2$	This represents the expense for Week 3
Week 4	$5e$		$+t^2$		This represents the expense for Week 4
Total:	$10e$	$+10t$	$+5t^2$	$7e^2$	This represents the expense for the month

Table 1.4 *The use of variables to represent the amount of each sandwich*

We use the "+" symbol to indicate that we are adding amount to each sandwich. Moreover, in a technical sense and to avoid ambiguity, when a variable has a coefficient of 1, it is not written.

From **Table 1.4**, we can see that the following are polynomials.

$$\left.\begin{array}{ll} \textbf{Week 1:} & 4e + 5t + 2e^2 \\ \textbf{Week 2:} & 3t + 2t^2 + e^2 \\ \textbf{Week 3:} & e + 2t + 2t^2 + 4e^2 \\ \textbf{Week 4:} & 5e + t^2 \end{array}\right\} \textbf{Polynomial}$$

Try to practice by transforming the tables used by Fidel and Charmaine into polynomial expressions! Remember, you may use any variables you like, even the initials of your name.

POLYNOMIALS

A **polynomial** is the sum or difference of algebraic expressions (consisting of constants and variables). Common variables used are x and y. Each algebraic expression in a polynomial is called a **term.** Some examples of terms are $2x$, $4y^3$, and -7.

Using $3x^5$ as an example, we identify the coefficient, variable, and exponent as shown:

Exponent

Coefficient → $3x^5$ ← Variable

If there is no exponent written in the variable, the exponent is understood to be 1. For instance,

$3x$ is the same as $3x^1$.

If a term does not have a variable, then it is called a **constant** term. For example, in the polynomial $2x^3 - 4x^2 + 3x + 10$, the constant term is 10.

Polynomials can be named based on the number of terms.

Examples:

$-2x$	has exactly one term, this is called a **monomial**
4	has exactly one term, a constant, and is also a **monomial**
$x + 1$	has exactly two terms, x and 1, this is called a **binomial**
$x^2 + 3x - 4$	has 3 terms x^2, $3x$, and -4, this is called a **trinomial**
$x3 - 3x^2 + 6x - 15$	has 4 terms, polynomials with 4 or more terms are generally referred to as **multinomials.**

TRY THIS QUICK: How many terms are there in the following polynomial expression?

$3x^3$	has	________	terms and is called a	________
$2x^3 + y^2 - 2x + y + 3$	has	________	terms and is called a	________
$x + 1$	has	________	terms and is called a	________
$x^2 + 3x - 4$	has	________	terms and is called a	________
-5	has	________	terms and is called a	________

The degree of a term is the sum of the exponents of the variables. To determine the degree of a term, we look at the exponents of the variables and add them.

Examples:

The degree of **$-2x^2y^2$** is **4** because the variable x has exponent 2 and variable y has exponent 2 as well. By adding them, we get $2 + 2 = 4$.

The degree of **5** is **0** because 5 can be written as $5x^0$ with exponent 0.

The degree of $\mathbf{-5z}$ is **1** because $-5z$ is the same as $-5z^1$ with exponent 1.

The degree of $\boldsymbol{x^2yz^3}$ is **6** because the variable x has exponent 2, the variable y has exponent 1, and the variable z has exponent 3. By adding them we get $2 + 1 + 3 = 6$.

The **degree of a polynomial** is the degree of the term with the highest degree.

To determine the degree of a polynomial, add the exponents for each term and choose the highest sum.

Examples:

Polynomial	Degree of the term	Degree of the polynomial (highest among the degrees in the 2nd column)
$2x + 3y^2$	$2x$ has a degree of 1 $3y^2$ has a degree of 2	2
$6x^4 - 3xy^2 + 2y^3$	$6x^4$ has a degree of 4 $-3xy^2$ has a degree of 3 $2y^3$ has a degree of 3	4
$-10x^4y^2 + x^2y^2 + 3xy^3$	$x^2\,y^2$ has a degree of 4 $3xy^3$ has a degree of 4 $-10x^4y^2$ has a degree of 6	6

As seen from the table, the degree of the polynomial is determined by the term with the highest degree.

TRY THIS QUICK: What is the degree of the polynomial?

$6x^4 - 9x^2 + x - 3$ has a degree of ________

$x^2 y^3 + x^3 y^2 + x4 - 2$ has a degree of ________

$6x - 5$ has a degree of ________

In writing a polynomial, the terms are placed in descending value of degree. Arranging the terms in descending order means that the exponents of the variable in each term decreases in value as you go from left to right.

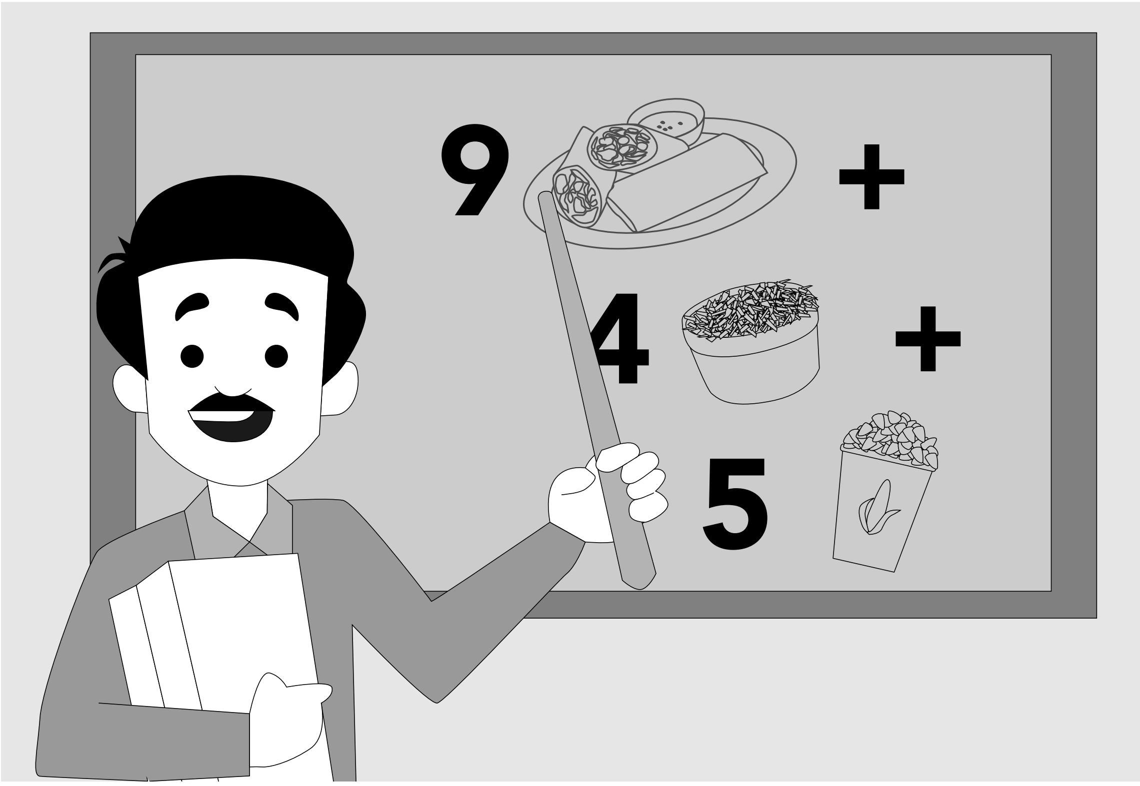
For instance, $x^3 - 3x^2 + 6x - 15$ is in descending order because the exponents go down in value: 3, 2, 1, 0. Other polynomials in descending order are shown below.

$$x^2 + 3x - 4 \qquad x^6 - 3x^5 + 2x^3 - 2x$$

Moreover, polynomials do not have **negative exponents** or **variables in the denominator** in any of its terms.

$\mathbf{2x + 3 - 4x^{-2}}$ Not a polynomial because of negative exponent	$\frac{2}{x+1}$ Not a polynomial because of a variable in the denominator

ADDING AND SUBTRACTING POLYNOMIALS



The total expenses for side dishes consumed for the month in Table 1.2 is a result of **adding** the polynomial for each week. Remember that the most important process you did to get the answer is to group the same items together. This is the same process we will follow in adding polynomials.

Terms that have the same exact variable and exponent are called **like terms** or **similar terms** while terms that do not have the same exact variable and exponent are called **unlike terms** or **dissimilar terms.**

In adding and subtracting polynomials, we can only add or subtract like/similar terms. Thus, before proceeding with the operation, make sure to sort or align similar terms.

Example 1: Find the sum of $\mathbf{(3x^2 + 4x + 1) + (2x^2 - 2x + 5)}$:

Make sure to line up the terms correctly. Here, $3x^2$ and $2x^2$ are aligned because they are similar terms; that is, they have the same variable x and exponent 2. Similarly, $4x$ and $-2x$ are aligned as well as 1 and 5 since they are like terms

$$\begin{array}{cccc} & 3x^2 & +4x & +1 \\ + & 2x^2 & -2x & +5 \\ \hline \end{array}$$

Then, apply the indicated operation.

$$\begin{array}{ccccc} & & 3x^2 & +4x & +1 \\ & + & 2x^2 & -2x & +5 \\ \hline \text{RESULT:} & & & & \end{array}$$

Add the coefficients and just copy the same variable and exponent, if there is any.

$$\begin{array}{ccccc} & & 3+2=5 & 4+(-2)=2 & 1+5=6 \\ & & 3x^2 & +4x & +1 \\ & + & 2x^2 & -2x & +5 \\ \hline \text{RESULT:} & & \mathbf{5x^2} & +\mathbf{2x} & +6 \end{array}$$

Therefore, $(3x^2 + 4x + 1) + (2x^2 - 2x + 5) = \mathbf{5x^2 + 2x + 6}$

Example 2: Find the difference of **(7*x* – 3)** – **(4*x* + 5).**

Sort out and line up the terms. Here, $7x$ and $4x$ are aligned because they are similar terms; that is, they have the same variable x and exponent 1. Similarly, −3 and 5 are aligned since they are both constants.

$$\begin{array}{rrr} & 7x & -\ 3 \\ - & 4x & +\ 5 \\ \hline \end{array}$$

Then, apply the indicated operation.

$$\begin{array}{rrr} & 7x & -\ 3 \\ - & 4x & +\ 5 \\ \hline \end{array}$$

RESULT:

Then, apply the indicated operation to the coefficients and copy the same variable.

$$\begin{array}{lrrr} & & 7-4=3 & (-3)-5=-8 \\ & & 7x & -\ 3 \\ & - & 4x & +\ 5 \\ \hline \text{RESULT:} & & 3x & -\ 8 \end{array}$$

Therefore, $(7x – 3) + (4x \qquad + 5) =$ $\mathbf{3x – 8x}$

LESSON 1

SHARPENING YOUR SKILLS

I. **Instructions:** From the box of algebraic expressions, write the like terms together. Do this activity on a separate sheet of paper.

$-3x$	$-4y^3$	$3y$	$-9w$	$17z$	$6y^2$
$4y$	$2x^3$	$-19z^2$	$-5x$	$13y^2$	$11x$
$6z^2$	$-3x^2$	$7x$	$-2z^3$	$27x^2$	$-2y^3$
$-2y^2$	$8z$	$13z^3$	$-12z$	$23z^2$	$10x^3$
$12w$	$-y$	$-15y$	$-6x^3$	$2y^3$	$21w$

1. x
2. y
3. z
4. w
5. x^2
6. y^2
7. z^2
8. x^3
9. y^3
10. z^3

II. **Instructions:** Classify the following polynomials as monomial, binomial, trinomial, or multinomial. Write your answers on a separate sheet of paper.

1. $5x^2$
2. 10
3. $y - 1$
4. $2x^2 - x + 3$
5. $13z + 10$
6. $x^3 + x^2 - x + 1$
7. y^2
8. -4
9. $2y^2 - 3y + 5$
10. $3y^3 + y^2$

III. Instructions: State the degree of each polynomial below. Do this activity on a separate sheet of paper. Item 1 serves as an example.

1. $x^2 - 2x + 1$ The degree is 2.
2. $x^3 + x^2 - 5$
3. $3x^2y - 2x^3y^5$
4. $z^3 + z^2 - 2z$
5. $15y + 84y^3 - 100 + 3y^2$
6. $3w^4 + w^3 - 2w^2 - w$
7. $25x^{10}y^2 + 3x^5y^5 - 15$
8. 13

IV. **Instructions:** Determine the sum or difference of each expression below. Do this activity on a separate sheet of paper. Item 1 serves as an example.

1. $(3x + 4) + (2x - 1) =$ **$5x + 3$**

$$\begin{array}{rcc} & 3 + 2 = 5 & 4 + (-1) = 3 \\ & 3x & +4 \\ + & 2x & -1 \\ \hline \text{RESULT:} & \multicolumn{2}{c}{5x + 3} \end{array}$$

2. $(-2x + 5) + (-x - 8)$
3. $(5x - 2) - (2x + 3)$
4. $(10x - 17) - (2 - 3x)$
5. $(4x - 7) - (- 5x + 2)$
6. $(2x^2 + 8x - 11) + (7x^2 - 5x + 10)$
7. $(x^2 - 4x - 6) - (x^2 - 5x + 2)$
8. $(6x^3 - 2x^2 + 5x) - (3x^3 - 9x^2 - 15x)$

LESSON 1

TREADING THE ROAD TO MASTERY

Instructions: Align similar terms in each expression and perform the indicated operation. Do this activity on a separate sheet of paper. Item 1 serves as an example.

1. $(x^3 - 2x + 1) + (3x^2 + 5x + 4) = \mathbf{x^3 + 3x^2 + 3x + 5}$

$$
\begin{array}{lccccc}
 & & 3+2=5 & 0+3=3 & -2+5=3 & 1+4=5 \\
 & & x^3 & & -2x & +1 \\
 & + & & +3x^2 & +5x & +4 \\
\hline
\text{RESULT:} & & x^3 & +3x^2 & +3x & +5
\end{array}
$$

2. $(2x^4 + 8x^2 - 5x + 3) - (3x^4 - 12x - 7)$

3. $(3x^7 - 8x^4 + 3x^3 - 2x) - (2x^5 + 9x^4 - 5x^2 + 3x)$

4. $(2x^3 - 4x^2 - 6x + 1) + (3x^2 - 7x + 5) - (x^3 - 2x - 6)$

5. $(6x^2 + 11x - 9) - (3x^2 - 12) + (-3x^3 - 2x^2 - 4x + 17)$

LESSON 2

SETTING THE PATH

LET'S DISTRIBUTE AND SHARE

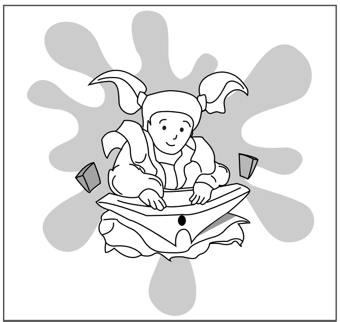
At the end of this lesson, you will be able to:

- perform operations on polynomials (LS3MP-PA-PSE-JHS-25)
- multiply and divide polynomials (LS3MP-PA-PSE-JHS-24); and
- factor polynomials using methods of factoring (LS3MP-PA-PSE-JHS-26).

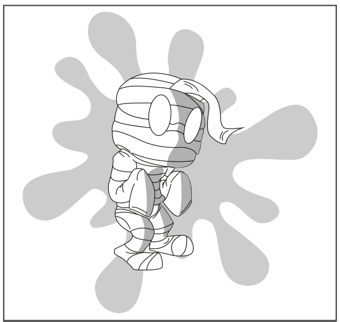
LESSON 2

TRYING THIS OUT

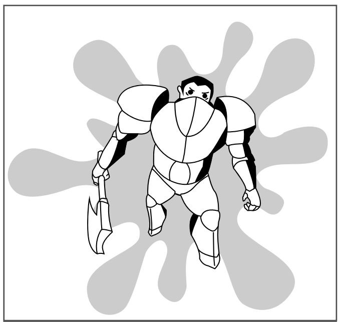
You want to use the game of playing hero cards to show your sister another application of mathematics. Let us look at some hero statistics. Note that SS stands for Super Skill.



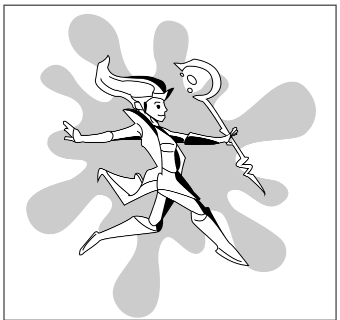
CHRISTINE	
HP	12
DEFENSE	5
ATTACK	8
MANA	9
SS DAMAGE: 25	



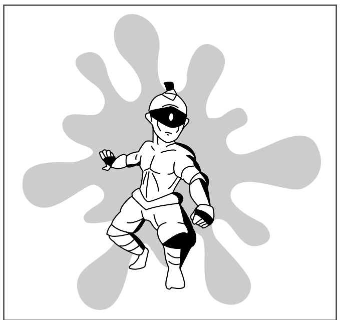
RUBEN	
HP	8
DEFENSE	9
ATTACK	15
MANA	13
SS DAMAGE: 30	



ANDRES	
HP	7
DEFENSE	11
ATTACK	17
MANA	12
SS DAMAGE: 27	



ROSE	
HP	10
DEFENSE	6
ATTACK	9
MANA	7
SS DAMAGE: 24	



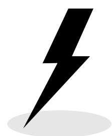
DANIEL	
HP	9
DEFENSE	9
ATTACK	13
MANA	11
SS DAMAGE: 35	

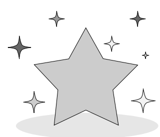
I. **Instructions:** Read the effect of each item to a specific Hero Stat. Calculate the resulting stat for all heroes when the item is applied. Do this activity on a separate sheet of paper.

MULTIPLIES MANA BY 3		
	CHRISTINE:	
	RUBEN:	
	ANDRES:	
	ROSE:	
MEDICINE KIT	DANIEL:	

MULTIPLIES HP BY 5		
	CHRISTINE:	
	RUBEN:	
	ANDRES:	
	ROSE:	
ENERGY BOOSTER	DANIEL:	

MULTIPLIES DEFENSE BY 6		
	CHRISTINE:	
	RUBEN:	
	ANDRES:	
	ROSE:	
SHIELD AND ARMOR	DANIEL:	

MULTIPLIES ATTACK BY 4		
	CHRISTINE:	
	RUBEN:	
	ANDRES:	
	ROSE:	
LIGHTNING BOLT	DANIEL:	

MULTIPLIES ATTACK BY 2		
	CHRISTINE:	
	RUBEN:	
	ANDRES:	
	ROSE:	
GOLD STAR ENABLER	DANIEL:	

II. How much damage will an enemy equally get from each hero when they use their super skill (use SS damage)? Explain your answer. Do this activity on a separate sheet of paper.

HEROES	NUMBER OF ENEMIES	SS DAMAGE EACH ENEMY RECEIVES
Example: Ruben Rose	3 enemies	Ruben – 10 Rose – 8
1. Ruben	6 enemies	
2. Andres	9 enemies	
3. Rose	3 enemies	
4. Daniel	7 enemies	
5. Christine	5 enemies	

UNDERSTANDING WHAT YOU DID

MULTIPLYING POLYNOMIALS

When you use items, the multiplier effect is distributed to the targeted stat for each hero in the group.

ITEM	EFFECT	HERO	MANA	RESULT
	MULTIPLIES MANA BY 3	CHRISTINE:	9	27
		RUBEN:	13	39
		ANDRES:	12	36
		ROSE:	7	21
		DANIEL:	11	33

ITEM	EFFECT	HERO	MANA	RESULT
	MULTIPLIES DEFENSE BY 6	CHRISTINE:	9	27
		RUBEN:	13	39
		ANDRES:	12	36
		ROSE:	7	21
		DANIEL:	11	33

The same process applies when we multiply polynomials. We must multiply each term of the polynomial to the other polynomial.

Example 1: Multiply $(x)(x + 5)$

Distribute (x) to each term in $(x + 5)$. Use lines to connect the terms you are multiplying.

$$(x)(x + 5) = x(x)$$

$$(x)(x + 5) = x(x) + \boldsymbol{x(5)}$$

Then, simplify the terms. Recall that when we multiply two terms, we multiply the coefficients of the variables and add the exponents. Hence, the product is

$$(x)(x + 5) = x^2 + 5x$$

Sometimes, using arrows to connect elements can become confusing if there are many elements involved. A better way to do this is by using the **box method.**

Example 2: Multiply $(x + 3)(x + 2)$

Write the first polynomial horizontally and the other polynomial vertically, as shown below. Use boxes to easily see which terms will be multiplied with each other.

	x	$+3$
x		
$+2$		

Pair a term from one column with the corresponding term from one row as indicated by the arrows shown below.

	x	$+3$
x	MULTIPLY	
$+2$		

Then, multiply the terms and write the answer on the space. Do this until all spaces are filled up. Again, when we multiply two terms, we multiply the coefficients of the variables and add the exponents.

	$\boldsymbol{x}$	**+3**
$\boldsymbol{x}$	x^2	$3x$
+2	$2x$	6

To get the final answer, add like terms (inside the box) diagonally, then add all the terms.

	$\boldsymbol{x}$	**+3**
$\boldsymbol{x}$	x^2	$3x$
+2	$2x$	6

$$x^2 \quad + \quad 5x \quad + \quad 6$$

Therefore, the product of $(x + 3)(x + 2)$ is $x^2 + 5x + 6$.

Example 3: Multiply $(x^2 + 3x - 4)(x - 3)$ using the box method.

Write the terms of the given polynomials accordingly.

	x^2	$+3x$	-4
x			
-3			

Then multiply each pair of terms.

	x^2	$+3x$	-4
x	x^3	$3x^2$	$-4x$
-3	$-3x^2$	$-9x$	$+12$

To get the result, add the like terms (inside the box) diagonally, then add all the terms

	x^2	$+3x$	-4
x	x^3	$3x^2$	$-4x$
-3	$-3x^2$	$-9x$	$+12$

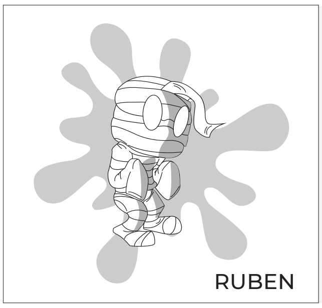
$x3 \quad + \quad 0x^2 \quad + \quad -13x \quad + \quad 12$

Therefore, $(x^2 + 3x - 4)(x - 3) = x^3 - 13x + 12$

DIVIDING POLYNOMIALS BY A MONOMIAL

To calculate the damage caused by the super skill of the heroes, we divide the damage caused by each hero by the number of enemies.

In number 1, Ruben encountered 6 enemies, so the SS damage must be divided equally to 6.

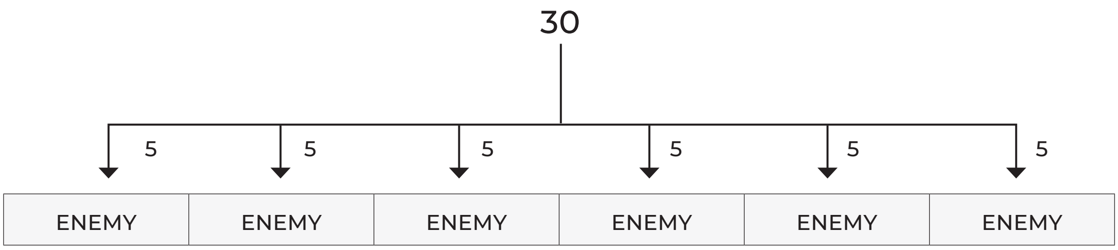


SS: 30

$$\frac{30}{6}$$

$\downarrow$

5 damage per enemy



The process of dividing the damage caused by each hero to the number of enemies is similar to the process done when we divide a monomial by a monomial.

If we are asked to simplify $\frac{(6x + 8)}{2}$, this means that we are dividing each term of **$(6x + 8)$** by **2.**

Look at the process below.

First, divide each term of the polynomial in the dividend by the divisor. In other words, both terms will share the same divisor.

$$\frac{(6x + 8)}{2} \longrightarrow \frac{6x}{2} + \frac{8}{2}$$

Then, simplify the quotient of each term.

$$\frac{6x}{2} = 3x \quad \text{and} \quad \frac{8}{2} = 4$$

Lastly, combine the terms with the appropriate operation.

$$\frac{(6x + 8)}{2} = 3x + 4$$

Let us have another example.

Example: Compute $(\mathbf{21x^4 - 15x^3 - 9x^2 + 30x}) \div \mathbf{3x}$.

Divide each term of the dividend by the divisor.

$$\frac{21x^4 - 15x^3 - 9x^2 + 30x}{3x} \longrightarrow \frac{21x^4}{3x} - \frac{15x^3}{3x} - \frac{9x^2}{3x} + \frac{30x}{3x}$$

Simplify the quotient of each term. Recall that when we divide two terms, we divide the coefficients of the variables and subtract the exponents.

$$\frac{21x^4}{3x} = 7x^3 \qquad \frac{-15x^2}{3x} = -5x^2 \qquad \frac{-9x^2}{3x} = -3x \qquad \frac{30x}{3x} = 10x^0 = 10\,(1) = 10$$

Combine the terms with the appropriate operation.

$$\frac{21x^4 - 15x^3 - 9x^2 + 30x}{3x} = 7x^3 - 5x^2 - 3x + 10$$

We can also determine the quotient of the previous example by using the **common monomial factor technique.**

Using the same example, $(21x^4 - 15x^3 - 9x^2 + 30x) \div 3x$, look at the process below:

First, find the greatest common factor (GCF) of the coefficients and variables (with exponents) in the dividend.

The coefficients are 21, −15, −9, and 30. Regardless of the sign, the GCF of these numbers is 3.

The variables (with exponents) are x^4, x^3, x^2, and x. The GCF is x.

Combine the GCF of coefficients and variables to get the **common monomial factor**.

The common monomial factor is $3x$.

Factor out the common monomial factor from the dividend. That is, we divide each term of the dividend by the common monomial factor. Observe that it is the first step in the previous example.

$$21x^4 - 15x^3 - 9x^2 + 30x = 3x\,(7x^3 - 3x^2 - 3x + 10)$$

Lastly, divide the simplified factors by the divisor. Here, we cancel out $3x$ because it appears in both numerator and denominator.

$$\frac{21x^4 - 15x^3 - 9x^2 + 30x}{3x} = \frac{3x(7x^3 - 3x^2 - 3x + 10)}{3x} = 7x^3 - 3x^2 - 3x + 10$$

Let us have another example showing the common monomial factor technique.

Example: Divide $\mathbf{14x^3 + 28x^2}$ **by** $\mathbf{7x}$**.**

Get the common monomial factor.

GCF of coefficients 14 and 28 is 7.
GCF of variables (with exponents) x^3 and $x^2$is x^2.
The common monomial factor is $7x^2$

Factor out the common monomial factor from the dividend.

$$14x^3 + 28x^2 = 7x^2(2x + 4)$$

Divide the simplified factors by the divisor.

$$\frac{14x^3 + 28x^2}{7x} = \frac{\overset{\boldsymbol{x}}{\cancel{7x^2}}(2x + 4)}{\cancel{7x}} = x\,(2x + 4) = 2x^2 + 4x$$

DIVIDING POLYNOMIALS BY ANOTHER POLYNOMIAL

We use the **long division** method in dividing a polynomial by another polynomial. It is clearly illustrated in the examples below.

Example 1: Divide **($-$ 4x^3 + 3x - 15)** by **(2x + 3)**.

Write the polynomial in descending order (from highest degree to lowest degree). Start the long division process in this form:	$x+5\overline{)x^2+9x+20}$
Divide the first term of the dividend to the first term of the divisor. In this example, divide x^2 by x.	$\begin{array}{r} x \\ x+5\overline{)x^2+9x+20} \end{array}$
Multiply (or distribute) the answer obtained in the previous step by the divisor. In this example, multiply x and x+5.	$\begin{array}{r} x \\ x+5\overline{)x^2+9x+20} \\ x^2+5x \quad\quad \end{array}$
Subtract the product then bring down the next term.	$\begin{array}{r} x \\ x+5\overline{)x^2+9x+20} \\ \underline{x^2+5x \quad\quad} \\ 4x+20 \end{array}$

Divide the first term of the result from the previous step to the first term of the divisor. In this example, divide $4x$ by x.	$\begin{array}{r} x+4 \\ x+5\,\overline{)\,x^2+9x+20} \\ \underline{x^2+5x} \\ 4x+20 \end{array}$
Multiply (or distribute) the answer obtained in the previous step to the divisor. In this example, multiply 4 and $x + 5$.	$\begin{array}{r} x+4 \\ x+5\,\overline{)\,x^2+9x+20} \\ \underline{x^2+5x} \\ 4x+20 \\ 4x+20 \end{array}$
Subtract the product. Observe that there are no terms left to bring down. The difference is the remainder. In this example, the remainder is 0.	$\begin{array}{r} x+4 \\ x+5\,\overline{)\,x^2+9x+20} \\ \underline{x^2+5x} \\ 4x+20 \\ \underline{4x+20} \\ 0 \end{array}$

Thus, the quotient when $(x^2 + 9x + 20)$ is divided by $(x + 5)$ is $(x + 4)$.

In the next example, we will learn how to divide polynomials that have missing terms. In this case, we must write a placeholder for a term using a zero coefficient.

Example 2: Divide **($-4x^3 + 3x - 15$)** by **($2x + 3$).**

Start the long division process in this form. Since the term x^2 is missing, use $0x^2$ as a placeholder.	$2x + 3\overline{)-4x^3 + 0x^2 + 3x - 15}$
Divide the first term of the dividend by the first term of the divisor. In this example, divide $-4x^3$ by $2x$.	$\begin{array}{r} -2x^2 \\ 2x + 3\overline{)-4x^3 + 0x^2 + 3x - 15} \end{array}$
Multiply (or distribute) the answer obtained in the previous step to the divisor. In this example, multiply $-2x^2$ and $2x + 3$. Subtract the product then bring down the next term.	$\begin{array}{r} -2x^2 \\ 2x + 3\overline{)-4x^3 + 0x^2 + 3x - 15} \\ \underline{-4x^3 + 6x^2 } \\ 6x^2 + 3x \end{array}$
Divide the first term of the result from the previous step by the first term of the divisor. In this example, divide $6x^2$ by $2x$.	$\begin{array}{r} -2x^2 + 3x \\ 2x + 3\overline{)-4x^3 + 0x^2 + 3x - 15} \\ \underline{-4x^3 + 6x^2 } \\ 6x^2 + 3x \end{array}$
Multiply (or distribute) the answer obtained in the previous step to the divisor. In this example, multiply $3x$ and $2x + 3$. Subtract the product then bring down the next term.	$\begin{array}{r} -2x^2 + 3x \\ 2x + 3\overline{)-4x^3 + 0x^2 + 3x - 15} \\ \underline{-4x^3 + 6x^2 } \\ 6x^2 + 3x \\ \underline{6x^2 + 9x } \\ -6x - 15 \end{array}$

Divide the first term of the result from the previous step by the first term of the divisor. In this example, divide $-6x$ by $2x$.	$\begin{array}{r} -2x^2+3x-3 \\ 2x+3\overline{)-4x^3+0x^2+3x-15} \\ \underline{-4x^3+6x^2} \\ 6x^2+3x \\ \underline{6x^2+9x} \\ -6x-15 \end{array}$
Multiply (or distribute) the answer obtained in the previous step to the divisor. In this example, multiply $3x$ and $2x + 3$.	$\begin{array}{r} -2x^2+3x-3 \\ 2x+3\overline{)-4x^3+0x^2+3x-15} \\ \underline{-4x^3+6x^2} \\ 6x^2+3x \\ \underline{6x^2+9x} \\ -6x-15 \\ -6x-9 \end{array}$
Subtract the product. Observe that there are no terms left to bring down. The difference is the remainder. In this example, the remainder is -6.	$\begin{array}{r} -2x^2+3x-3 \\ 2x+3\overline{)-4x^3+0x^2+3x-15} \\ \underline{-4x^3+6x^2} \\ 6x^2+3x \\ \underline{6x^2+9x} \\ -6x-15 \\ \underline{-6x-9} \\ -6 \end{array}$

Thus, the quotient will be written as $-2x^2 + 3x - 3 - \frac{6}{2x+3}$.

LESSON 2

SHARPENING YOUR SKILLS

I. Instructions: Find the product of the polynomials by distribution. Do this activity on a separate sheet of paper.

1. $x\,(x^2 - 2x)$
2. $-2x\,(3x^2 - 5x + 7)$
3. $4y\,(2y^3 + 5y - 3)$
4. $(x + 2)(x^2 - 7)$
5. $(z + 1)(3z^2 + 4z - 9)$

II. Instructions: Divide the polynomials by the given monomial. Do this activity on a separate sheet of paper.

1. $\dfrac{4x^2 - 12x}{4}$

2. $\dfrac{-15x^4 - 21x^2 + 18x^2}{-3}$

3. $\dfrac{-5x^8 - 25x^4 + 10x^3}{-5x^2}$

4. $\dfrac{27x^6 + 36x^5 - 45x^3 + 18x}{9x}$

5. $\dfrac{12x^3 + 6x^2 - 24x}{6x}$

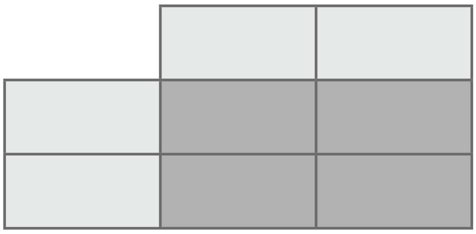
LESSON 2

TREADING THE ROAD TO MASTERY

I. **Instructions:** Determine the product of the polynomials using the box method. Write your answer on a separate sheet of paper.

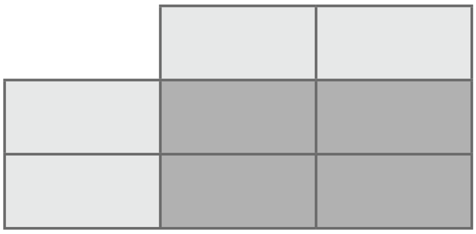
1. $(x^2 + 1)\ (x - 5)$

Final Answer:



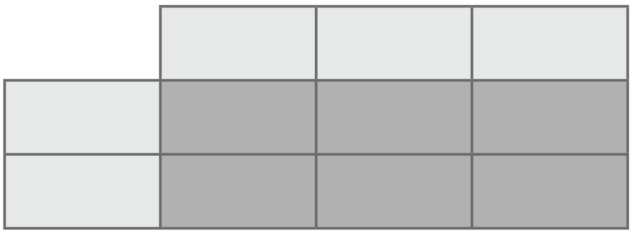
2. $(3y - 2)\ (2y + 7)$

Final Answer:



3. $(x^2 + 4x + 9)\ (-6x^3 - 2x)$

Final Answer:



4. $(2x+11)(-3x^2 - 2x - 6)$ Final Answer:

5. $(5x^3+x^2 - 4x + 12)(x+3)$ Final Answer:

II. Use long division to determine the quotient of the polynomials. Do this activity on a separate sheet of paper.

1. $(x^2 + 2 + 3x) \div (x + 1)$
2. $(x^2 + 4x + 3) \div (x + 3)$
3. $(2x^2 + 3x - 6) \div (x - 2)$
4. $(1 - 4x^2 + x^3) \div (x - 2)$
5. $(-2x^3 + 3x - 10) \div (x + 3)$

LESSON 3

SETTING THE PATH

FRACTIONS OF YOUR X

At the end of this lesson, you will be able to:

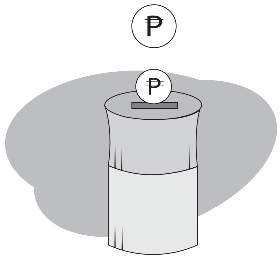
- illustrate rational algebraic expression (LS3MP-PA-PSE-JHS-27);
- perform addition of rational algebraic expressions (both with the same and different denominators) (LS3MP-PA-PSE-JHS-34);
- perform subtraction of rational algebraic expressions (both with the same and different denominators) (LS3MP-PA-PSE-JHS-35); and
- represent real-life situations using rational algebraic expressions (LS3MP-PA-PSE-JHS-36).
  - amount of work done per unit
  - speed per unit

LESSON 3

TRYING THIS OUT

You went with your sister around the barangay to show her more applications of mathematics in every day life. Now that she is more familiar with variables and polynomials, help her represent each scenario with a mathematical expression using variables for the missing values.

a. Your savings in a coin bank is increased by 10 and you shared it with your sister equally.



Let x = ______________________________

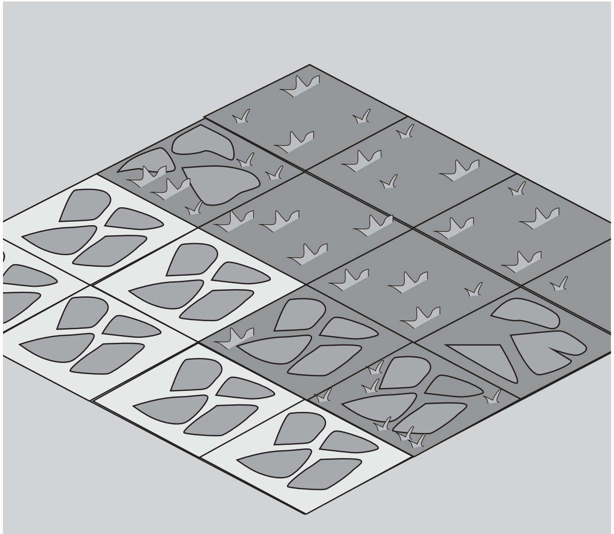
Mathematical Expression: ____________________

b. Five vendors shared the remaining fishes from the supplier after being subtracted with 7 pieces.

Let x = ______________________________

Mathematical Expression: ____________________

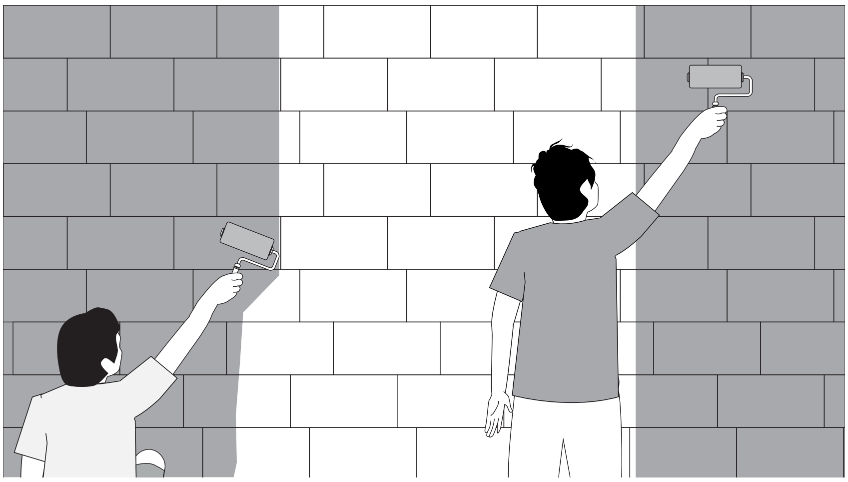
c. Divide the area of a square land to the 4 farmers.



Let x = ____________________

Mathematical Expression: ____________________

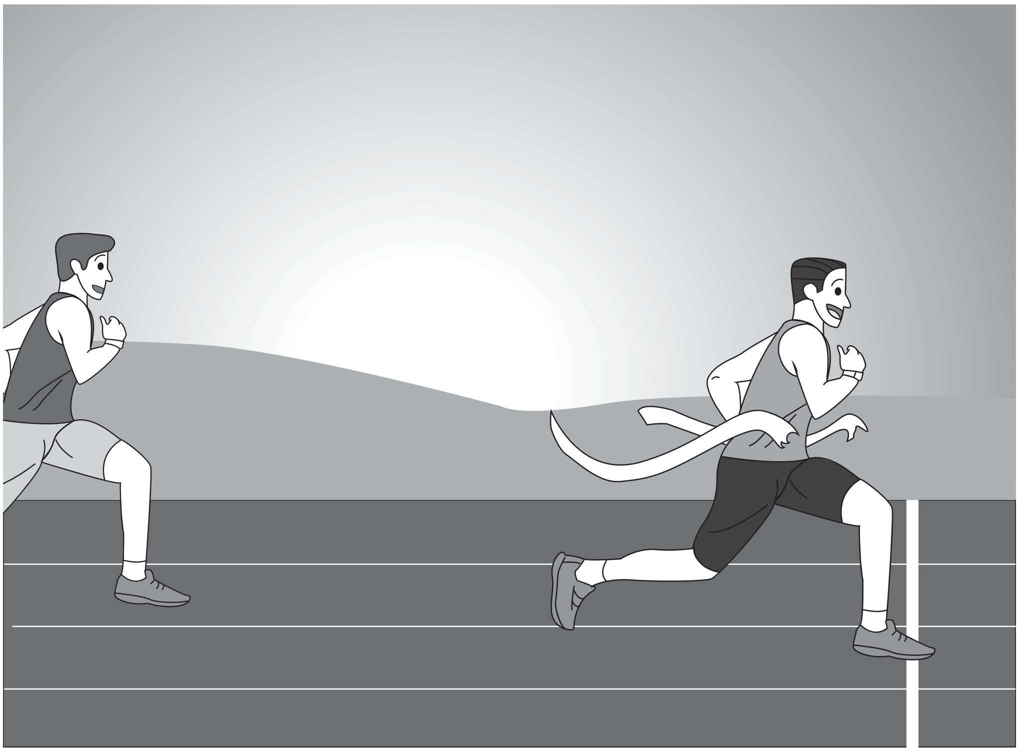
d. Pipoy finished painting a fifth of the wall, while Tikboy painted a third of the same wall. Combine the work they have finished together.



Let x = ____________________

Mathematical Expression: ____________________

e. Coach Rivo computed the difference of the speed of two runners. The first runner ran 1 meter more than the other. The first ran for 2 minutes while the second ran for 5 minutes. $\text{speed} = \dfrac{\text{distance}}{\text{time}}$



Let x = ____

Mathematical Expression: ________________________________

LESSON 3

UNDERSTANDING WHAT YOU DID

Translating each situation in the activity using variables, results in the following:

a. If x = amount of savings in the coin bank, then we have $\frac{x + 10}{2}$.

b. If x = number of fishes, then we have $\frac{x - 7}{5}$.

c. If x = length of one side of a square, then its area is x^2. Thus, we have $\frac{x^2}{4}$.

All of these expressions are called **rational expressions.** These expressions are simply fractions whose numerator and denominator are made up of polynomials.

These **numerators** are all polynomials.

$$\frac{x + 10}{2} \qquad \frac{x - 7}{5} \qquad \frac{x^2}{4}$$

These **denominators** are all polynomials.

ADDITION AND SUBTRACTION OF RATIONAL EXPRESSIONS

Rational expressions act the same way as fractions. The same process for operations on fractions apply to rational expressions.

Recall: For similar fractions (fractions with same denominators),

Addition: $\frac{1}{5} + \frac{3}{5} = \frac{1+3}{5} = \frac{3}{5}$

Subtraction: $\frac{6}{7} - \frac{5}{7} = \frac{6-5}{7} = \frac{1}{7}$

In adding or subtracting rational expressions, we add or subtract the polynomials in the numerators and copy the same denominator.

Examples:

$$\frac{x+2}{x} + \frac{3x-5}{x} = \frac{x+2+3x-5}{x} = \frac{x+3x+2+(-5)}{x} = \frac{4x-3}{x}$$

$$\frac{5x+6}{x-2} - \frac{2x+1}{x-2} = \frac{(5x+6)-(2x+1)}{x-2} = \frac{5x-2x+6-1}{x-2} = \frac{2x+1}{x-2}$$

Recall: For dissimilar fractions (fractions with different denominators), we use the least common denominator (LCD).

Addition:

$\frac{2}{3}$ and $\frac{3}{4}$ → LCD of 3 & 4 is 12. $\frac{2}{3} + \frac{3}{4} = \frac{4(2) + 3(3)}{12} = \frac{8 + 9}{12} = \frac{17}{12}$

Subtraction:

$\frac{3}{2}$ and $\frac{3}{5}$ → LCD of 2 & 5 is 10. $\frac{3}{2} - \frac{3}{5} = \frac{5(3) - 2(3)}{10} = \frac{15 - 6}{10} = \frac{9}{10}$

In items d and e of the previous activity, we set up some rational expressions with operations:

d. Pipoy finished painting a fifth of the wall, while Tikboy painted a third of the same wall.

Combine the work they have finished together.

Let x = amount of wall to be painted. Since Pipoy finished painting a fifth of the wall and Tikboy finished a third of the same wall,

Pipoy's work done	Tikboy's work done
$\frac{1}{5}x$ or $\frac{x}{5}$	$\frac{1}{3}x$ or $\frac{x}{3}$

To compute the work they have finished together, we add Pipoy's work and Tikboy's work. We have:

$$\frac{x}{5} + \frac{x}{3}$$

These rational expressions have different denominators. To add them, we get the LCD of 5 and 3 which is 15. Thus,

$$\frac{x}{5} + \frac{x}{3} = \frac{3(x) + 5\,(x)}{15} = \frac{8x}{15} = \frac{8}{15}\,x$$

This means that together, Pipoy and Tikboy have already painted $\frac{8x}{15}$.

e. Coach Rivo computed the difference in speed of two runners. First runner ran one meter more than the other. First runner ran for 2 minutes, while the other ran for 5 minutes.

let = the distance the second runner ran

$x + 1$ = the distance the first runner ran

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

speed of the first runner $= \frac{x+1}{2}$

speed of the second runner $= \frac{x}{5}$

Computing the difference between their speed, we have:

$$\frac{x+1}{2} - \frac{x}{5}$$

These rational expressions have different denominators. To subtract them, we get the LCD of 2 and 5 which is 10. Therefore,

$$\frac{x+1}{2} - \frac{x}{5} = \frac{5(x+1) - 2(x)}{10} = \frac{5x + 5 - 2x}{10} = \frac{3x+5}{10}$$

This means that the difference in their speeds is represented by

$$\frac{3x+5}{10}$$

Let us have another example.

Add $\frac{5}{2} + \frac{2x-6}{3x}$

Since the denominators are different, we first get the LCD by multiplying the denominators 2 and $3x$. This will give us $2(3x) = 6x$.

Next, we transform the given into similar rational expressions by dividing the LCD to the denominator then multiplying its product to the numerator, as shown below.

$$\frac{5}{2} + \frac{2x-6}{3x} = \frac{3x(5)}{6x} + \frac{2(2x-6)}{6x}$$

Since the fractions are now similar, we proceed to the rule for adding similar rational expressions. Therefore, as shown below.

$$\frac{5}{2} + \frac{2x-6}{3x} = \frac{3x(5)}{6x} + \frac{2\,(2x-6)}{6x} = \frac{15x + 4x - 12}{6x} = \frac{19x - 12}{6x}$$

LESSON 3
SHARPENING YOUR SKILLS

I. Instructions: Add or subtract the given rational expressions. Write your answers on a separate sheet of paper.

1. $\frac{x+1}{2x} + \frac{5x}{2x}$

2. $\frac{3x+17}{x-2} + \frac{5x+10}{x-2}$

3. $\frac{2x^3+3x}{5x+8} - \frac{5x^2-4x}{5x+8}$

4. $\frac{4y^2+2}{3y-4} - \frac{5y^2-2}{3y-4}$

5. $\frac{7x^2-3x+1}{5x+7x} - \frac{3x^2-8}{5x^2+7x}$

II. Instructions: Apply the indicated operation to the given rational expressions. Write your answers on a separate sheet of paper.

1. $\frac{2}{3} + \frac{x+1}{7x}$

2. $\frac{8-3x}{5} - \frac{9}{2}$

3. $\frac{2}{3x+4} + \frac{6}{2x}$

4. $\frac{2x}{x-2} + \frac{5x^2-2}{6x-1}$

5. $\frac{x^2-3x-1}{2x} + \frac{6x+3}{5}$

LESSON 3
TREADING THE ROAD TO MASTERY

Instructions: Solve the following word problems using rational expressions. Write your answers on a separate sheet of paper. Item 1 serves as an example.

1. Mang Abdul gave his daughter a quarter of his square farm with unknown length of one side while his son received a fifth of the same land. Find the difference between the area of the lands he gave to his children.

Solution:
Let x be the unknown length of one side of the square farm. Then the area of the square farm is x^2.
Since Mang Abdul gave his daughter a quarter of his square farm, his daughter received one-fourth of the area of the land, which is $\frac{x^2}{4}$. Moreover, his son received a fifth of the area of the land which is $\frac{x^2}{5}$.
Since we are looking for the difference between the areas of the land, we subtract the two areas. We have $\frac{x^2}{4} - \frac{x^2}{5}$
Using the LCD of 4 and 5 which is 20, we get $\frac{x^2}{4} - \frac{x^2}{5} = \frac{5(x^2) - 4\,(x^2)}{20} = \frac{5x^2 - 4\,x^2}{20} = \frac{x^2}{20}$
Thus, the difference between the lands Mang Abdul gave his children is $\frac{x^2}{20}$

2. Totoy and Nene earned the same amount of money. If Nene only kept half of the original amount, while Totoy's money decreased by five and he only kept one-third of what remained, how much of the original amount do they have in total?

3. Apo Lilia was able to finish weaving $\frac{2}{5}$ of the area of a fabric while her daughter, Manang Banak, helped her finish $\frac{1}{4}$ of the same fabric. How much have they finished weaving so far?

4. Pedro and Jose are runners. Pedro ran for 3 minutes while Jose ran for 5 minutes. If Jose ran 4 more kilometers than Pedro, what is the difference between their speeds? $\text{speed} = \frac{\text{distance}}{\text{time}}$

MODULE 2

DON'T FORGET

- A **variable** is a letter that may represent a number whose value may vary or change. A **constant** is a value that cannot change.

- A **polynomial** is the sum or difference of algebraic expressions (consisting of constants and variables).

- Each algebraic expression in a polynomial is called a **term.**

- The **degree of a polynomial** is the degree of the term with the highest degree.

- Polynomials can be classified based on the number of terms.

  a. A **monomial** is a polynomial that has exactly one term.
  b. A **binomial** is a polynomial that has exactly two terms.
  c. A **trinomial** is a polynomial that has exactly three terms.
  d. A **multinomial** is a general name for a polynomial that has four or more terms.

- When adding or subtracting polynomials, sort out and combine like terms. Then, perform the indicated operation of addition or subtraction. Remember to only add or subtract the coefficient (number) and copy the variable. For example,

$$(2x^2 - x + 1) + (2x^2 + 3x - 4) = 4x^2 + 2x - 3$$
$$(7x - 2) - (3x - 6) = 4x + 4$$

- When multiplying polynomials, distribute each term in the first multiplier to each term in the second multiplier. Then, combine like terms, if there are any. For instance,

$$(x + 1)(x - 2) = x^2 - 2x + x - 2 = x^2 - x - 2$$

- Box method can also be used when multiplying polynomials with several terms to see the distribution clearly. Write each term in the first polynomial above the first row and write each term of the second polynomial on the left side of the first column. For example,

	x^2	$+3x$	$+1$
x^3	x^5	$+3x^4$	$+x^3$
$+5x^2$	$+5x^4$	$+15x^3$	$+5x^2$
$-3x$	$-3x^3$	$-9x^2$	$-3x$
-1	$-x^2$	$-3x$	-1

$$x^5 \quad + \quad 8x^4 \;+\; 13x^3 \;+\; (-5x^2) \;+\; (-6x) \;+\; (-1)$$

Therefore,
$(x^2 + 3x + 1)(x^3 + 5x^2 - 3x - 1) = x^5 + 8x^4 + 13x^3 - 5x^2 - 6x - 1$

- When dividing polynomial by a monomial, divide each term of the dividend by the divisor. Do not forget to write all the resulting answers together. For instance,

$$\frac{6x^3+12x^2+3x}{3x} \longrightarrow \frac{\overset{2}{\cancel{6}}x^3}{\cancel{3}x}+\frac{\overset{4}{\cancel{12}}x^2}{\cancel{3}x}+\frac{\overset{1}{\cancel{3}}x^3}{\cancel{3}x}=2x^2+4x+1$$

  Cancellation process is used in this solution . The exponent of the variable x in the denominator is subtracted from the exponent of the variable x in the denominator.

- When dividing polynomial by another polynomial, long division is often used. For example,

$$\begin{array}{r|l} & x^2+4x+3 \\ \hline x+3 & x^3-x^2+9x+9 \\ & x^3+3x^2 \\ \hline & -4x^2-9x \\ & -4x^2-9x \\ \hline & 3x+9 \\ & 3x+9 \\ \hline & 0 \end{array}$$

- **Rational Expression**s are fractions whose either numerators or denominators are polynomials. These expressions act the same way as fractions. The denominator of a rational expression should not be zero (0).

- When adding or subtracting rational expressions with the same denominators (similar), combine the numerators and copy the denominators.

- When adding or subtracting rational expressions with different denominators (dissimilar), find the least common denominator (LCD), then transform the expressions into similar fractions. Proceed with addition or subtraction.

MODULE 2

EXPLORE MORE

If you wish to study further, here are some additional materials you can refer to:

"Multiplying Polynomials by Binomials"
https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:poly-arithmetic/x2ec2f6f830c9fb89:bi-by-poly/v/more-multiplying-polynomials

"How Do You Graph a Set on a Number Line?"
https://www.youtube.com/watch?v=FHOE4EvYCm8

"Adding & subtracting rational expressions: like denominators"
https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:rational/x2ec2f6f830c9fb89:rational-add-sub-intro/v/adding-and-subtracting-rational-expressions-with-like-denominators

"Adding rational expressions: unlike denominators"
https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:rational/x2ec2f6f830c9fb89:rational-add-sub-intro/v/adding-rational-expression-w-unlike-denominators

"Subtracting rational expressions: unlike denominators"
https://www.khanacademy.org/math/algebra2/x2ec2f6f830c9fb89:rational/x2ec2f6f830c9fb89:rational-add-sub-intro/v/subtracting-rational-expressions-w-unlike-denominators

MODULE 2

REACH THE TOP

Instructions: Choose the letter of the correct answer by writing it on a separate sheet of paper.

1. Which of the following are like terms?

   a. $3ab$, $2bc$, $-ac$
   b. $3abc$, $-2abc$
   c. $2x^2y$, $-xy^2$
   d. $4xy$, $-xy$, $3x$

2. Which is not a binomial?

   a. $x^3 - 5x^2$ b. $3x^2 + 1$ c. $2x^3 - x + 1$ d. $3x - 4$

3. Which of the following polynomial is written in descending order?

   a. $6x^3 + 2x^2$ b. $2x + 3x^2$ c. $7x - 5x^3 + x^5$ d. $1 - x + 5x^3$

4. What is the sum of $(2x + 3x^2) + (2x - y)$?

   a. $3x^2 + 4x$ b. $3x^2 - y$ c. $4x - y$ d. $3x^2 + 4x - y$

5. Find the difference of $(x^2 - 3x + 7) - (3x^2 + 6x + 4)$.

   a. $-2x^2 - 3x + 11$
   b. $-2x^2 - 9x + 11$
   c. $-2x^2 - 9x + 3$
   d. $-2x^2 - 3x + 3$

6. Perform the indicated operations: $(4x - 3y) + (2x - 7y) - (3x + 5y)$

   a. $9x - 5y$ b. $9x - 15y$ c. $3x - 15y$ d. $3x + 15y$

7. What is the product of $(3x - 2y)(2x - y)$?

   a. $5x^2 - 7xy - y^2$
   b. $5x^2 + 3xy - y^2$
   c. $6x^2 - 7xy - y^2$
   d. $6x^2 - 7xy + 2y^2$

8. What is the quotient of $(8x^4 + 12x^2 - 4x) \div 4x$?

a. $2x^2 - 3xy + 1$
b. $2x^3 + 3x$
c. $2x^2 + 3x$
d. $2x^3 + 3x$-1

9. Divide $(x^3 - x^2 - 3x - 1)$ by $(x + 1)$.

a. $x^2 - 2x - 1$
b. $x^2 + 2x - 1$
c. $x^2 - 2x + 1$
d. $x^2 + 2x + 1$

10. Determine the quotient: $(x^3-1)\div(\text{x}-1)$

a. $x^2 - x + 1$
b. $x^2 + x + 1$
c. $x^2 - 2x + 1$
d. $x^2 + 2x + 1$

11. Perform the indicated operation: $\frac{3}{x+1} + \frac{4x-1}{x+1}$

a. $\frac{2}{x+1}$ **b.** $\frac{4x}{x+1}$ **c.** $\frac{4x+2}{x+1}$ **d.** $\frac{4x-2}{x+1}$

12. What is the sum of $\frac{2}{x} + \frac{3}{y}$?

a. $\frac{4x+2y}{xy}$ **b.** $\frac{3x-2y}{xy}$ **c.** $\frac{3x+2y}{x+y}$ **d.** $\frac{3x-2y}{x+y}$

13. Jun can clean the garage in 5 hours while Pip can do the same job for 3 hours. How much work will they finish in an hour if they work together.

a. $\frac{2}{15}$ **b.** $\frac{8}{15}$ **c.** $\frac{7}{15}$ **d.** $\frac{6}{15}$

14. Find the quotient when the sum of $(x^2 - 3x + 4) + (2x^2 + 5)$ is divided by 3

a. $2x^2 + 3$ **b.** $x^2 - x + 3$ **c.** $x^2 + x + 3$ **d.** $2x^2 - 3$

15. May can paint the room in 6 hours. Kate can do the same job in 4 hours and Angel in 3 hours. How much work will they finish if they work together for an hour?

a. $\frac{3}{4}$ **b.** $\frac{1}{3}$ **c.** $\frac{1}{2}$ **d.** $\frac{1}{4}$

ANSWER KEY

PRE-ASSESSMENT PAGE 1

1. a
2. a
3. b
4. a
5. a
6. d
7. d
8. b
9. c
10. c
11. c
12. b
13. d
14. c
15. b

LESSON I: TO COMBINE OR NOT TO COMBINE

SHARPENING YOUR SKILLS
ACTIVITY I PAGE 20

1. $-3x$, $7x$, $-5x$, $11x$
2. $4y$, $-y$, $3y$, $-15y$
3. $8z$, $-12z$, $17z$
4. $12w$, $-9w$, $21w$
5. $-3x^2$, $27x^2$
6. $-2y^2$, $13y^2$, $6y^2$
7. $6z^2$, $-19z^2$, $23z^2$
8. $2x^3$, $-6x^3$, $10x^3$
9. $-4y^3$, $2y^3$, $-2y^3$
10. $13z^3$, $-2z^3$

ANSWER KEY

ACTIVITY II PAGE 20

1. monomial
2. monomial
3. binomial
4. trinomial
5. binomial
6. multinomial
7. monomial
8. monomial
9. trinomial
10. binomial

ACTIVITY III PAGE 21

1. 2
2. 3
3. 8
4. 3
5. 3
6. 4
7. 12
8. 0

ACTIVITY IV PAGE 24

1. $5x + 3$
2. $-3x - 3$
3. $3x - 5$
4. $13x - 19$
5. $9x - 9$
6. $9x^2 + 3x - 1$
7. $x - 8$
8. $3x^3 + 7x^2 + 20x$

TREADING THE ROAD TO MASTERY PAGE 22

1. $x3 + 3x^2 + 3x + 5$
2. $-x^4 + 8x^2 + 7x + 10$
3. $3x^7 - 2x^5 - 17x^4 + 3x^3 + 5x^2 - 5x$
4. $x^3 - x^2 - 11x + 12$
5. $-3x^3 + x^2 + 7x + 20$

ANSWER KEY

LESSON 2: LET'S DISTRIBUTE AND SHARE

SHARPENING YOUR SKILLS

ACTIVITY I PAGE 41

1. $x^3 - 2x^2$
2. $-6x^3 + 10x^2 - 14x$
3. $8y^4 + 20y^2 - 12y$
4. $x^3 + 2x^2 - 7x - 14$
5. $3z^3 + 7z^2 - 5z - 9$

ACTIVITY II PAGE 41

1. $x^2 - 3x$
2. $5x^4 + 7x^3 - 6x^2$
3. $x^6 - 5x^2 + 2x$
4. $3x^5 + 4x^4 - 5x^2 + 2$
5. $2x^2 - x - 4$

TREADING THE ROAD TO MASTERY

ACTIVITY I PAGE 42

1. $x^3 - 5x^2 + x - 5$
2. $6y^2 + 17y - 14$
3. $-6x^5 - 24x^4 - 56x^3 - 8x^2 - 18x$
4. $-6x^3 - 37x^2 - 34x - 66$
5. $5x^4 + 16x^3 - x^2 + 36$

ACTIVITY II PAGE 43

1. $x + 2$
2. $x + 1$
3. $2x + 7 + \frac{8}{x-2}$
4. $x^2 - 2x - 4 - \frac{7}{x-2}$
5. $-2x^2 + 6x - 15 + \frac{35}{x+3}$

ANSWER KEY

LESSON 3: FRACTIONS OF YOUR X

SHARPENING YOUR SKILLS

ACTIVITY I

PAGE 53

1. $\dfrac{x + 6}{2x}$
2. $\dfrac{5x + 27}{x - 2}$
3. $\dfrac{2x^3 - 5x^2 + 7x}{5x + 8}$
4. $\dfrac{-y^2 + 4}{3y - 4}$
5. $\dfrac{10x^2 - 3x + 7}{5x^2 + 7x}$

ACTIVITY II

PAGE 53

1. $\dfrac{17x + 13}{21x}$
2. $\dfrac{-6x - 29}{10}$
3. $\dfrac{22x + 24}{6x^2 + 8x}$
4. $\dfrac{-5x^3 + 22x^2 - 2x - 4}{6x^2 - 13x + 2}$
5. $\dfrac{17x^2 - 9x - 5}{10x}$

ANSWER KEY

TREADING THE ROAD TO MASTERY PAGE 54

1. $\frac{x^2}{4} - \frac{x^2}{5} = \frac{x^2}{20}$

   Thus, the difference between the lands Mang Abdul gave his children is $\frac{x^2}{20}$

2. $\frac{x}{2} + \frac{x-5}{3} = \frac{5x-10}{6}$

   Totoy and Nene have $\frac{5x-10}{6}$ of the original amount.

3. $\frac{x^2}{4}x - \frac{1}{4}x = \frac{13}{20}x$

   Apo Lilia and Manang Banak have finished $\frac{13}{20}$ of the fabric.

4. $\frac{x}{3} - \frac{x+4}{5} = \frac{2x-12}{15}$

   The difference between Pedro and Jose's speeds is $\frac{2x-12}{15}$.

REACH THE TOP PAGE 61

1. b
2. c
3. a
4. d
5. c
6. c
7. d
8. d
9. a
10. b
11. c
12. a
13. b
14. b
15. a

GLOSSARY

Binomial	a polynomial that has exactly two terms
Degree of a polynomial	degree if the term with the highest degree
Like terms (similar terms)	terms that have exactly the same variable, exponents, and constants
Long division	method of dividing a polynomial by another polynomial
Monomial	a polynomial that has exactly one term
Multinomial	a polynomial that has four or more terms
Polynomial	sum or difference of algebraic expressions (consisting of constants and variables)
Rational Expressions	fractions whose either numerators or denominators are polynomials

GLOSSARY

Term	each algebraic expression in a polynomial
Trinomial	a polynomial that has exactly three terms
Unlike terms (dissimilar terms)	terms that do not have the exact variables, exponents, or constants
Variable	a letter that may represent a number whose value may vary or change

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