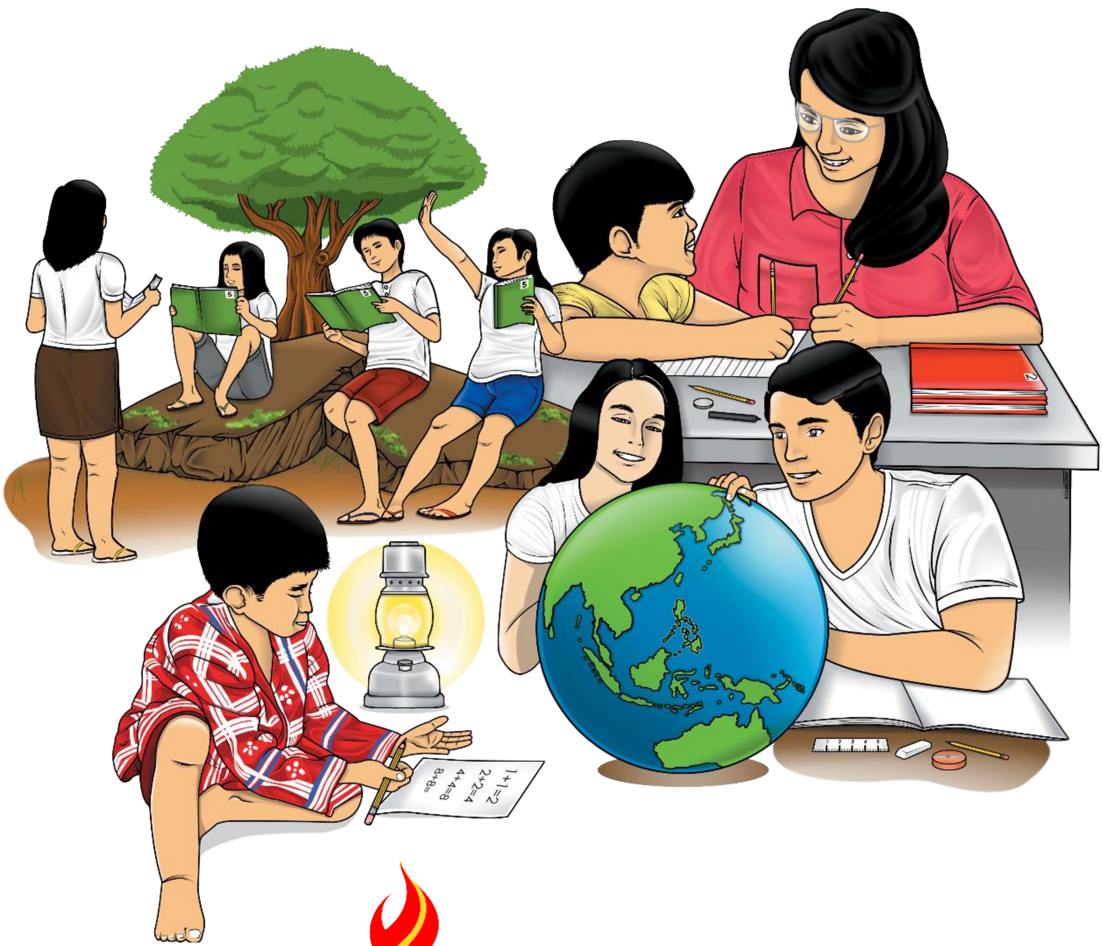
9

Mathematics

Quarter 4-Module 7

Law of Sines and Its Applications



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Mathematics

Quarter 4-Module 7

Law of Sines and Its Applications

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Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

What I Need to Know

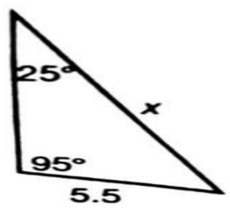
In the previous modules, you have learned how to solve right triangle using trigonometric functions. Now you will learn to solve non-right triangles called oblique triangles. Any triangle, right or oblique, may be solved using the Law of Sines and the Law of Cosines. If any three of the six measures of a triangle are given, provided at least one measure is a side, then the other three measures can be found. An oblique triangle is a triangle that does not have a right angle. There are laws or formulas that describe the relationships between the angles and the sides of an oblique triangle. These are the Law of Sines and the Law of Cosines.

After going through with this module, you are expected to be able to illustrate law of sines.

What I Know

A. Find out how much you already know about the module. Write the letter of the best answer to each question from 1 – 6 and solve the items 7 – 15 on a sheet of paper. Answer all items. After taking and checking this short test, take note of the items that you were not able to answer correctly and look for the right answer as you go through this module.

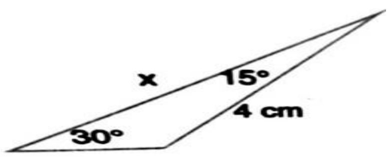
1. Refer to the triangle below. Which of the following statements is the correct equation to solve for the value of x?



a. $\frac{5.5}{\sin 95°} = \frac{x}{\sin 25°}$ c. $\frac{x}{\sin 25°} = \frac{5.5}{\sin 95°}$

b. $\frac{x}{\sin 95°} = \frac{5.5}{\sin 25°}$ d. $\frac{5.5}{\sin 95°} = \frac{\sin 25°}{x}$

2. Use the Law of Sines to solve for the value of x. Round the answer to two decimal places.



a. 5.33 units c. 5.44 units
b. 5.44 units d. 5.66 units

For items 3 – 6, In each ΔABC, sides $a, b,$ and c are opposite sides of angles $A, B,$ and C, respectively.

3. If $m\angle A = 40°, m\angle C = 70°$, and $b = 10\ cm$, use the Law of Sines to solve the triangle.

a. $m\angle$B = 70°, a = 10 cm, c = 6.84 cm c. $m\angle$B = 65°, a = 10 cm, c = 6.84 cm
b. $m\angle$B = 75°, a = 6.84 cm, c = 10 cm d. $m\angle$B = 70°, a = 6.84 cm, c = 10 cm

4. In ΔABC, if $m\angle A = 50°$, $m\angle B = 79°$, and c = 8 cm, then a = _____.
   a. 7.89 cm b. 7.77 cm c. 7.55 cm d. 7.51 cm

5. In ΔABC, if $m\angle A = 35°$, $m\angle B = 25°$, and $a = 5$ cm then b = _____.
   a. 3.68 cm b. 3.78 cm c. 3.88 cm d. 3.98 cm

6. In ΔABC, if a = 16 cm, $m\angle A = 13°$, and $m\angle B = 44°$, then c = ______.
   a. 49.4 cm b. 56.3 cm c. 59.65 cm d. 65.1 cm

B. *In each ΔABC, sides $a, b,$ and c are opposite sides of angles $A, B,$ and C, respectively. Solve each triangle.*

7. If $m\angle A = 42°, m\angle B = 96°, and\ b = 12$ cm
8. If $m\angle A = 33°, m\angle C = 128°, and\ a = 16$ cm
9. If $m\angle A = 42°, m\angle B = 48°, and\ c = 12$ cm
10. If $m\angle A = 48°, m\angle B = 37°, and\ a = 100$ cm

C. *In each ΔABC, sides $a, b,$ and c are opposite sides of angles $A, B,$ and C, respectively. Determine whether each ΔABC has no solution, one solution, or two solutions, then solve each triangle.*

11. $m\angle A = 50°, a = 15\ cm, and\ b = 10\ cm$
12. $m\angle A = 30°, a = 5\ cm, and\ b = 20\ cm$
13. $m\angle A = 136°, a = 115\ cm, and\ b = 99.6\ cm$
14. $m\angle A = 150°, a = 10\ cm, and\ b = 30\ cm$
15. $m\angle A = 112°, a = 84.2\ cm, and\ c = 74\ cm$

Lesson 1 Law of Sines and Its Applications

What's In

LET'S RECALL

A. What are the special angles?

B. Without using a scientific calculator, find the values of each of the following.

1. sin 45°, cos 45°
2. sin 30°, cos 30°
3. sin 60°, cos 60°
4. sin 90°, cos 90°
5. sin 0°, cos 0°

C. Using a scientific calculator, find the values of each of the following.

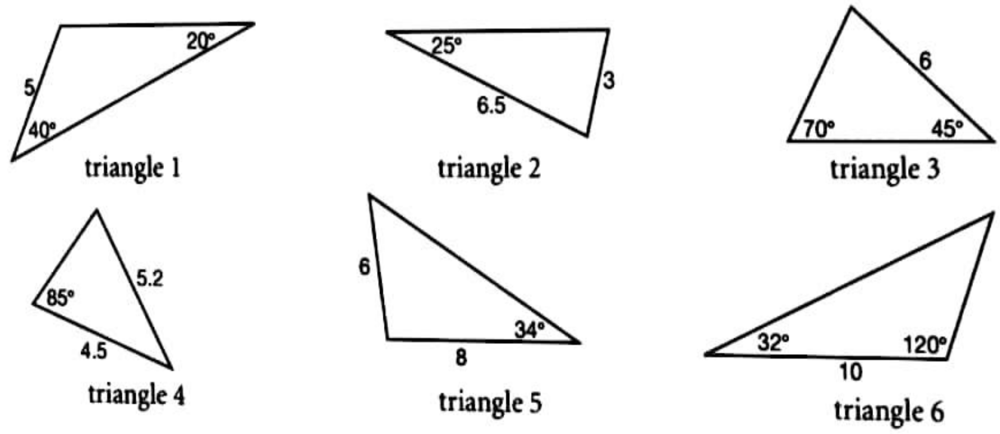
1. sin 20°
2. sin 50°
3. sin 125°
4. sin 162°

D. Using a scientific calculator, find the values of X to the nearest whole number of degree.

1. sin X = 0.1115
2. sin X = 0.2436
3. sin X = 0.3219
4. sin X = 0.5432

What's New

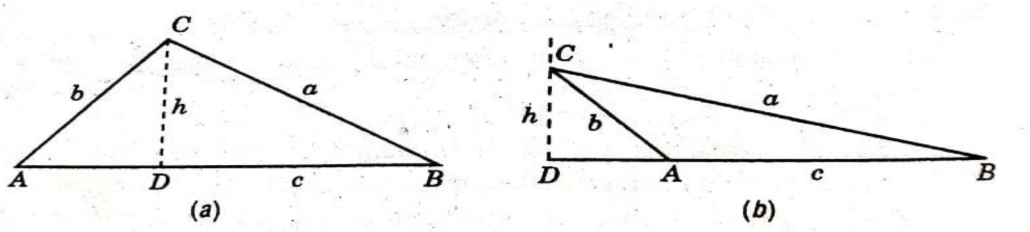
Look at the triangles below.



Questions:
1. What have you noticed about each of the given triangles?
2. Can you use trigonometric ratios to solve for the missing parts of these triangles? Why?

Examine closely the triangles. Can you solve the missing parts of these triangles using the previous concepts you have learned?

Consider two oblique triangles shown below.



The two triangles in (a) and (b) each has angles A, B, and C and corresponding opposite sides a, b, and c.

Draw an altitude of length h from vertex C of each of the triangles. Notice that two right triangles are formed, and these are triangles CDA and CDB. Using the definition of the sine of an angle, you have

$$\sin B = \frac{\text{length of the opposite side}}{\text{length of the hypotenuse}} = \frac{h}{a}, \qquad \sin A = \frac{\text{length of the opposite side}}{\text{length of the hypotenuse}} = \frac{h}{b}$$

Note that the equations are true for triangles in (a) and (b).
Solving for h in the two equations, you will get
$h = a \sin B \qquad and \qquad h = b \sin A$

Therefore, equating the two expressions for h gives you

$$a \sin B = b \sin A$$

Dividing both sides of the equation by $\sin A \sin B$ results to

$$\frac{a \sin B}{\sin A \sin B} = \frac{b \sin A}{\sin A \sin B}$$

Simplifying both sides of the equation gives you

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

That is, side a divided by the sine of its opposite angle ($\sin A$) is equal to side b divided by the sine of its opposite angle ($\sin B$).

Repeating the process of drawing an altitude of length h from vertex A of each triangle, the following equation is obtained.

$$\frac{b}{\sin B} = \frac{c}{\sin C}.$$

Combining the two equations, you get

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

The above equation gives the *Law of Sines*.

The Law of Sines

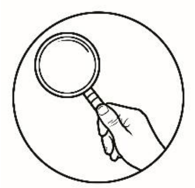
In any triangle, a side divided by the sine of its opposite angle is equal to any other side divided by the sine of the corresponding opposite angle.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

This can also be expressed as

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Note that the derived formula for oblique triangles is also applicable to right triangles.



What is It

The Law of Sines can be used in solving problems involving oblique triangles, given the measures of two angles and one side. It can be used when the given are two angles and the included side (ASA) or two angles and a non-included side (SAA).

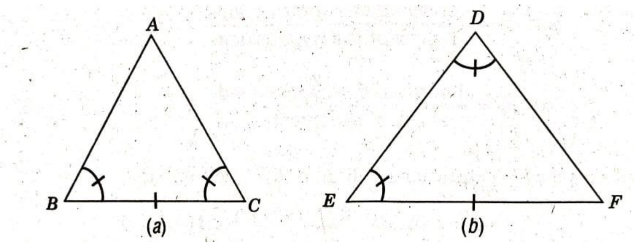


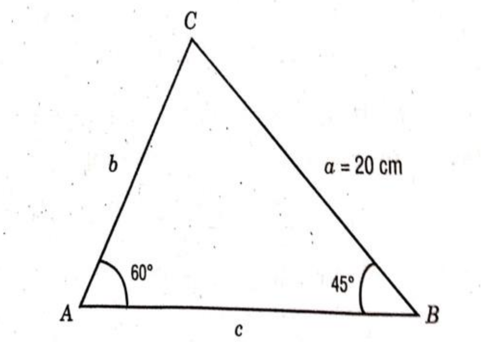
Figure (a) shows triangle ABC given its two angles and the included side (ASA) while (b) shows triangle DEF given its two angles and a non-included side (SAA).

Example 1:

Solve $\Delta ABC\ given\ m\angle A = 60°, m\angle B = 45°, and\ a = 20cm.$

Solution:

You are given two angles and a non-included side (SAA)



First, find the measure of angle C. Since the sum of the measures of the interior angles of any triangle equals 180°, that is,

$$m\angle A + m\angle B + m\angle C = 180°$$
$$60° + 45° + m\angle C = 180°$$
$$m\angle C = 180° - (60° + 45°)$$
$$m\angle C = 180° - 105°$$
$$m\angle C = 75°$$

Use the Law of Sines to find b.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{20}{\sin 60°} = \frac{b}{\sin 45°}$$

$$b = \frac{20 \sin 45°}{\sin 60°}$$

$$b = 16.33\ cm.$$

You can also use the Law of Sines to find c.

$$\frac{a}{\sin a} = \frac{c}{\sin c}$$

$$\frac{20}{\sin 60°} = \frac{c}{\sin 75°}$$

$$c = \frac{20 \sin 75°}{\sin 60°}$$

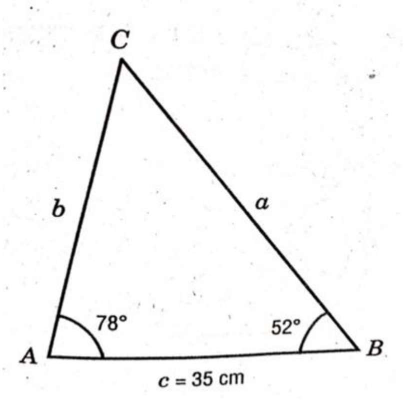
$$c = 22.31\ cm$$

Thus, $m\angle C = 75°, b = 16.33\ cm, and\ c = 22.31\ cm.$

Example 2:

Solve $\Delta ABC\ given\ m\angle A = 78°, m\angle B = 52°, and\ c = 35\ cm.$

Solution:
You are given two angles and an included side (ASA)



First, find the measure of angle C. Since the sum of the measures of the interior angles of any triangle equals 180°, that is,

$$m\angle A + m\angle B + m\angle C = 180°$$
$$78° + 52° + m\angle C = 180°$$
$$m\angle C = 180° - (78° + 52°)$$
$$m\angle C = 180° - 130°$$
$$m\angle C = 50°$$

Use the Law of Sines to find a.

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{a}{\sin 78°} = \frac{35}{\sin 50°}$$

$$a = \frac{35 \sin 78°}{\sin 50°}$$

$$a = 44.69\ cm.$$

You can also use the Law of Sines to find b.

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$

$$\frac{b}{\sin 52°} = \frac{44.69}{\sin 78°}$$

$$b = \frac{44.69 \sin 52°}{\sin 78°}$$
$$b = 36\ cm.$$

Thus,

$$m\angle C = 50°, a = 44.69\ cm, and\ b = 36\ cm$$

The Law of Sines can also be used when two sides and a non-included angle are given (SSA). In this case, there may be no triangle having the given measurements or there may be one or two triangles that satisfy the given conditions. This case is often referred to as the *ambiguous case*.

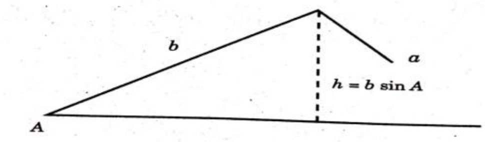
Now, suppose that in ΔABC, angle A and sides a and b are given (SSA). Based on the Law of Sines,

$$\frac{b}{\sin B} = \frac{a}{\sin A} \qquad \sin B = \frac{b \sin A}{a}$$

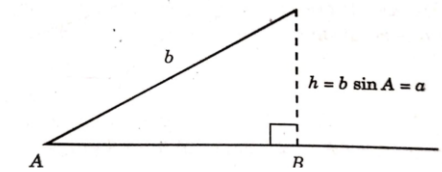
Consider the following cases:

Case 1. $0° < A < 90°$

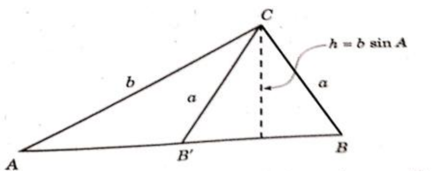
a. If $a < b\sin A, then \sin B > 1$. This means that no angle B is determined; hence, no triangle is formed and there is no solution.



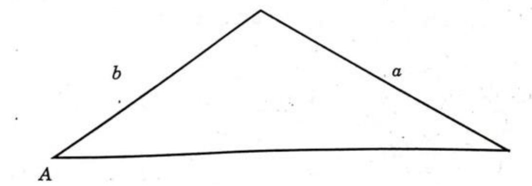
b. If $a = b\ sin\ A$ then $sin\ B = 1$. This means $m\angle B = 90°$ and a right triangle is determined.



c. If $a > b\sin A\ and\ a < b$, then two angles are formed: an acute angle B in triangle ABC and an obtuse angle B' in triangle AB'C. Hence, there are two solutions.

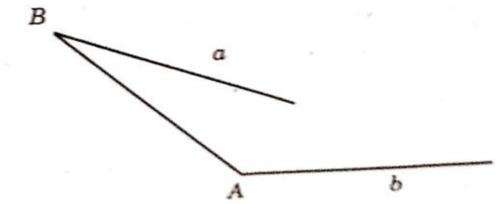


d. If $a > b\sin A$ and $a \geq b$ then there is exactly one angle determined. Hence, there is only one solution.

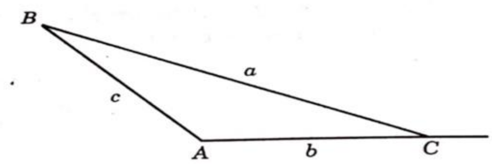


Case 2: $90° < A < 180°$

a. If $a \leq b$ then it can be seen from the figure below that there is no solution.



b. If $a > b$ then there is exactly one triangle formed. Hence, there is exactly one solution.



Example 3:

Solve ΔABC given $m\angle A = 60°, a = 8\ cm,\ and\ b = 53\ cm$.

Solution:

To solve for B, use the Law of Sines.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{8}{\sin 60°} = \frac{53}{\sin B}$$

$$\text{Sin}\ B = \frac{53\sin 60°}{8}$$

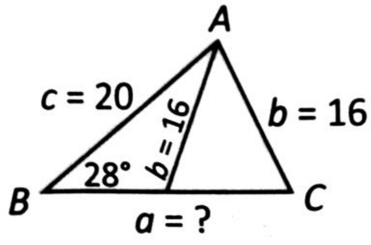
$$\sin B = 5.7374$$

Notice that $\sin B = 5.7374$, which is greater than 1. This is not possible since $-1 \leq \sin B \leq 1$ for any angle B. This means that there is no solution, that is, there is no triangle having the given measurements. This is an example of *ambiguous case*.

Example 4:

Solve ΔABC given $m\angle B = 28°, b = 16\ cm,\ and\ c = 20\ cm.$

Solution:



$m\angle A = ?$ $a = ?$
$m\angle B = 28°$ $b = 16$
$m\angle C = ?$ $c = 20$

Compare the values of $b\ and\ c\sin B$.

$$b\ ?\ c\sin B$$

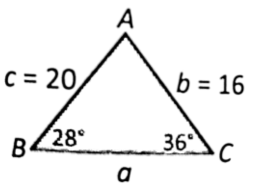
$$16 > 20\sin 28°$$

So, $b > c\sin B$.

Since B is an acute angle and $c > b$, then this is Case 1.c. This *is an ambiguous* case and there are two possible solutions.

The two possible solutions are shown below.

Solution No. 1:



Solving for $m\angle C$

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{16}{\sin 28°} = \frac{20}{\sin C}$$

$$\sin C = \frac{20\sin 28°}{16}$$

$$\sin C = .5868$$

$$m\angle C = 36°$$

Solving for $m\angle A$

$$m\angle A = 180° - (28° + 36°)$$
$$m\angle A = 116°$$

Solving for a $\frac{b}{\sin B} = \frac{a}{\sin A}$

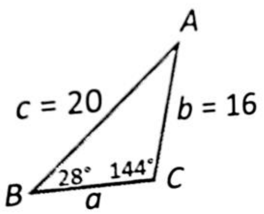
$$\frac{16}{\sin 28°} = \frac{a}{\sin 116°}$$

$$a = \frac{16\sin 116°}{\sin 28°}$$

$$a = 31$$

Thus, $m\angle A = 116°, m\angle C = 36°, and\ a = 31\ cm.$

Solution No. 2:



Solving for the other value of C,

$$m\angle C = 180° - 36°$$
$$m\angle C = 144°.$$

Remember:
For $0° < C < 180°$, *there are two angles with a sine value of* $\frac{20\sin 28°}{16}$:
one acute angle and one obtuse angle.

Solving for $m\angle A$,
$m\angle A = 180° - (28° + 144°)$
$m\angle A = 8°.$

Solving for a,

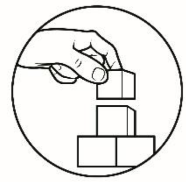
$$\frac{b}{\sin B} = \frac{a}{\sin A}$$

$$\frac{16}{\sin 28°} = \frac{a}{\sin 8°}$$

$$a = \frac{16\sin 8°}{\sin 28°}$$

$$a = 5$$

Thus, $m\angle A = 8°, m\angle C = 144°, and\ a = 5\ cm.$

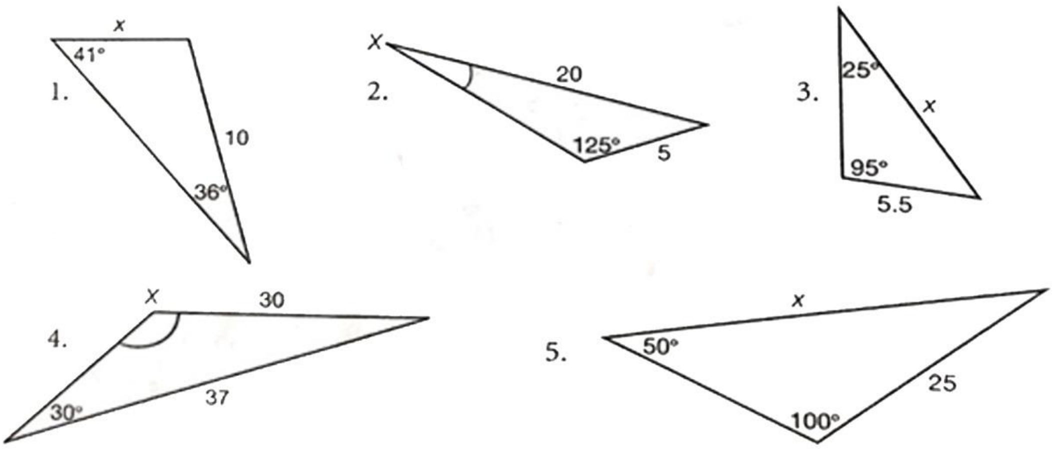


What's More

A. Determine whether each statement is *true* or *false.*

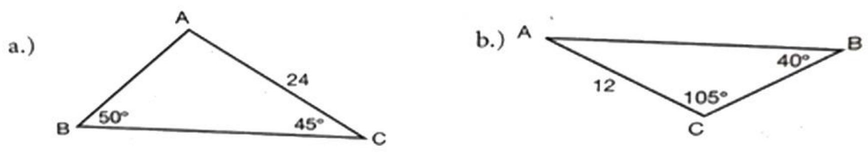
1. It is possible to solve a triangle if the only given information consists of the measures of the three angles of the triangle.
2. In general, it is not possible to use the Law of Sines to solve a triangle for which the given are the lengths of all the sides.
3. Given ΔABC with $m\angle A = 30°, c = 3\ cm, and\ a = 2.5\ cm.$ There can be more than one triangle that can be drawn with the given dimensions.
4. In a scalene triangle, the longest side is always opposite the largest angle and the shortest side is always opposite the smallest angle.
5. Given ΔABC with $A = 57°, a = 15\ m,$ and $c = 20\ m$. There is no triangle that can be formed for these values of $A, a,$ and c.

B. Write an equation that will solve for the value of x.

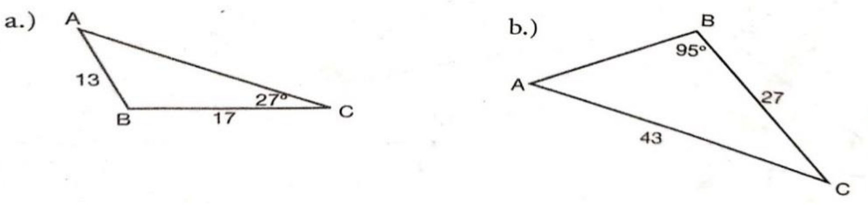


C. For each triangle given in Exercise B, solve for the value of x. Round answer to the nearest tenth.

D. Find the length of $\overline{AB}$. Round answer to the nearest tenth.

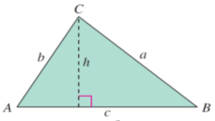


E. Find measure of angle A. Round answer to the nearest tenth.



What I Have Learned

Given Triangle ABC, with angles $A, B,$ and C and corresponding opposite sides $a, b,$ and c.
The Law of Sines states that



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{or} \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

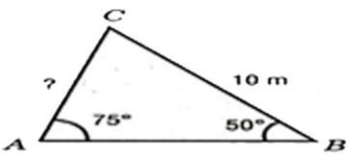
To solve an oblique triangle using the Law of Sines, you need to know the measure of one side and the measures of two other parts of the triangle: two angles, or one angle and another side.

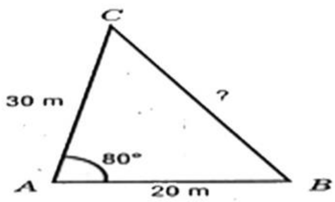
This breaks down into the following cases:
1. Two angles and any side (AAS or ASA)
2. Two sides and an angle opposite one of them (SSA or Angle Side Side)

What I Can Do

A. Determine the unknown part of the triangle marked with"?". Round answer to the nearest tenth.

1.





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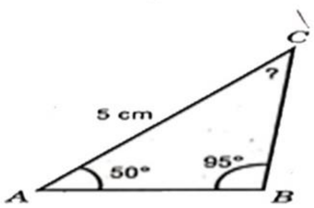

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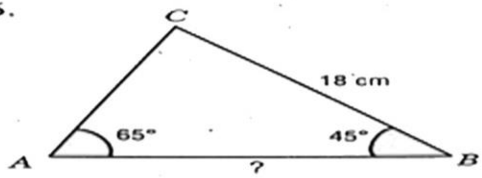
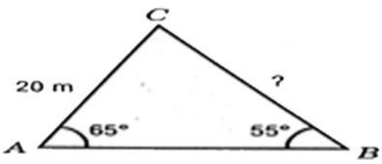


3.



B. Solve each ΔABC given that $a, b, and\ c$ are opposite sides of $\angle A, \angle B, and\ \angle C$, respectively.

6. SSA

   m∠A = 73° a = 18 cm
   m∠B = ____ b = 11 cm
   m∠C = ____ c = _____

7. SSA

   m∠A = _____ a = 17 cm
   m∠B = _____ b = ______
   m∠C = 27° c = 13 cm

8. ASA

   m∠A = 26° a = _____
   m∠B = _____ b = 13 cm
   m∠C = 35° c = _____

9. AAS

   m∠A = _____ a = ______
   m∠B = 50° b = 14 cm
   m∠C = 45° c =

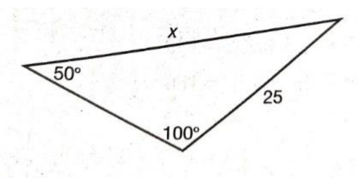
Assessment

Read and answer each question carefully.

A. Write the letter of the correct answer on a sheet of paper.

1. Two sides and an angle opposite one of the given sides of a triangle are known. To find the measure of the angle opposite the other given side, which one should be used?
   a. Law of Sines b. Heron's Formula c. Law of Cosines d. All of the above

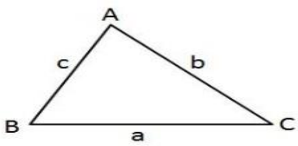
2. Which of the following is the correct equation to solve for the value of x in the given triangle below?



   a. $\frac{25}{\sin 100°} = \frac{x}{\sin 50°}$ c. $\frac{x}{\sin 100°} = \frac{25}{\sin 50°}$

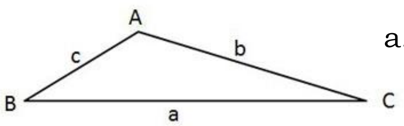
   b. $\frac{x}{\sin 50°} = \frac{25}{\sin 100°}$ d. $\frac{25}{\sin 50°} = \frac{\sin 100°}{x}$

3. In the given triangle below, m∠A = 81° and m∠B = 67°. If side a is 34 cm long, approximately how long is side b?



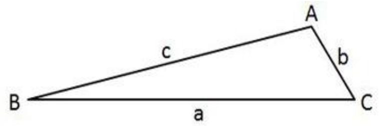
   a. 32 cm b. 30 cm c. 22 cm d. 20 cm

4. In the given triangle below, m∠A = 137° and m∠B = 28°. If side b is 71 cm long, approximately how long is side a?



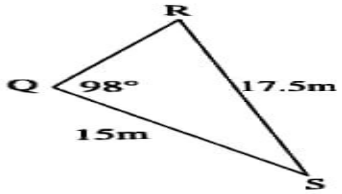
   a. 77 cm b. 84 cm c. 98 cm d. 103 cm

5. In the given triangle below, m∠A = 98° and m∠B = 12°. If side a is 84 cm long, approximately how long is side b?



a. 20 cm b. 18 cm c. 16 cm d. 14 cm

6. In the given triangle below, $\mathrm{m}\angle Q = 98°, q = 17.5\ m, and\ r = 15\ m$. Find $\mathrm{m}\angle R$.



a. 78° b. 68° c. 58° d. 44°

B. In each ΔABC, sides $a, b,$ and c are opposite sides of angles $A, B,$ and C, respectively. Solve each triangle.

7. $\mathrm{m}\angle A = 49°, \mathrm{m}\angle B = 57°, and\ a = 8\ cm$

8. $\mathrm{m}\angle C = 110°, c = 18\ cm, and\ a = 13\ cm$

9. $\mathrm{m}\angle B = 49°, a = 11.6\ cm, and\ b = 14.9\ cm$

10. $\mathrm{m}\angle A = 105°, a = 18\ cm, and\ b = 14\ cm$

C. In each ΔABC sides $a, b,$ and c are opposite sides of angles $A, B,$ and C, respectively. Determine whether ΔABC has no solution, one solution, or two solutions. Then solve each triangle.

11. $\mathrm{m}\angle A = 20°, a = 15\ cm, and\ b = 20\ cm$

12. $\mathrm{m}\angle A = 37°, a = 24\ cm, and\ b = 32.2\ cm$

13. $\mathrm{m}\angle A = 31°, a = 9\ cm, and\ b = 20\ cm$

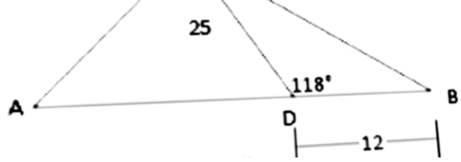
14. $\mathrm{m}\angle A = 90°, a = 12\ cm, and\ b = 14\ cm$

15. $\mathrm{m}\angle A = 25°, a = 125\ cm, and\ b = 150\ cm$

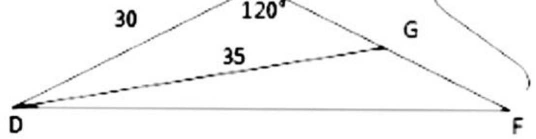
Additional Activities

Challenge Problems:

1. Find $m\angle A$ to the nearest whole number of degree.



2. Find angle $m\angle DGF$ to the nearest whole number of degree.



E-Search

To further explore the concept learned today and if it possible to connect the internet, you may visit the following links:

- www.onlinemath learning.com>law of sine 2
- www.math is fun .com>algebra>trig-sine-law
- www.teacherspayteachers .com>Browse>Search law of sines
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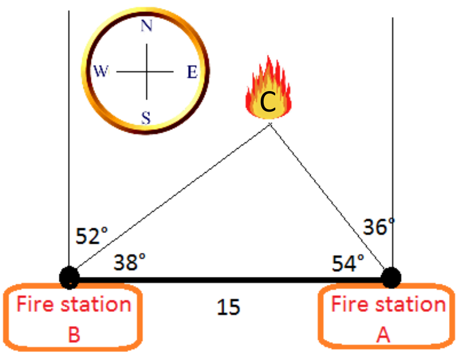
- www.mathopenref.com/lawofcosinesproof.html
- https://cdn.kutasoftware.com
- https://www.buffaloschool.org

PROBLEM – BASED LEARNING WORKSHEET

Problem

Two fire-lookout stations are 15 miles apart, with station A directly east of station B. Both stations spot a fire at position C. The angular direction of the fire from station B is N52°E and the angular direction of the fire from station A is N36°W. How far is the fire from station A?



Draw the triangle and use A, B, *and* C as angles and a, b, *and* c as the opposite sides, respectively.

Questions:

1. How do you find A?
2. How about B?
3. What is C?
4. What law are you going to use to find the distance from *Fire Station A* to point C where the fire is *or* what law are you going to use to solve for b?
5. Write the equation to solve for b.
6. How far is the fire from station A?

Answer Key

WHAT I KNOW

1. b 2. d 3. d 4. a 5. a 6. c

7. m$\angle C = 42°, a = 8.07\ cm, and\ c = 8.07\ cm$

8. m$\angle B = 19°, b = 9.56\ cm, and\ c = 23.15\ cm$

9. m$\angle C = 90°, a = 8.03\ cm\ and\ b = 8.92\ cm$

10. m$\angle C = 95°, b = 80.98\ cm, and\ c = 134.05\ cm$

11. one solution: m$\angle B = 30.71°$, m$\angle C = 99.29°, and\ c = 19.32\ cm$

12. No solution

13. one solution: m$\angle B = 37°$, m$\angle C = 7°, and\ c = 20.18$ cm

14. No solution

15. one solution: m$\angle B = 13.43°$, m$\angle C = 54.57°, and\ b = 21.1\ cm$

WHAT'S IN

A. The specials angle are $30°, 45°, and\ 60°$angles.

B. 1. $\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}$ 2. $\frac{1}{2}, \frac{\sqrt{3}}{2}$ 3. , $\frac{\sqrt{3}}{2}, \frac{1}{2}$ 4. 1, 0 5. 0,1

C. 1. 0.3420 5. 6°
2. 0.7660 6. 14°
3. 0.8192 7. 19°
4. 0.3090 8. 33°

WHAT'S MORE

A. 1. False 2. True 3. True 4. True 5.True

B. 1. $\frac{x}{\sin 36°} = \frac{10}{\sin 41°}$ 4. $\frac{37}{\sin x} = \frac{30}{\sin 30°}$

2. $\frac{5}{\sin x} = \frac{20}{\sin 125°}$ 5. $\frac{x}{\sin 100°} = \frac{25}{\sin 50°}$

3. $\frac{x}{\sin 95°} = \frac{5.5}{\sin 25°}$

C. 1. $x = 9$ 4. m$\angle X = 38.1°$
2. m$\angle X = 11.8°$ 5. $x = 32.1$
3. $x = 13$

D. a. $|AB| = 22.2$ b. $|AB| = 18$

E. a. $m\angle A = 36.4°$ b. $m\angle A = 38.7°$

WHAT I CAN DO

A.1. 7.9 4. 33.04
2. 35° 5. 18.7
3. 22.1

B. 6. m$\angle B = 35.8°$, m$\angle C = 71.2°, and\ c = 17.8\ cm$
7. m$\angle A = 36.4°$, m$\angle B = 116.6°, and\ b = 25.6\ cm.$
8. m$\angle B = 119°, a = 6.5\ cm, and\ c = 8.5\ cm.$
9. m$\angle A = 85°, a = 18.2\ cm, and\ c = 12.9\ cm.$

ASSESSMENT

A. 1. A　　4. D
2. C　　5. B
3. A　　6. C

B.7. $m\angle C = 74°, b = 8.9\ cm, and\ c = 10.2\ cm$
8. $m\angle A = 43°,\ m\angle B = 27°, and\ b = 8.7\ cm$
9. $m\angle A = 36°, m\angle C = 95°, and\ c = 19.7\ cm$
10. $m\angle B = 48.7°, m\angle C = 26.3°, and\ c = 8.3\ cm$
11. Two Solutions
(1) $m\angle B = 27.1°, m\angle C = 132.9°, and\ c = 32.1\ cm$
(2) $m\angle B = 152.9°, m\angle C = 7.1°, and\ c = 5.4\ cm$
12. Two Solutions
(1) $m\angle B = 53.8°, m\angle C = 89.2°, and\ c = 39.9\ cm$
(2) $m\angle B = 126.2°, m\angle C = 16.8°, and\ c = 11.7\ cm$
13. No solution
14. No Solution
15. Two Solutions
(1) $m\angle B = 30.5°,\ m\angle C = 124.5°,\ c = 242.3\ cm$
(2) $m\angle B = 149.5°,\ m\angle C = 5.5°,\ c = 25.8\ cm$

Additional Activities

Challenge Problems: 1) $m\angle A = 39^0$　　2) $m\angle DGF = 132^0$

Problem-Based Learning Worksheet

9.24 miles – the distance of the fire from station A.

References:

A. Book

1. Gladys C. Nivera and Minie Rose C. Lapinid, *Grade 9 Mathematics, Patterns and Practicalities* (Makati City, Philippines: Salesiana BOOKS by Don Bosco Press, Inc., 2013).

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3. Orlando A. Oronce and Marilyn O. Mendoza, *Worktext in Mathematics, E-Math Advanced Algebra and Trigonometry* (Manila, Philippines: Rex Bookstore, 2013).

4. Regina Macarangal Tresvalles and Wilson Cordova, *Math Ideas and Applications Series Advanced Algebra, Trigonometry, and Statistics* (Quezon City, Philippines: Abiva Publishing House, Inc., 2010).

5. Merden L. Bryant, et. al., *MATHEMATICS, Learner's Material 9* (Pasig City, Philippines: Vibal Group, Inc., 2014).

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