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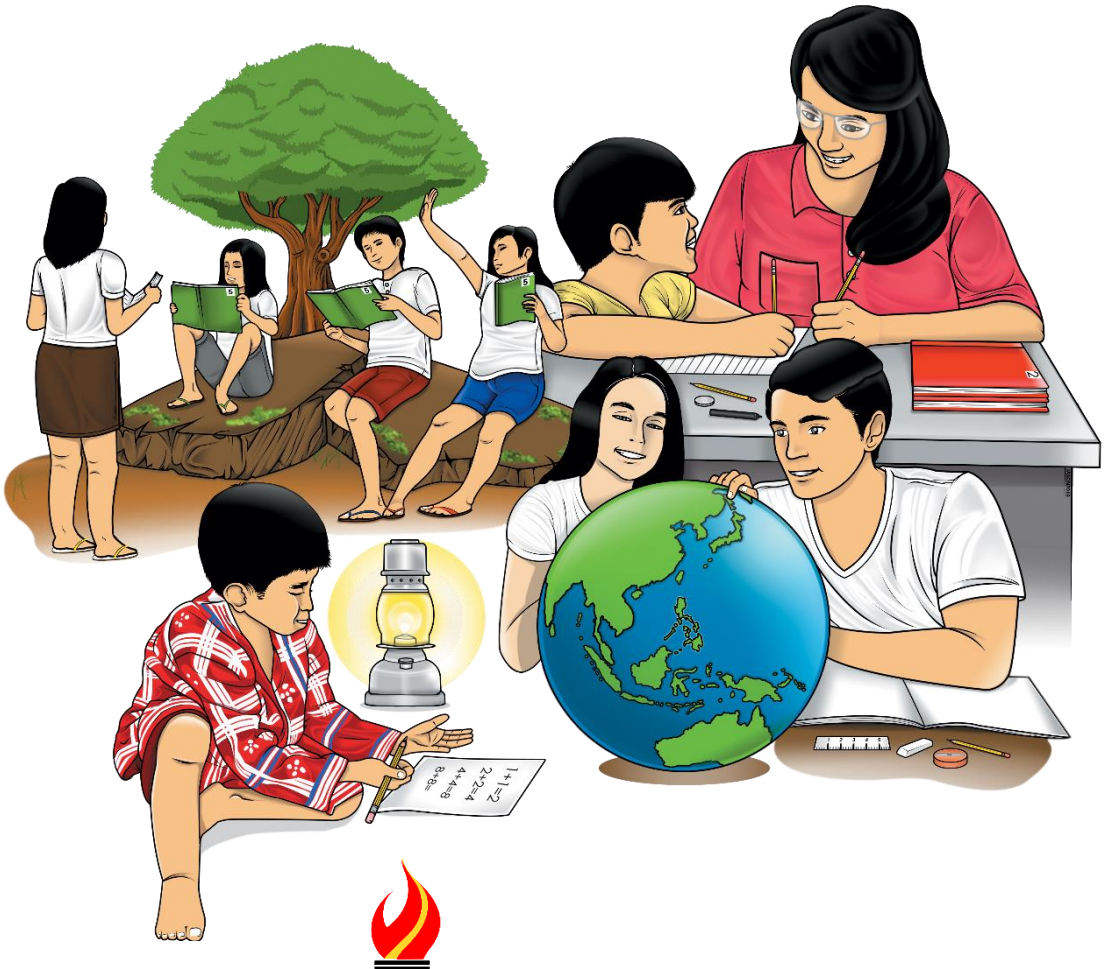
DEPARTMENT OF EDUCATION

Mathematics

Quarter 4-Module 9
Solve Problems Involving Oblique Triangles



ALTERNATIVE DELIVERY MODE
ADM

Mathematics – Grade 9
Alternative Delivery Mode
Quarter 4 – Module 9: Solve Problems Involving Oblique Triangles
First Edition, 2020

Published by the Department of Education
Secretary: Leonor Magtolis Briones
Undersecretary: Diosdado M. San Antonio

Development Team of the Module

Writers:	Marissa S. Penaflor	
Editor:	Edwin M. Yap Analynn M. Argel	
Validators:	Remylinda T. Soriano Angelita Z. Modesto George B. Borromeo	
Illustrator:	Marissa S. Penaflor	
Layout Artist:	Marissa S. Penaflor	
Management :	Malcolm S. Garma	Genia V. Santos
	Dennis M. Mendoza	Maria Magdalena M. Lim
	Aida H. Rondilla	Lucky S. Carpio

Printed in the Philippines by _____

Department of Education - National Capital Region (NCR)

Office Address: Misamis St., Brgy. Bago Bantay, Quezon City
Telefax: (632) 8926-2213 /8929-4330 /8920-1490 and 8929-4348
E-mail Address: ncr@deped.gov.ph

9

Mathematics

Quarter 4-Module 9
Solve Problems Involving Oblique Triangles

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Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

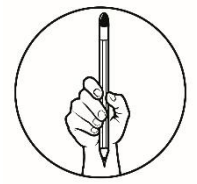
If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

What I Need to Know

The learners will be able to:

- Solve word problems involving oblique triangles .**M9GE-IVj-48.1**

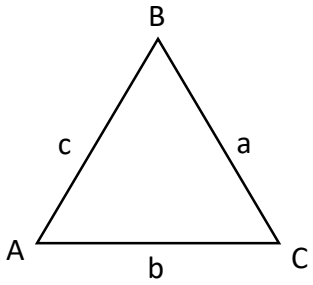


What I Know

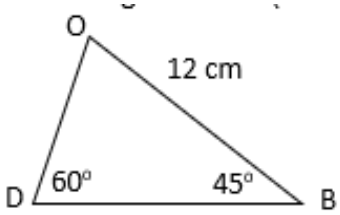
Find out how much you already know about the topic. On a sheet of paper, write the letter that you think is the best answer to each question. Answer all items as honestly as possible. After answering this short test, take note of the items that you missed and try to find right answers as you go through this module.

For items 1 – 5, given ΔABC, determine what Law will be used **FIRST** to solve the missing measures of the parts of the oblique triangle, a) Law of Sine, b) Law of Cosine.

1. $m\angle A = 50^o$, b = 7 cm, c = 8 cm
2. c = 10 cm, $m\angle B = 30°$, $m\angle C = 45°$
3. c = 10 cm, a = 15 cm, $m\angle B = 70°$
4. c = 10 cm, a = 15 cm, $m\angle C = 50°$
5. a = 8 cm, b = 10 cm, c = 15 cm

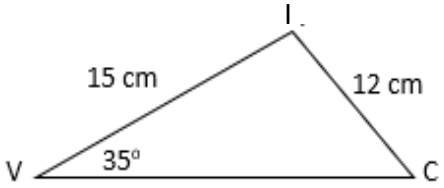


6. Given ΔDOB below, find the length of $\overline{DO}$.



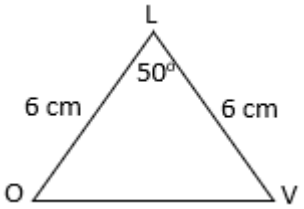
a. 9.8 cm b. 14.7 cm c. 16.97 cm d. 24 cm

7. Given ΔVIC at the right, find the length



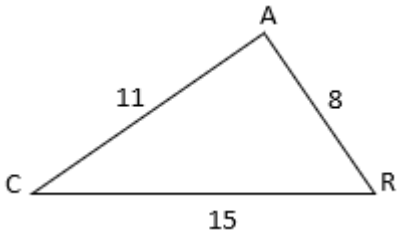
a. 20.65 cm b. 15 cm c. 10.74 cm d. 6.97 cm

8. Given ΔLOV at the right, find the length of



a. 6 m b. 5.70 cm c. 5.07 cm d. 4.72 cm

9. Given ΔCAR at the right, find m∠R.



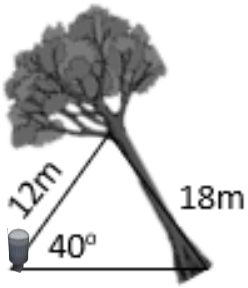
a. 31.29° b. 45.57° c. 59.4° d. 103.14°

For items 10 -13, consider the situation that follows:

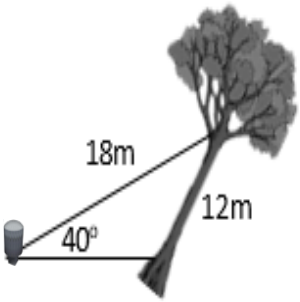
A large mango tree was partially blown by heavy storm. To keep the tree from falling further, an 18-meter cable was attached to it 12 meters from its base and a steel stake on the ground. The cable makes a 40−*degree* angle with the ground.

10. Which one is the appropriate illustration for the problem?

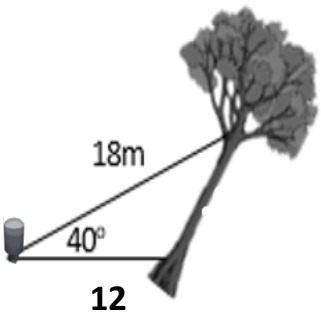
a.



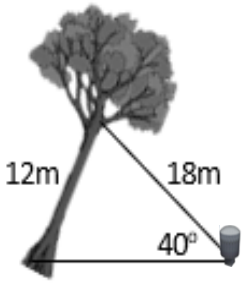
b.



c.



d.



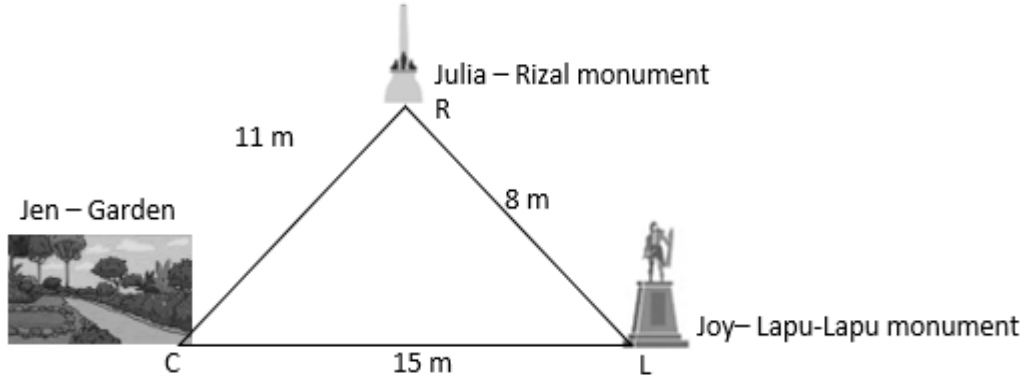
11. What is the measure of the angle formed by the tree and the ground in the triangle?
    a. 25.37^0 b. 74.62^0 c. 105.38^0 d. 154.63^0

12. What is the measure of the angle formed by the tree and the cable?
    a. 25.37^0 b. 34.62^0 c. 35^0 d. 40^0

13. How far is the stake from the base of the tree?
    a. 8 m b. 10.61 m c. 13.58 m d. 15.12

For items 14-15, consider the situation that follows:

Three sisters (Julia, Joy and Jen) went to a park to answer their Math exercises in their module. Each one chose a part of the park where to stay as shown in the illustration below.



14. What is the measure of ∠C?
    a. 31.29° b. 44.43° c. 45.57° d. 58.71°

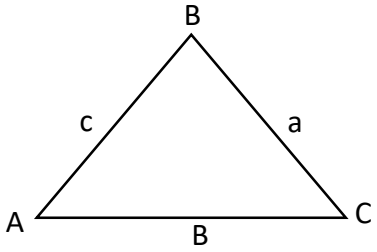
15. What is the measure of ∠L?
    a. 31.29° b. 44.43° c. 45.57° d. 58.71°

Lesson 1 Solve Problems Involving Oblique Triangles

Solving right triangles was your focus during the past lessons. But in this module, you will learn how to solve oblique triangles or triangles that do not have a right angle. In solving this kind of triangles, the Law of Sines and the Law of Cosines are used. At the end of this module, there is an assurance that you can find whether to use the Law of Sines or the Law of Cosines and use these concepts in solving real life problems.

What's In

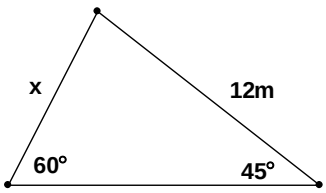
A. The measures of the three parts of triangle ABC are given. Determine which between the Law of Sines and the Law of Cosines is appropriate to use to solve for the triangle. **Do not solve.**



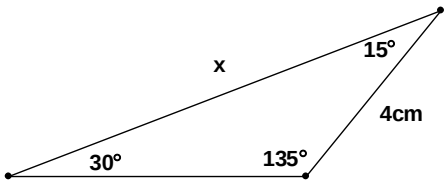
1. a =14 cm, b = 15 cm, c = 16 cm	6. m∠A = 40°, m∠C = 70°, c = 14 cm
2. m∠C = 35°, a = 11 cm, b = 10.5 cm	7. c = 10.3 cm, a = 21 cm, b = 16.7 cm
3. a = 10 cm, m∠A = 40°, c = 8 cm	8. c = 14 cm, m∠A = 29°, b = 8 cm
4. m∠A = 40°, b = 6 cm, c = 7 cm	9. b = 17 cm, m∠B = 45°28', a = 12 cm
5. c = 21 cm, a = 14 cm, m∠B = 60°	10. m∠A = 28°50', b = 6 cm, c = 3.9 cm

B. Use the Law of sines to find the value of x in each triangle.

1.

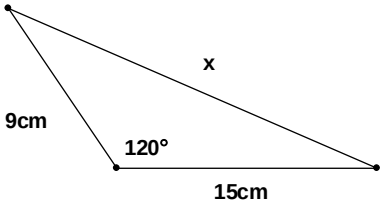


2.

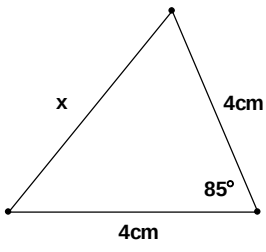


C. Use the Law of cosines to find the value of x in each triangle.

3.



4.



What's New

Communication, Creativity, Character Building and Collaboration

Read the selection below.

The subject Trigonometry is a branch of Mathematics which was developed for both practical and intellectual needs – computing the measures of angles and distances in surveying, map-making, and navigation. Problems involving angles and distances in one plane are covered in plane trigonometry.

The Alexandrian mathematicians triangulated the universe and gave precise knowledge about the earth and heaven.

With this we cannot ignore the importance of triangle in our life. If there is a single most important shape in engineering, it is the triangle. Triangulation of materials adds strength by eliminating lateral movement.

The trigonometric functions can also be used to solve problems involving triangles that are not right triangles which are called oblique triangles.

An oblique triangle has either three acute angles or one obtuse and two acute angles. In any case, as in any triangle, the sum of the measures of the three angles is equal to 180^0.

It is described in the situation above that concepts in trigonometry are applied in many real-life situations. These includes the Law of Sines and the Law of Cosines which can be used to solve oblique triangles.

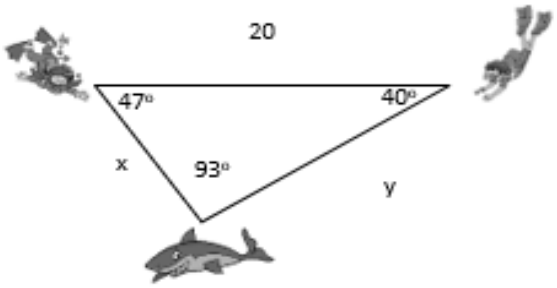
Now that we can solve oblique triangles, we can use this concept in solving real-life problems. To solve word problems involving oblique triangle, the following steps are suggested:

1. Draw a picture representing the situation in the problem.
2. Label the picture with the give information and a variable for the unknown measure/s of the part/s.
3. Identify which case of the triangle is determined by the give information, such as ASA, AAS, or SSA which requires the Law of Sines or an SAS or SSS that requires the Law of Cosines. Sometimes you will have to use these laws first and then followed by any appropriate mathematical procedure for the other parts of the triangle.
4. Solve for the required answer/s with appropriate unit of measure.

What is It

Illustrative Examples:

1. In the same depth of the water, two scuba divers who are 20 meters apart spot a shark that is at the deeper part of the sea. The angles of depression of the shark from diver 1 and diver 2 are 47^0 and 40^0, respectively. How far is each diver from the shark?



Solution:

In this problem, you are given with the measures of one side and two angles of the triangle that is determined by the locations of the two divers and the shark as shown in the illustration above. When the measures of two angles of a triangle are given, the measure of the third angle can always be computed using the fact that the sum of the measures of the three angles of any triangle is always 180^0.

Since the given information define the ASA case of a triangle, hence, to solve for either the value of x or the value of y, you can use the Law of Sines.

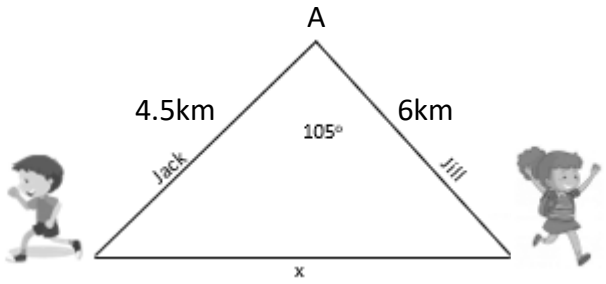
Solving for the value of y,

$$\frac{y}{\sin 47°} = \frac{20}{\sin 93°} \rightarrow \quad y = \frac{20 \sin 47°}{\sin 93°} \quad \rightarrow \quad y = \frac{(20)(0.731354)}{0.998630} \rightarrow y = 14.65 \text{ m}$$

Solving for the value of x,

$$\frac{x}{\sin 40°} = \frac{20}{\sin 93°} \rightarrow \quad x = \frac{20 \sin 40°}{\sin 93°} \quad \rightarrow \quad x = \frac{(20)(0.642788)}{0.998630} \rightarrow x = 12.87 \text{ m}$$

2. Jack and Jill both start at point A. Each walk in a straight line and their paths form an angle of 105°. If after 45 minutes Jack has walked 4.5 km and Jill has walked 6 km, how far apart are they from each other?



Solution:

In this problem, you are given with the measures of two sides and the included angle of the triangle that is determined by their paths from the starting point and their locations after 45 minutes as shown in the illustration above.

Since the given information define the SAS case of a triangle, hence, to solve for the value of x which is the distance between them after 45 minutes, you can use the Law of Cosines.

Solving for the value of x in the figure,

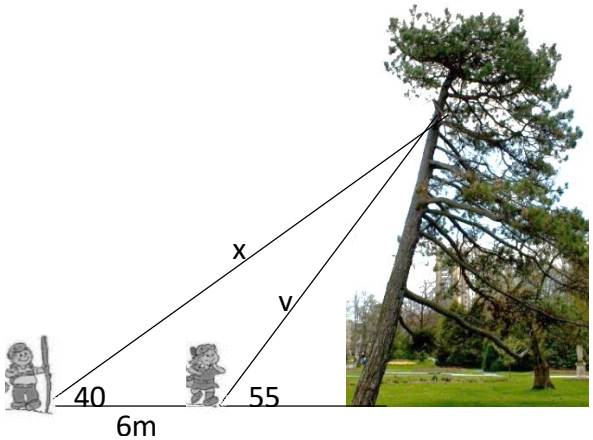
$$x^2 = 6^2 + 4.5^2 - 2\,(6)(4.5)\cos 105°$$

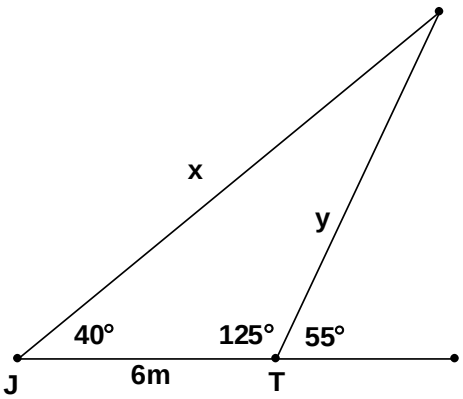
$$x^2 = 70.2262$$

$$x = 8.38 \text{ km}$$

The distance between Jack and Jill after 45 minutes of walking is 8.38 km.

3. Due to old age, a tree leans as shown below. Trish and Joey decided to attach ropes from a point on the tree to two different points on the ground to keep the tree from falling further. From Joey's location, the rope makes an angle of 40° with the ground and from Trish's spot, the rope makes an angle of 55° with the ground. If the distance between Joey and Trish is 6 meters, find the length of each rope.





Solution:

Given in the problem are measures of the interior and exterior angles and one side, with respect to the obtuse triangle formed by the two ropes and the ground. If you will analyze the figure, the angle adjacent to 55-degree angle measures 125°. Since two angles and one side are given, we will use the Law of Sines.

Solving for the value of x ;

$$\frac{x}{sin\ 125°} = \frac{6}{sin\ 15°} \rightarrow x = \frac{6\ sin\ 125°}{sin\ 15°} \rightarrow x = \frac{6(0.819152)}{0.258819} \rightarrow x = 18.99 \text{ m}$$

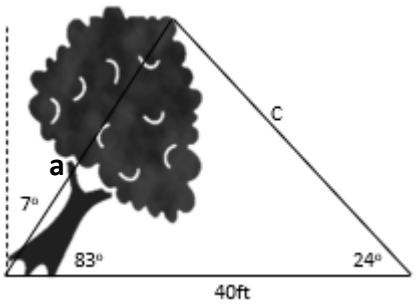
Solving for the value of y:

$$\frac{y}{sin\ 40°} = \frac{6}{sin\ 15°} \rightarrow y = \frac{6\ sin\ 40°}{sin\ 15°} \rightarrow y = \frac{6(0.642788)}{0.258819} \rightarrow y = 14.90 \text{ m}$$

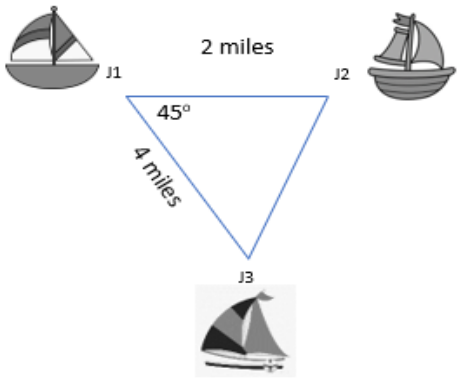
What's More

Solve the following problems and show your complete solution.

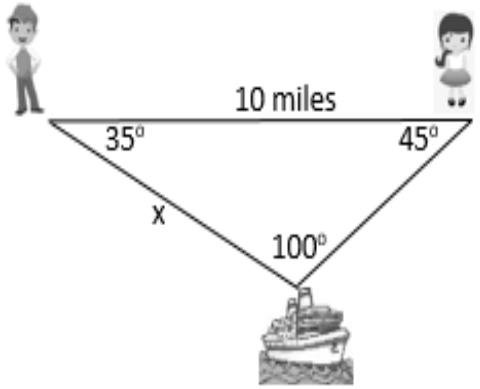
1. A tree leans and forms a 7-degree angle with the vertical. At a point on the ground 40 feet away from the foot of the tree, the angle of elevation of the top of the tree is 24°. What is the length of the tree?



2. Three boats are at sea, Jackson (J_1), Jinyoung (J_2) and Jaebom (J_3). The crew of J_1 can see both J_2 and J_3. The angle between the line of sight to J_2 and the line of sight to J_3 is 45 degrees. If the distance between J_1 and J_2 is 2 miles and the distance between J_1 and J_3 is 4 miles, what is the distance between J_2 and J_3?



3. Juan and Romella are standing at the seashore 10 miles apart. Both can see a same ship on the water. The angle between the coastline and Juan's line of sight to the ship measures 35 degrees. The angle between the coastline and Romella's line of sight to the ship measures 45 degrees. How far is the ship from Juan?



What I Have Learned

- In solving problems involving oblique triangles, the Law of Sines and the Law of Cosines are used.

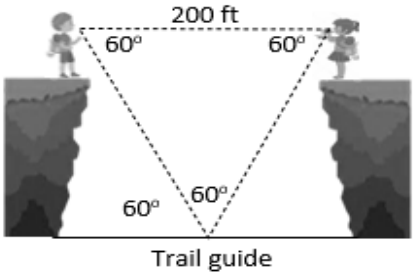
- In applying the law of Sines, any of the two conditions must hold true:
  1. two angles and a side are known (AAS or ASA), or
  2. two sides and an angle opposite one of them are known (SSA).

- In applying the Law of Cosines, any of two conditions must hold true:
  1. two sides and the included angle are given (SAS), or
  2. three sides are given (SSS).

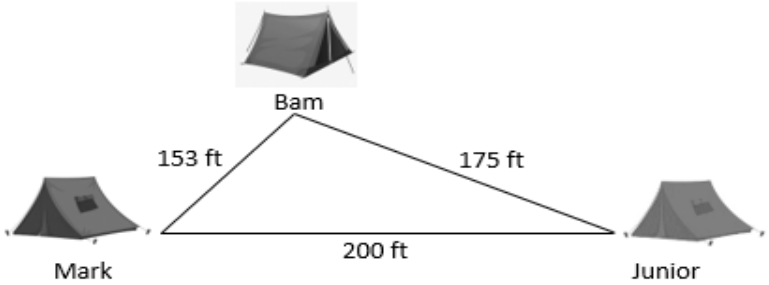
What I Can Do

Read and answer the following problems below and show your complete computation.

1. Jack is on one end of a 200-foot-wide canyon and Jill is on the other end. Jack and Jill can both see the trail guide at an angle of depression of 60 degrees. How far is each from the trail guide?



2. Mark, Bam and Junior are camping in their tents. If the distance between Mark and Bam is 153 feet, the distance between Mark and Junior is 200 feet, and the distance between Bam and Junior is 175 feet, find the measure of each angle of the triangle formed by the locations of their tents.



3. An electric post leans and makes an angle of about 85° with the ground. Marissa stands 171 feet away from the base of the post and observes that the angle of elevation of the top of the post to be 50°. Find the length of the post to the nearest tenth of a foot.

Assessment

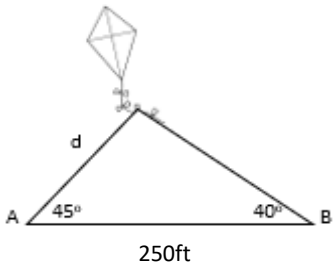
Read and answer each of the following problems accurately. Write the letter of the correct answer on your answer sheet.

For items 1 – 5, consider situation A.

A. Aiza and Bona, standing in the field 250 feet apart, observe that the angles of elevation of a kite are 40^0 and 45^0 respectively.

1. Which diagram below best illustrates situation A?

a.



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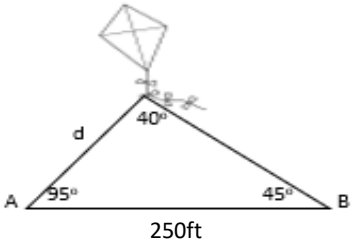

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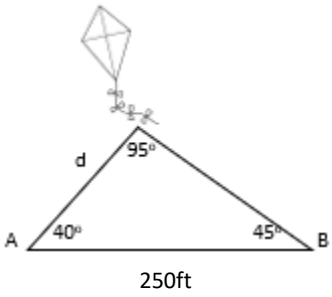

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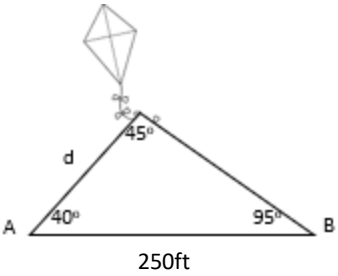


2. What is the measure of the angle formed by the lines of sight of Aiza and Bona to the kite?

a. 85^0 b. 90^0 c. 95^0 d. 105^0

3. How far is Aiza from Bona?

a. 250 ft b. 200 ft c. 150 ft d. 125 ft

4. Which equation gives the distance of Aiza to the kite?

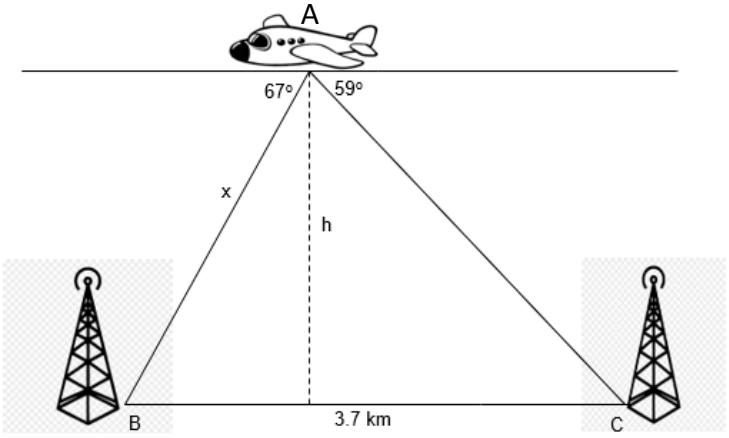
a. $\frac{d}{sin\,95^\circ} = \frac{250}{sin\,45^\circ}$ b. $\frac{sin\,40^\circ}{250} = \frac{sin\,45^\circ}{d}$ c. $\frac{d}{sin\,45^\circ} = \frac{250}{sin\,95^\circ}$ d. $\frac{d}{sin\,40^\circ} = \frac{250}{Sin\,95^\circ}$

5. What is the distance of Aiza to the kite?

a. 161.31 ft. b. 177.45 ft. c. 275.02 ft d. 352.21 ft

For items 6 – 10, consider situation B:

B. An airplane is flying over a straight highway. The pilot of the plane observes that the angles of depression of two towers that are 3.7 km. apart to be 67° and 59° as shown in the figure below.



6. Using the given figure, what is the measure of $\angle ACB$?
   a. 54° b. 59° c. 67° d. 90°

7. What is the measure of $\angle BAC$?
   a. 54° b. 59° c. 67° d. 90°

8. To find the length of $\overline{AB}$, which equation is appropriate to use?
   a. $\frac{Sin\ 54^\circ}{|AB|} = \frac{Sin\ 59^\circ}{3.7}$ b. $\frac{Sin\ 59^\circ}{|AB|} = \frac{Sin\ 54^\circ}{3.7}$ c. $\frac{Sin\ 67^\circ}{|AB|} = \frac{Sin\ 54^\circ}{3.7}$ d. $\frac{Sin\ 67^\circ}{|AB|} = \frac{Sin\ 59^\circ}{3.7}$

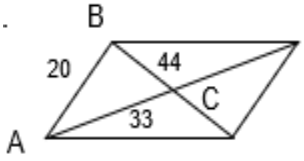
9. What is the length of $\overline{AB}$?
   a. 3.49 km b. 3.61 km c. 3.92 km d. 4.21 km

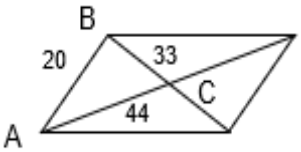
10. What is the height of the plane?
    a. 3.49 km b. 3.61 km c. 3.92 km d. 4.21 km

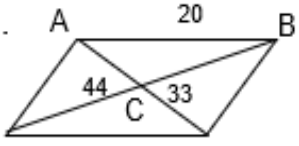
For items 11 – 15, consider situation C:

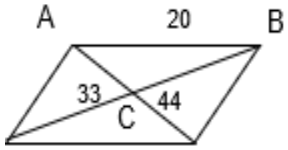
C. The lengths of the diagonals of a parallelogram are 88 cm and 66 cm. The shorter side measures 20 cm.

11. Which of the following parallelograms is correctly labelled?

a. 

b. 

c. 

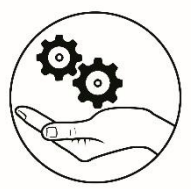
d. 

12. What case of a triangle in the parallelogram is illustrated?
    a. SSS b. AAA c. ASA d. SAS

13. Which law is appropriate to use to find the measure of the acute angle formed by the diagonals of the parallelogram?
    a. Pythagorean law c. Law of cosines
    b. Law of sines d. Law of tangent

14. Which of the following is the correct equation?
    a. $\cos C = \frac{33^2+44^2-20^2}{2(33)(44)}$ c. $\cos C = \frac{33^2+20^2-44^2}{2(33)(20)}$
    b. $\cos C = \frac{20^2+44^2-33^2}{2(20)(44)}$ d. $\cos C = \frac{33^2+44^2-20^2}{2(20)(44)}$

15. What is the measure of the acute angle formed by the two diagonals of the parallelogram?
    a. 70.21° b. 64.68° c. 44.89° d. 25.32°

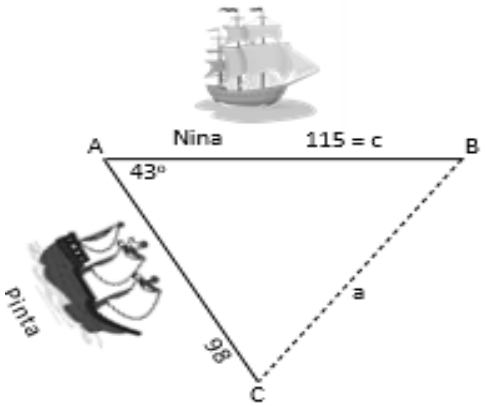


Additional Activities

Critical Thinking and Collaboration

A. Think it up!

Using the figure to create your own problem using sine and cosine law.



B. Reflect!

Answer the following.

1. What are your thoughts about the application of the law of sines and the law of cosines?
2. Are the given illustrations helpful? How it helps to solve the problem?
3. Do you find other way to solve the problems? Share it?

PROBLEM-BASED LEARNING WORKSHEET

How well do you make an illustration given the information on a problem? Study the problem below, draw an illustration using the given information, and then solve the problem.

Will the Tree Hit the House?

Dear Friend,

There is a tree out in front of our yard. It stands obliquely with the ground at 70 degrees. Our house is 66 1/2 feet away from the tree. The angle of elevation of the top of the tree from our house is 40 degrees.

My family is worried that if we have a big storm the tree will fall and might hit the house. I read somewhere that if you have 2 angles and a side you can figure out the dimensions of the triangle. I have not taken trigonometry yet, so could you please help me out? Thanks.

Billy

E-Search

You may also check the following link for your reference and further learnings on solving quadratic equations.

- https://www.youtube.com/watch?v=_gWjLKsFOPE
- https://www.youtube.com/watch?v=1aYMo1NIfFA
- https://www.youtube.com/watch?v=OVEK6JI8Mcc
- https://www.mathsisfun.com/algebra/inequality-quadratic-solving.html

Answer Key

What I Know

1. Cos law	9. B
2. Sin law	10. B
3. Cos law	11. C
4. Sin law	12. B
5. Cos law	13. B
6. A	14. A
7. A	15. C
8. C	

What's In

A.
1. Law of Cosines
2. Law of Cosines
3. Law of Sines
4. Law of Cosines
5. Law of Cosines
6. Law of Sines
7. Law of Cosines
8. Law of Cosines
9. Law of Sines
10. Law of Cosines

B.
1. x = 9.80 cm
2. x = 5.66 cm

C.
3. x = 21cm
4. x = 5.4 cm

What's More

1. 17.01 ft
2. 2.95 miles
3. 7.18 miles

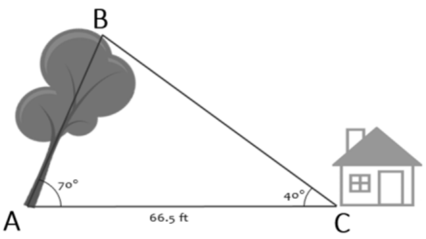
What I Can Do

1. 200 ft – 200 ft
2. m∠J=47.58^0
   m∠B=74.81^0
   m∠M=57.61^0
3. 185.25

Assessment

1. C
2. C
3. A
4. C
5. B
6. B
7. A
8. B
9. C
10. B
11. B
12. A
13. C
14. A
15. D

Problem-Based Learning Worksheet



$$\frac{|AB|}{\sin C} = \frac{|AC|}{\sin B}$$

$$|AB| = \frac{|AC|(\sin C)}{\sin B}$$

$$|AB| = \frac{66.5 \cdot (\sin 40°)}{\sin 70°}$$

$$|AB| = 45.49\ ft.$$

The length of the tree is 45.49 ft. If it falls towards the house, the house will not be hit because the top of the tree will fall at a point on the ground 21 ft away from the house. (66.5 ft – 45.49 ft = 21 ft).

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For inquiries or feedback, please write or call:

Department of Education - Bureau of Learning Resources (DepEd-BLR)

Ground Floor, Bonifacio Bldg., DepEd Complex
Meralco Avenue, Pasig City, Philippines 1600

Telefax: (632) 8634-1072; 8634-1054; 8631-4985

Email Address: blr.lrqad@deped.gov.ph * blr.lrpd@deped.gov.ph