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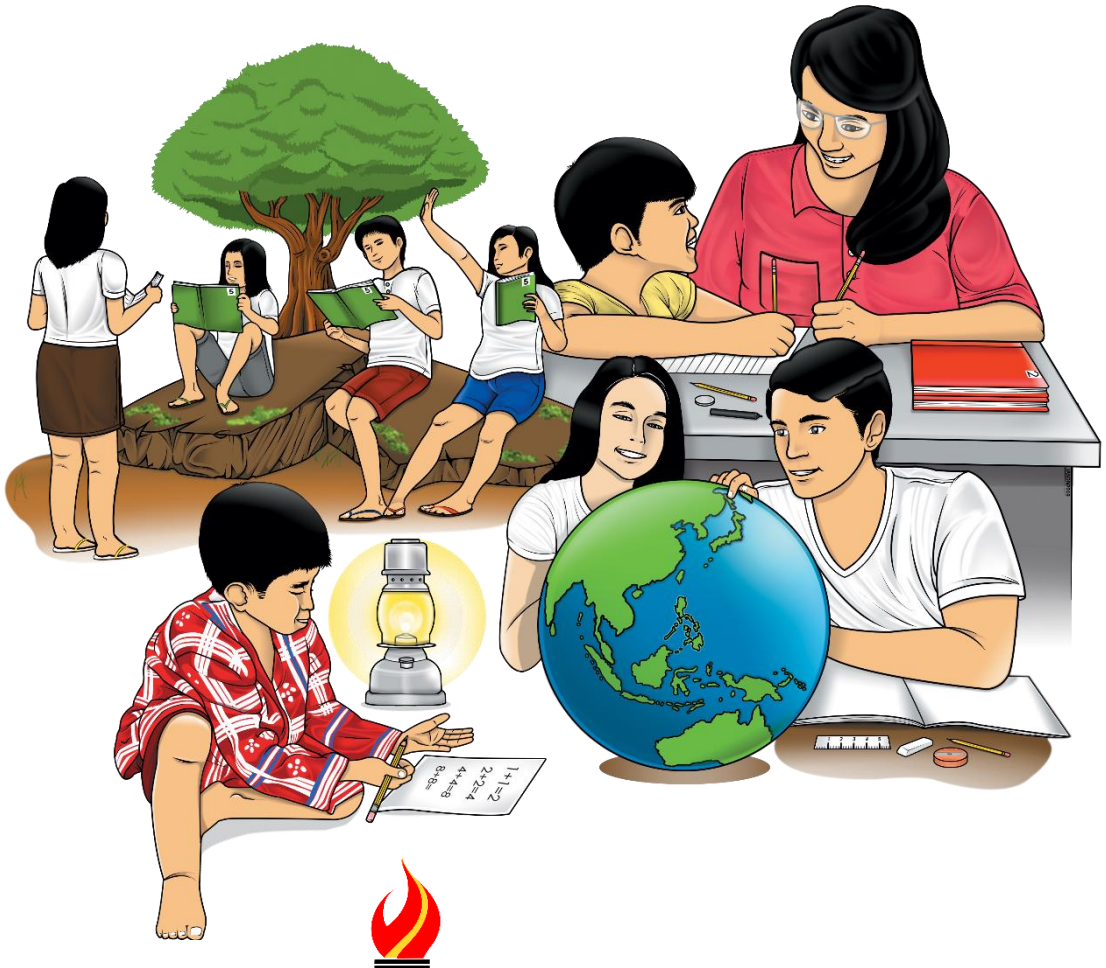
Mathematics

Quarter 4 – Module 4:
Angle of Elevation



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Mathematics

Quarter 4 – Module 4:
Angle of Elevation

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Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

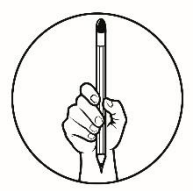
Thank you.

What I Need to Know

This module was designed and written with you in mind. It is here to help you master **Angle of Elevation.** The scope of this module permits it to be used in many different learning situations. The language used recognizes the diverse vocabulary level of students. The lessons are arranged to follow the standard sequence of the course. But the order in which you read them can be changed to correspond with the textbook you are now using.

After going through this module, you are expected to:

- solve problems involving angles of elevation **(M9GE-IVd-45.1)**



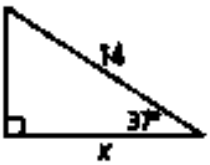
What I Know

DIRECTIONS: Answer each of the following items and write the letter of the correct answer on your answer sheet.

1) SOH-CAH-TOA is the mnemonic used in solving the solutions of _____ using the trigonometric functions.

A. Obtuse Triangle
B. Acute Triangle
C. Right Triangle
D. All the above

2) Using the figure at the right, which of the following equations is correct to find the value of x?

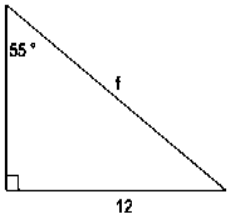


A. $\sin 37^0 = \frac{x}{14}$
B. $\tan 37^0 = \frac{x}{14}$
C. $\cos 37^0 = \frac{x}{14}$
D. $\sin 37^0 = \frac{14}{x}$

3) It is an acute angle formed by the eye level (horizontal line) and the line of sight when the observer looks at an object that is located higher than him.

A. Angle of Depression
B. Angle of Elevation
C. Inclination
D. Reference angle

4) Using the figure at the right, which of the following ratios is equal to the tangent of the given angle of the right triangle?

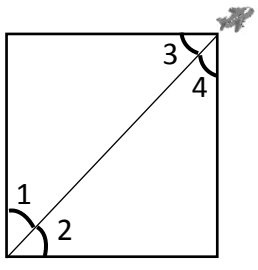


A. $\frac{f}{12}$
B. $\frac{12}{f}$
C. $\frac{12}{a}$
D. $\frac{a}{12}$

5) Using the figure in item 4, which of the following is correct to find the value of f?

A. $\cos 55^0 = \frac{12}{f}$
B. $\sin 55^0 = \frac{f}{12}$
C. $\cos 55^0 = \frac{f}{12}$
D. $\sin 55^0 = \frac{12}{f}$

6) Using the figure at the right. Which is the angle of elevation of the plane?



A. $\angle 1$
B. $\angle 2$
C. $\angle 3$
D. $\angle 4$

For items 7 – 8, A car traveled 100 feet up a ramp to a bridge. The angle of elevation of the ramp was 10°.

7) How high was the bridge above road level?
   A. 17.36 ft. B. 17.63 ft. C. 98.5 ft. D. 100 ft.

8) How far is the foot of the ramp to a point on the level road that is directly below the other end of the ramp?
   A. 98.5 ft. B. 95.8 ft. C. 17.36 ft. D. 10 ft.

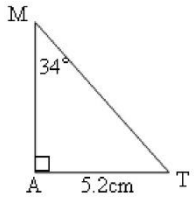
9) What is the measure of $\angle X$ if $\sin X = \frac{4}{9}$?
   A. 7° B. 26° C. 64° D. 83°

10) Which of the following is equal to $\sin 64°$?
   A. cos 64° B. cos 26° C. tan 64° D. tan 26°

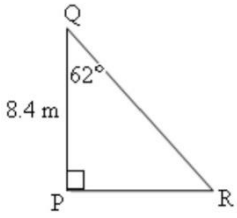
11) A student sees a bird on top of a 12-m high pole. The student is standing 20 m away from the foot of the pole. At what angle must the student incline her camera to take a picture of the bird?
   A. 31° B. 37° C. 53° D. 73°

12) An airplane approches an airport. At a certain time, the plane is 1,020 m high and its angle of elevation measured from the aiport is 20.5°. How far is the plane from the aiport?
   A. 1,089 m B. 2,728 m C. 2,913 m D. 3,000 m

13) Using the ΔMAT at the right, what is the length of $\overline{MA}$?
   A. 2.9 cm C. 4.3 cm
   B. 3.5 cm D. 7.7 cm



14) What is the area of ΔQPR at the right?
   A. 15.8 m^2 C. 66.4 m^2
   B. 33.2 m^2 D. 132.7 m^2



15) You are standing 25 m from the foot of a tree and you observe that the angle of elevation of the top of the tree is 48 degrees. How tall is the tree?
   A. 27.8 m B. 22.5 m C. 18.6 m D. 16.7 m

Lesson 1 Angle of Elevation

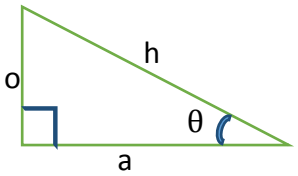
In this lesson you will study how to illustrate and solve problems involving angle of elevation. You will be given an opportunity to do activities that will help you identify and define angle of elevation through illustrations. Also, you will learn that in solving problems you need to draw a detailed diagram to help you visualize them. Those concepts that you have just learned about right triangles and trigonometric functions will help you to get through the next lesson.

What's In

Sine, Cosine and Tangent are the three main functions in trigonometry. They are often shortened as sin, cos, and tan. The calculation is simply the length of one side of a right-angled triangle divided by the length of another side, we just have to know which sides to use, and that is where "SOH – CAH – TOA" helps.

For a right triangle with an angle θ, the functions are calculated this way.

Sine	**SOH**	**S**in $\theta = \frac{Opposite}{Hypotenuse} = \frac{o}{h}$
Cosine	**CAH**	**C**os $\theta = \frac{Adjacent}{Hypotenuse} = \frac{a}{h}$
Tangent	**TOA**	**T**an $\theta = \frac{Opposite}{Adjacent} = \frac{o}{a}$



The other trigonometric functions of angle θ which are the reciprocals of sine, cosine, and tangent can be correspondingly and mnemonically written as "CHO-SHA-CAO".

Cosecant	**CHO**	**C**sc $\theta = \frac{Hypotenuse}{Opposite} = \frac{h}{o}$
Secant	**SHA**	**S**ec $\theta = \frac{Hypotenuse}{Adjacent} = \frac{h}{a}$
Cotangent	**CAO**	**C**ot $\theta = \frac{Adjacent}{Opposite} = \frac{a}{o}$

TRY IT!

Use figures 1 and 2 below, match each trigonometric function of the indicated angle in ***Column A*** with the correct ratio in ***Column B***. Write only the letter of the correct answer on your answer sheet.

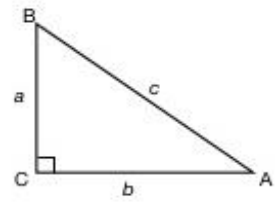


Fig. 1

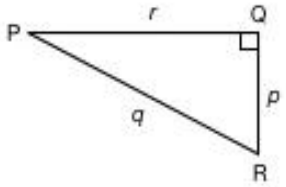


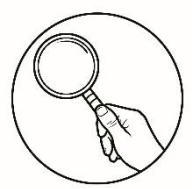
Fig. 2

Column A	**Column B**
1) tan A	A. $\frac{b}{c}$
2) cos A	B. $\frac{p}{q}$
3) sin P	C. $\frac{q}{r}$
4) cot P	D. $\frac{r}{p}$
5) sec P	E. $\frac{a}{b}$
	F. $\frac{c}{b}$
	G. $\frac{p}{r}$

What's New

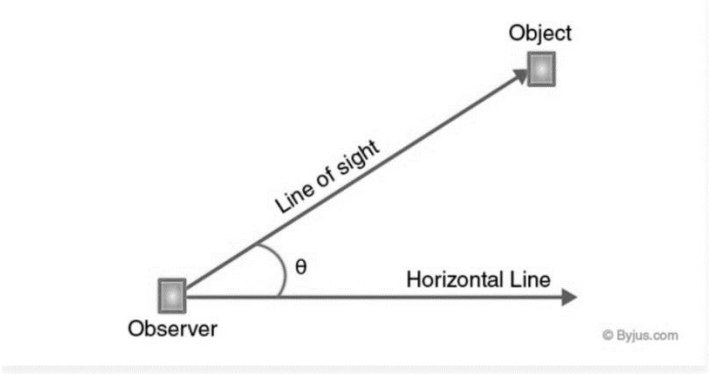
Read and analyze the selection below.

Suppose that an airplane takes off from a ruway of an airport and flies along a straight path to reach a certain height, does the path of the plane make an angle with the runway? Is it possible to find directly the distance from the point where the plane takes off to the endpoint of the runway when the plane is directly above it using a ruler? To answer these questions, there is a need to understand first the term Angle of Elevation. It is a widely used concept related to height and distance. The trigonometric functions as you have learned in the previous lesson will help you answer these questions. Perform the succeeding activities to apply these concepts in solving real-life problems.



What is It

The **Angle of Elevation** of an object as seen by an observer is the acute angle formed by the horizontal line (eye level) and the line from the observer's eye to the object (line of sight) that is located higher than the observer.



In the above figure, you can see an observer on the ground who is looking at the object located higher than him, an angle θ is formed by his line of sight and horizontal line which is his eye level. The angle is called angle of elevation of the object. Here, if we connect the object to the point directly below it along the horizontal line, a right triangle is formed. Thus we can use a trigonometic function to find the distance of the observer from the foot of the vertical line. The line segment representing the height of the object is the side opposite the angle θ and the horizontal line is the adjacent side to the same angle θ.

The trigonometic function to use in finding the angle of elevation of an object depends on the given measures of the sides of the formed right triangle. There is a need for you to recall the ratio of the lengths of the sides that defines each trigonometric function with referece to an acute angle of the right triangle. The mnemonic SOH-CAH-TOA-CAO-SHA-CHO may be of great help to you in identifying the appropriate trigonometric function based on the given measures of the right triangle.

You may consider the following as reminders in determining the function to use.

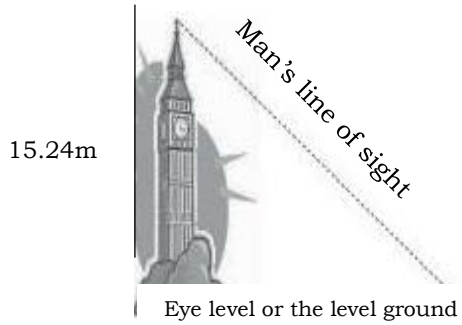
1. If the measures of either leg and the hypoteuse of the right triangle are given, then sine or cosine function may be used.

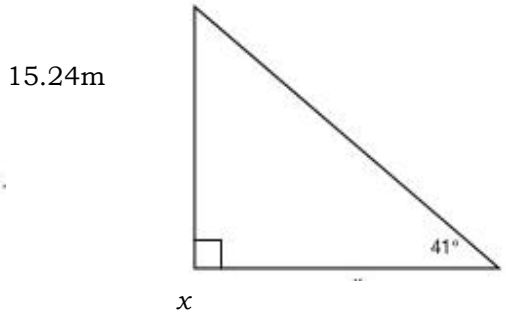
2. If the measures of the legs of the right triangle are given, then tangent function is to be used.

Let us have some situational problems that involve angle of elevation. Proper understanding and interpretation to the problem is necessary for you to come up with proper illustration. Usually, limited understanding and wrong interpretation to the problem lead to incorrect illustration which gives you lower chance to answer and solve the give problem correctly.

EXAMPLE 1

A tower is 15.24 m high. At a certain distance away from the tower, a man on the level ground observes that the angle of elevation of the top of the tower is 41°. How far is the man from the tower?





Solution:

Let *x* be the distance of the man from the tower (adjacent side to $\theta = 41^0$)

$$\tan \theta = \frac{opposite\ side}{adjacet\ side}$$

$$\tan 41^0 = \frac{15.24}{x}$$

$$0.87\ (x) = 15.24$$

$$x = \frac{15.24}{0.87}$$

$$x = 17.52 \text{ m}$$

Since the given 15.24 m is the opposite side of the 41° angle and the unknown side (x) is the adjacent side of the given angle, then we will use tangent ratio.

tan | 41 = 0.87

17.52 meters *is the distance of the man from the tower.*

EXAMPLE 2

Suppose that when the angle of elevation of the sun is 63.4°, a building casts a shadow of 37.5 feet. How tall is the building?

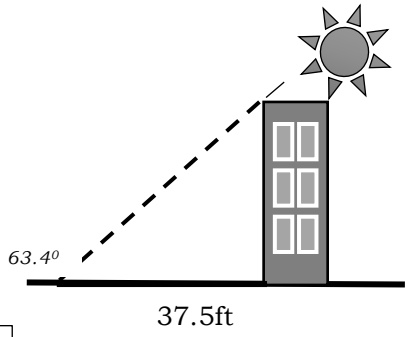
Solution: Let *y* be the height of the building (opposite side to $\theta = 63.4^0$)

$$\tan\theta = \frac{opposite\ side}{adjacent\ side}$$

$$\tan 63.4° = \frac{y}{37.5}$$

$$y = 1.997\ (37.5)$$

$$y = 74.89$$



Therefore, 74.89 ft. *is the height of the building.*

EXAMPLE 3

A passenger on a ship, observes a 1,800-m high lighthouse that stands on a hill which is 3,700 meters away from the ship. Find the angle of elevation of the top of the lighthouse from the ship.

Solution:

Let θ be the angle of elevation of the top of the lighthouse from the ship.

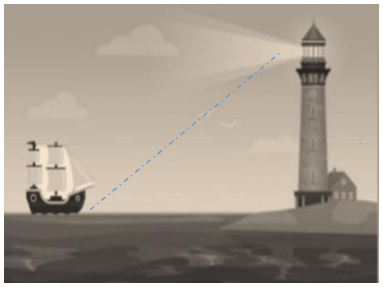
$$\tan\theta = \frac{opposite\ side}{adjacent\ side}$$

$$\tan\theta = \frac{1{,}800}{3{,}700}$$

$$\tan\theta = 0.48\overline{648}$$

$$\theta = \tan^{-1}(0.48\overline{648})$$

$$\theta = 25.94°$$

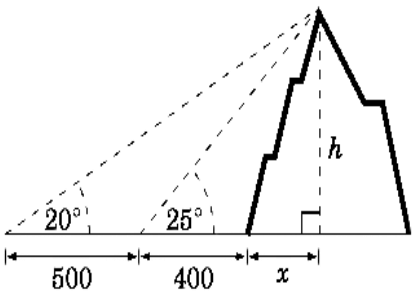


shift | tan | 0.48 | = 26.10 °

θ = 25.94° *is the angle of elevation of the top of the lighthouse from the ship.*

EXAMPLE 4

A person standing 400 ft away from the base of a mountain measures the angle of elevation of the top of the mountain to be 25˚ He then walks 500 ft straight farther and finds the angle of elevation of the top of the mountain to be 20˚. How tall is the mountain?



Solution:

We will assume that the ground is flat and not inclined relative to the base of the mountain. Let h be the height of the mountain, and let x be the distance from the base of the mountain to the point directly beneath the top of the mountain, as shown in the figure at the right. Then we see that:

$$\tan 25^o = \frac{h}{x+400} \rightarrow h = \tan 25^0\ (x + 400) \text{ (eq. 1)}$$

$$\tan 20^o = \frac{h}{x+400+500} \rightarrow h = \tan 20^0\ (x + 400 + 500) \text{ (eq. 2)}$$

Since ***h*** is common to the two triangles, then $h = h$.

$$\tan 25^0\ (x + 400) = \tan 20^0\ (x + 400 + 500)$$

Use this equation to solve for the value of x.

$$\tan 25^0\ (x + 400) = \tan 20^0\ (x + 400 + 500)$$
$$0.466(x + 400) = 0.364(x + 900)$$
$$0.466x + 186.52 = 0.364x + 327.57$$
$$0.466x - 0.364x = 327.57 - 186.52$$
$$0.102x = 141.05$$
$$x = 1{,}382.84$$

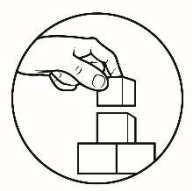
Finally, substitute x by 1,382.84 ***in either equation for h to get the height of the mountain:***

Eq 1: $h = \tan 25^\circ\ (1{,}382.84 + 400) = 0.466(1{,}782.84) = 830.8$

Eq 2: $h = \tan 20^0\ (1{,}382.84 + 400 + 500) = 0.364(2{,}282.84) = 830.95$

Both obtained values of h are rounded off to 831 ft.

Therefore, 831 ft. is the height of the mountain.

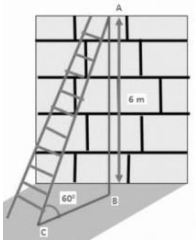


What's More

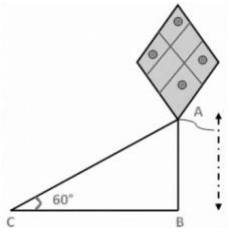
A. In a separate sheet, solve each problem involving angle of elevation.

Show your computations and label your final answer properly.

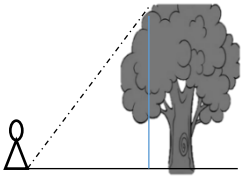
1) A ladder leans against a 6-m high wall and reaches the top. If the ladder is inclined with the ground at an angle of 60 degrees, how far is the foot ladder from the wall?



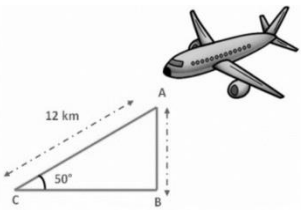
2) A string of a kite is 100 meters long and it makes of an angle of 60° with the horizontal. Find the height of a kite if there is no slack in the string?



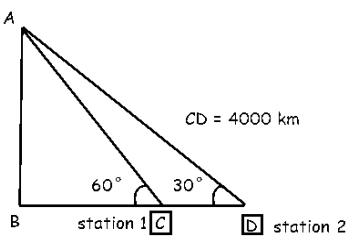
3) Grace wants to measure the height of a tree. She walks exactly 125 feet away from the foot of the tree and looks back at the top of the tree. She finds that the angle of elevation of the top of the tree as 33°. How tall is the tree?



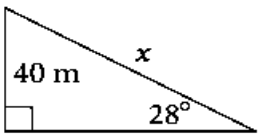
4) An airplane is observed to be approaching the airport. What is the height of the plane if it is 12 km away from the point of observation and its angle of elevation is 50 degrees?



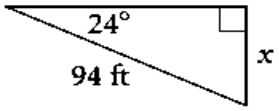
5) The angles of elevation of an artificial satellite measured from two earth stations, situated on the same side of the satellite, are found to be 30° and 60°. The two earth stations and the satellite are in the same vertical plane. If the distance between the earth stations is 4000 km, find the distance of the satellite from the earth.



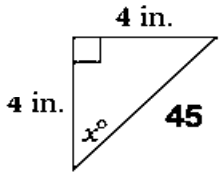
B. Find the value of x. Round off the answers to the nearest tenths.

1. 

85.2 m

2. 

38.2 ft

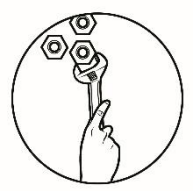
3. 

What I Have Learned

Angle of elevation is an acute angle formed by the eye level (horizontal line) and the line of sight of the observer when he looks at an object that is located higher than him.

Steps in solving math problems involving angle of elevation.

1) Draw an illustration that visualizes the problem and label it using the given information.
2) Formulate a ratio of a given measure and the variable representing what is being asked in the problem.
3) Form an equation using the formulated ratio and the trigonometric function defined by the ratio.
4) Solve the solution of the equation and the solution of the problem.
5) Label your final answer correctly.



What I Can Do

DIRECTIONS: Illustrate and solve the following problems. Use the diagram below.

	PROBLEM 1	PROBLEM 2	PROBLEM 3
Draw an appropriate illustration. ⇨			
What is/are given? What is asked? ⇨			
Formulate the equation. ⇨			
Solve ⇨			

Word Problems:

1) From a boat on the sea, the angle of elevation of the top of a 92-meter cliff is 12°. How far (meter) is the boat from the foot of the cliff?

2) A hiker is 400 meters away from the base of a radio tower. The angle of elevation of the top of the tower is 46°. How high (meter) is the tower?

3) A 12-meter flagpole casts a 19-meter shadow. Find the angle of elevation of the sun.

Questions:

1) How did you find the activity?
2) How did you Illustrate the situation presented in each problem?
3) What mathematical concept did you apply to have an accurate illustration?
4) How and why are these concepts important in your daily life?
5) Give one example of your experiences where these concepts are applicable.

Assessment

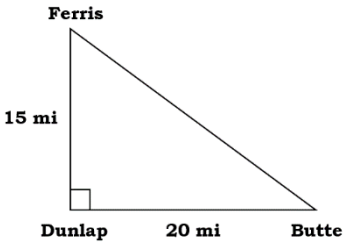
DIRECTIONS: Read and answer each of the following items accurately. Write the letter of the correct answer on your answer sheet.

1) Cos P of the right triangle PQR with right angle at Q is _____.
   A. $\frac{|QR|}{|PQ|}$ B. $\frac{|QR|}{|PR|}$ C. $\frac{|PQ|}{|PR|}$ D. $\frac{|PR|}{|PQ|}$

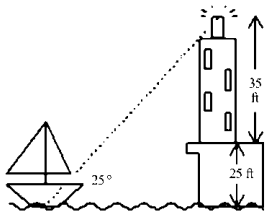
2) Consider a right triangle PQR with right angle at Q, if |PQ|=27 cm and |QR|=17 cm, then the measure of angle P is _____.
   A. 32.19˚ B. 45.19˚ C. 49.58˚ D. 62.46˚

3) Consider a right triangle ABC with right angle at B, if |AB|=5 cm, |AC|=13 cm, and |BC|=12 cm, what is the value sin A?
   A. $\frac{5}{12}$ B. $\frac{12}{13}$ C. $\frac{5}{13}$ D. $\frac{13}{12}$

4) Wayne used the diagram at the right to compute the distance from Ferris to Butte. How much shorter is the distance from Ferris to Dunlap than the distance Wayne computed?
   A. 10 mi C. 25 mi
   B. 20 mi D. 35mi

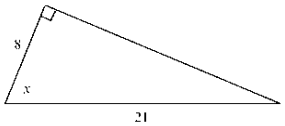


5) The line of sight from a small boat to the light at the top of a 35-foot lighthouse built on a cliff 25 feet above the water makes a 25° angle with the water. To the nearest foot, how far is the boat from the cliff?
   A. 141 ft. C. 75 ft.
   B. 129 ft. D. 27 ft.

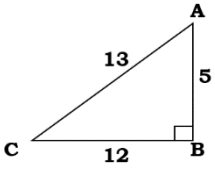


6) The corporate office building is 50 m high and angle of elevation of the top of the building is 52° when seen from a point A on the level ground. What is the distance from point A to the foot of the building?
   A. 30.78 m B. 39.06 m C. 63.45 m D. 81.21 m

7) Find the value of x in the right triangle at the right.
   A. 20.85˚ C. 67.6˚
   B. 22.39˚ D. 69.15˚



8) Given at the right is ΔABC, what $\overline{BC}$ in relation to ∠C?
   A. Adjacent side C. Hypotenuse
   B. Altitude D. Opposite side



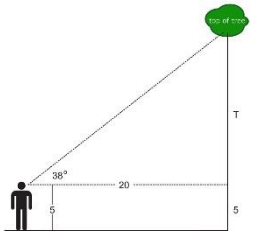
For numbers 9-10, consider the problem that follows:

A firefighter's ladder leans against a building. The ladder is 10 m long and makes an angle of 37˚ with the ground.

9) How high up the building does the ladder reach?
   A. 6 m B. 7.5 m C. 8 m D. 16.6 m

10) How far is the foot of the ladder from the base of the building?
   A. 6 m B. 7.5 m C. 8 m D. 16.6 m

11) A 5-footer man standing 20 feet away from the tree, finds the angle of elevation of the top of the tree to be 38°. How tall is the tree?
   A. 15.63 ft. C. 25.63 ft.
   B. 20.63 ft. D. 30.63 ft.



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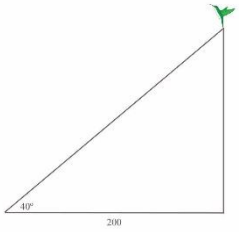
12) A man, sitting on the ground, finds the angle of elevation of a bird in the sky to be 40°. The shadow of the bird is directly below it, and 200 feet away from the man on the ground. How high is the bird in the sky?
   A. 128.56 ft. C. 167.82 ft.
   B. 153.21 ft. D. 238.35 ft.



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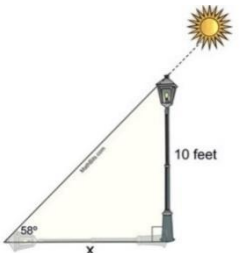
13) Find the length of the shadow cast by a 10-foot lamp post when the angle of elevation of the sun is 58°.
   A. 6.25 ft. C. 16 ft
   B. 11.8 ft. D. 18.87 ft



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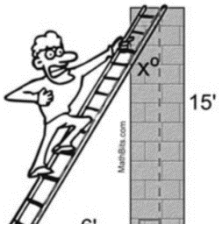
14) A ladder leans against a brick wall. The foot of the ladder is 6 feet away from the wall. The ladder reaches a height of 15 feet on the wall. Find the measure of the angle the ladder makes with the wall.
   A. 21.80° C. 66.42°
   B. 23.58° D. 68.20



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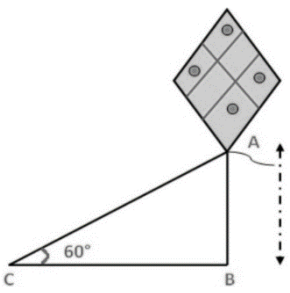
15) A string of a kite is 100 meters long and it makes an angle of 60° with horizontal. Find the height of the kite, assuming that there is no slack in the string.
   A. 84.6 m C. 88.6 m
   B. 86.6 m D. 90.6 m



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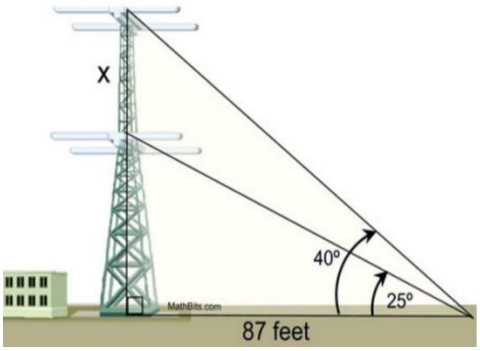
Additional Activities

Consider the given situation and answer the following questions below.

A radio station tower was built in two sections. From a point 87 feet from the base of the tower, the angle of elevation of the top of the first section is 25°, and the angle of elevation of the top of the second section is 40°.



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a. What is the height of the first section of the tower?
b. What is the height of the second section of the tower?
c. What is the total height of the tower?

Notice, in this problem, the trigonometric functions could not work directly on the side labeled "x" because that side is not a side of a right triangle.

PROBLEM – BASED LEARNING WORKSHEET

A 6-ft man stands 12 ft. away from the foot of a tree. The angle of elevation of the top of the tree from his eyes is 76°.

1) Describe how ∠1 relates to the situation.
2) About how tall is the tree?
3) If the man releases a pigeon that flies directly to the top of the tree, about how far will it fly?

Answer Key

WHAT I KNOW

1. C	6. B	11. A
2. C	7. A	12. C
3. B	8. A	13. D
4. C	9. B	14. C
5. D	10. B	15. A

WHAT'S IN

1. E
2. A
3. B
4. D
5. C

WHAT'S MORE

A.
1. 3.464 m
2. 86.60 m
3. 81.18 ft
4. 9.192 km
5. 3464 km

B.
6. 85.2 m
7. 38.2 ft.
8. 45^0

ASSESSMENT

1. C	6. B	11. B
2. A	7. C	12. C
3. B	8. A	13. A
4. A	9. A	14. A
5. B	10. C	15. B

ADDITIONAL ACTIVITIES

A. 40.57 ft.
B. 32.43 ft.
C. 73 ft.

PROBLEM - BASED LEARNING WORKSHEET

1. Angle of elevation of the top of the tree
2. 54.13 ft.
3. 49.60 ft.

References

Gladys C. Nivera, Minie Rose C. Lapinid (2013) *Grade 9 Mathematics Patterns and Practicalities,* Salesiana BOOKS by Don Bosco Press, Inc. Chino Roces Avenues, Makati City, Philippines

Orlando A. Oronce, Marilyn O. Mendoza (2010) *Work text in Mathematics e-math Advanced Algebra and Trigonometry,* Rex Book Store 856 Nicanor Reyes, Sr. Street 1977 C.M. Recto Avenue, Manila, Philippines

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https://www.onlinemathlearning,com/angle-of-elevation.html

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