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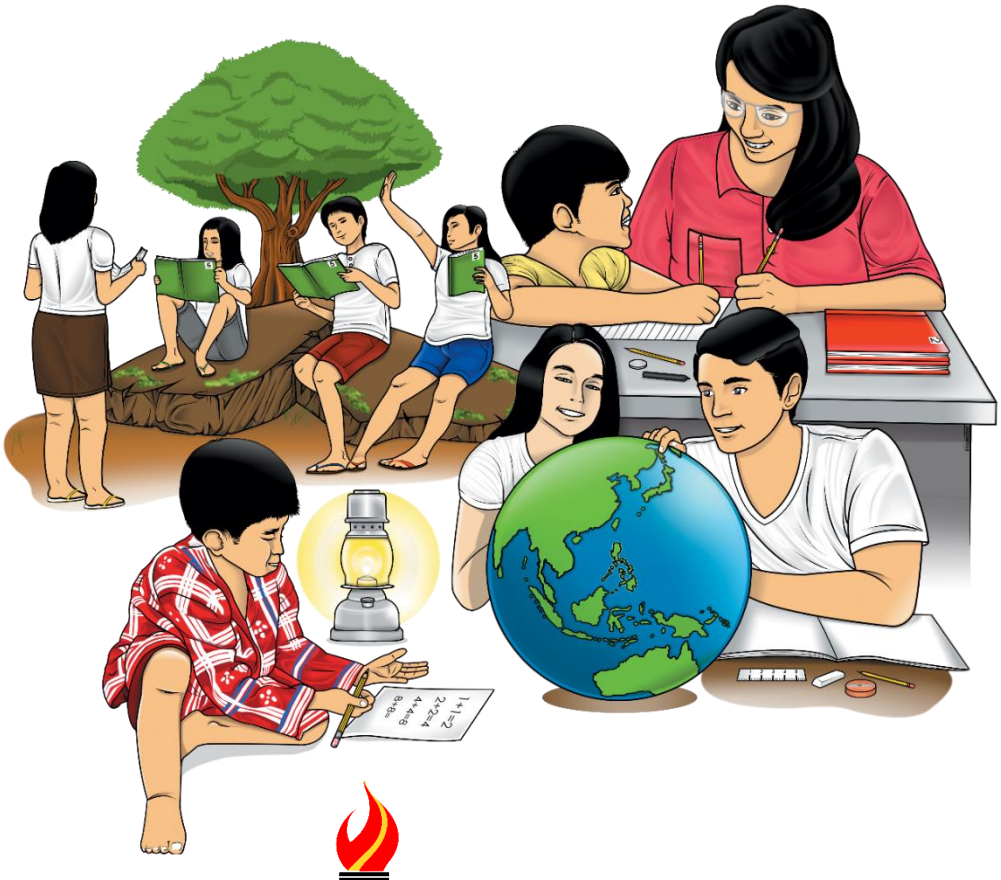
DEPARTMENT OF EDUCATION

Mathematics

Quarter 4 – Module 4

Proving Properties of Parallel Lines Cut by a Transversal



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Mathematics

Quarter 4 – Module 4

Proving Properties of Parallel Lines Cut by a Transversal

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Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

What I Need to Know

This module was designed and written with you in mind. It will help you to learn the properties of parallel lines. This guides you in discovering the postulates and theorems of parallel lines. Moreover, it introduces the idea of angle pairs when two parallel lines are cut by a transversal line. The lessons are arranged to follow the standard sequence of the course. But the order in which you read them can be changed to correspond with the textbook you are now using.

This module contains lesson on proving properties of parallel lines cut by a transversal (M8GE-IVd-1).

After going through this module, you are expected to:

1. identify the different angle pairs if parallel lines are cut by a transversal,
2. determine the properties of parallel lines when cut by a transversal,
3. find the measures of angles using the properties of parallel lines cut by a transversal,
4. prove properties of parallel lines that are cut by a transversal, and
5. relate real-life problems involving parallel lines cut by a transversal.

What I Know

Directions: **Answer each of the following items.** Write the letter of the correct answer on a separate sheet of paper.

1. What can be said about the alternate interior angles if two parallel lines are cut by a transversal?
   A. complementary
   B. congruent
   C. parallel
   D. perpendicular

2. What do you call those two coplanar lines that do not intersect?
   A. congruent
   B. parallel
   C. perpendicular
   D. supplementary

For items 3 – 4, refer to Figure 1 below. $e \parallel f$ and d is a transversal line.

3. Which of the following are corresponding angles?
   A. $\angle 1$ *and* $\angle 8$, $\angle 2$ *and* $\angle 3$
   B. $\angle 2$ *and* $\angle 7$, $\angle 1$ *and* $\angle 6$
   C. $\angle 4$ *and* $\angle 8$, $\angle 1$ *and* $\angle 8$
   D. $\angle 5$ *and* $\angle 3$, $\angle 2$ *and* $\angle 8$

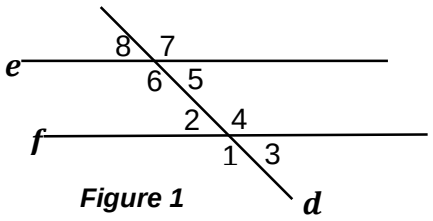
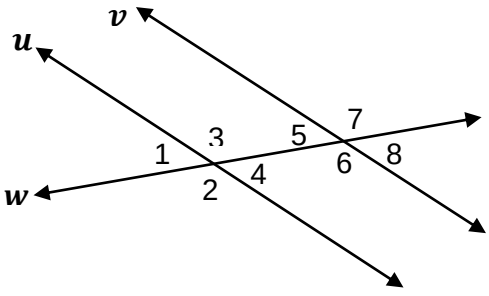


Figure 1

4. Which of the following statements is **NOT** correct?
   A. $\angle 2$ and $\angle 3$ are congruent.
   B. $\angle 3$ and $\angle 8$ form a linear pair.
   C. $\angle 4$ and $\angle 7$ are congruent angles.
   D. $\angle 5$ and $\angle 6$ are supplementary angles.

For items 5 – 9, refer to the figure at the right.

Given: $u \parallel v$ **and** w **is a transversal line.**



5. Which of the following could be the reason to prove that $\angle 1 \cong \angle 4$?
   A. Vertical angles are congruent.
   B. Corresponding angles are congruent.
   C. Supplementary angles are congruent.
   D. Alternate interior angles are congruent.

6. Which of the following could be the reason to prove that $\angle 2 \cong \angle 7$?
   A. Vertical angles are congruent.
   B. Corresponding angles are congruent.

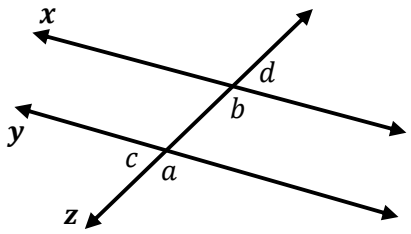
C. Alternate exterior angles are congruent.
D. Alternate interior angles are congruent.

7. Which of the following statements is justified by the reason, "same side interior angles are supplementary?"
   A. $m\angle 4 + m\angle 7 = 180^0$
   B. $m\angle 5 + m\angle 8 = 180^0$
   C. $m\angle 4 + m\angle 6 = 180^0$
   D. $m\angle 2 + m\angle 7 = 180^0$

8. What would be the reason to prove that $\angle 3 \cong \angle 7$?
   A. Vertical angles are congruent.
   B. Corresponding angles are congruent.
   C. Alternate interior angles are congruent.
   D. Alternate exterior angles are congruent.

9. In a two-column proof, "vertical angles are congruent" is written under the reason column. Which of the following is appropriate to write under the statement column?
   A. $\angle 5 \cong \angle 8$
   B. $\angle 4 \cong \angle 3$
   C. $\angle 1 \cong \angle 8$
   D. $\angle 3 \cong \angle 6$

For items 10 – 11, refer to the figure at the right.



Given: $x \parallel y$ **and** z **is a transversal line.**

10. If $m\angle a = 99^0$, what is the measure of $\angle d$?
    A. 45^0
    B. 81^0
    C. 90^0
    D. 99^0

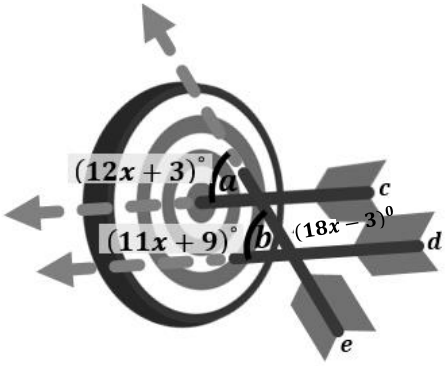
11. Rona was asked by her teacher to find the value of y given that $m\angle a = 6y$ and $m\angle c = 13y - 10^0$. She wrote her computation in this way:

$$(6y) + (13y - 10^0) = 180^0$$
$$19y - 10^0 + 10^0 = 180^0 + 10^0$$
$$19y = 190^0$$
$$y = 10^0$$

Did she arrive at the correct answer?
A. Yes. Common variables must be combined.
B. No. The equation must be $(6y) = (13y - 10)$.
C. No. Multiplying both sides of the equation by $\frac{1}{19}$ is not equal to $y = 10$.
D. Yes. The equation she formulated is correct since $\angle a$ and $\angle c$ are supplementary angles.

For items 12 – 13, please refer to the situation below and the figure at the right.

Peter, James, and John were practicing archery. One day, they decided to play the target with their arrow. The figure at the right shows the orientations of their arrows. Arrow c is Peter's arrow, arrow d is James', and arrow e is John's. Peter's and James' arrows are parallel to each other while John's cut the other two. The angle formed by Peter's and John's arrows denoted by $\angle \boldsymbol{a}$ is $12x + 3^0$, James' and John's denoted by $\angle \boldsymbol{b}$ is $11x + 9^0$, and the other angle formed by James' and John's arrows is $18x - 3°$.



12. Which of the following shows the mathematical equation of the measures of corresponding angles?
    A. $12x + 3^0 = 11x + 9^0$
    B. $12x + 3^0 + 11x + 9^0 = 180$
    C. $12x + 3^0 + 18x - 3° = 180$
    D. $12x + 3^0 = 18x - 3°$

13. A Grade 8 student wanted to find the value of x in the situation. His computation is as follows:

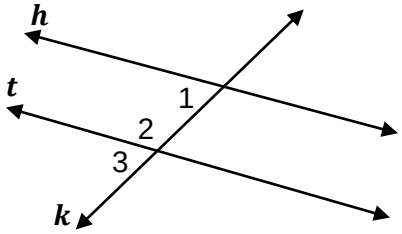
$$(12x + 3°) + (11x + 9°) = 180°$$
$$(12x + 11x) + (3° + 9°) = 180°$$
$$23x + 12° - 12° = 180° - 12°$$
$$23x = 168°$$
$$x = 7.30°$$

Is his computation correct?

A. Yes. Common terms must be combined.
B. Yes. The value $x = 7.30°$ gives the measure of $\angle \boldsymbol{a}$ as 90.6°.
C. No. The equation should be $(12x + 3°) + (18x - 3°) = 180°$, since they are alternate angles.
D. No. The equation should be $(12x + 3°) = (11x + 9°)$, since they are corresponding angles.

For items 14 – 15, complete the proof presented in the table below by choosing the answer from any of the following statements or reasons:

A. $m\angle 1 \cong m\angle 3$
B. Transitive Property of Congruence
C. Substitution Property of Equality
D. $m\angle 2 + m\angle 3 = 180^0$



In the figure above, $\boldsymbol{h} \parallel \boldsymbol{t}$ and $\boldsymbol{k}$ is a transversal of $\boldsymbol{h}$ and $\boldsymbol{t}$.

Prove that $m\angle 1 + m\angle 2 = 180^0$.

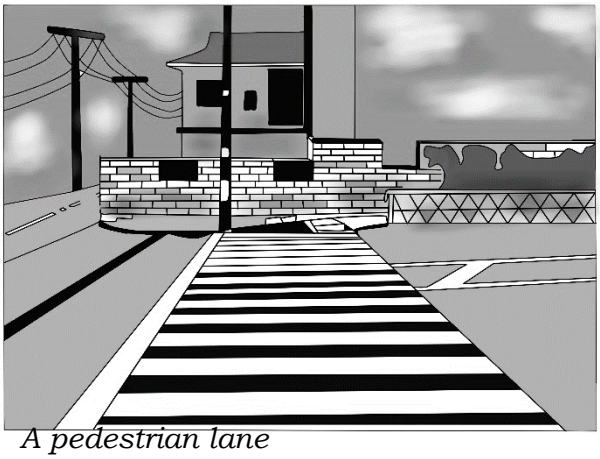
Proof:

Statements	**Reasons**
1. $\boldsymbol{h} \parallel \boldsymbol{t}$, $k \text{ is a transversal of } h \text{ and } t$	1. Given
2. $\angle 1 \cong \angle 3$	2. Parallel – Corresponding Angles Theorem
3. $m\angle 1 = m\angle 3$	3. Congruent angles have the same measure.
4. (14)	4. Linear Pair Theorem
5. $m\angle 1 + m\angle 2 = 180^0$.	5. (15)

Lesson 1 Proving Properties of Parallel Lines Cut by a Transversal

When crossing the street, have you noticed the lines used on pedestrian lanes? How about in designing ***vintas***? Imagine that you are in the bustling city and you have been standing in the middle of tall buildings and magnificent skyscrapers. Have you ever wondered how the architects, engineers and carpenters made these designs? How are parallel lines applied in these designs?

We know that parallel lines exist everywhere in our everyday lives like on the edges of a piece of paper and in the way shelves of books are positioned in a library. The concept of parallel lines is a key component of geometry that is applied in real life in designs and structures.



A pedestrian lane

A vinta

This lesson culminates with various activities such as solving real-life situations and proving properties of parallel lines.

What's In

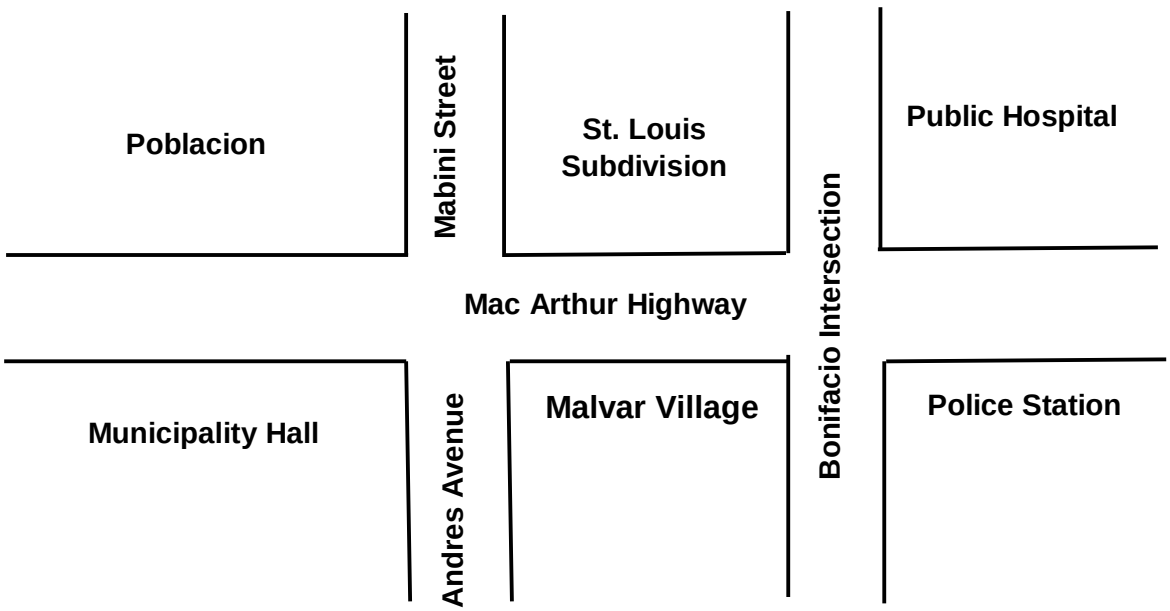
Activity: Arrange Me!

You have learned in the previous quarter the different properties and theorems of angles. This time, let us check your prior knowledge on the types of lines. This activity encourages you to remember the terms your learned from the previous lessons.

A. Directions: Arrange the jumbled letters to form a word.

1. Two lines that share a common point.
   G N T C I T E N R I E S
2. Two coplanar lines that do not intersect.
   A L L A P E R L
3. These are lines that lie on the same plane.
   C A P R O A L N
4. Two lines that intersect and form right angles.
   P R E P D E N L I U R A C
5. Two lines that are not coplanar and do not intersect.
   E W S K
6. A line that intersects two or more coplanar lines at two or more distinct points.
   R A N T S S V E L A R

B. Road Map! Below is the road map of the city of San Juaquin**.**



Questions:

1. Which roads intersect?
2. Which roads are parallel?
3. Which roads are perpendicular?

What's New

Let us now begin discovering the world of parallel lines, uncover its properties and how they play an important role in our daily lives. Read and do the tasks that follow.

Activity 1: Uncover Me!

Directions: Perform the simple activity and answer the questions that follow.

Materials needed:

- Broom Stick or Straws measuring 8 inches, 10 inches and 12 inches

Steps:

1. Get the sticks or straws measuring 8 and 10 inches.
2. Position the sticks or straws horizontally (8-inch stick above the 10-inch stick).
3. Place the remaining 12-inch stick or straw on top of the two horizontal sticks in slant position so that it intersects the two lines at two different points.

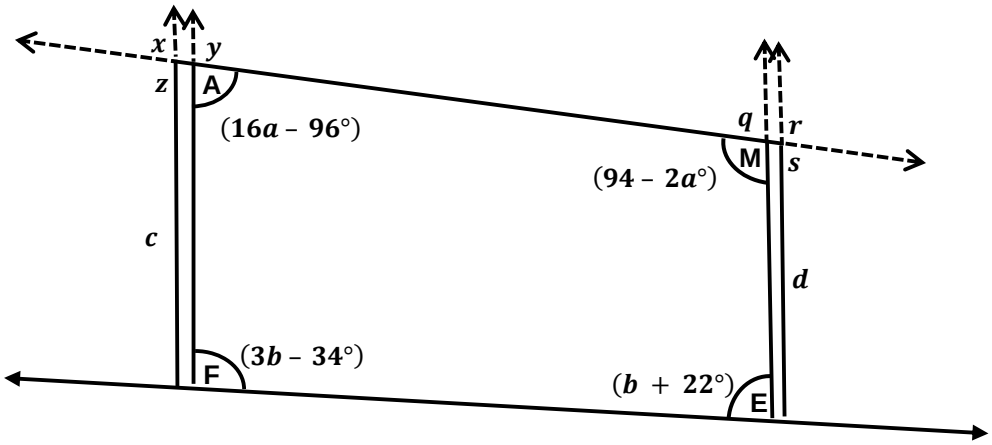
Questions:

1. How was the activity?
2. What was the activity about?
3. Did you find any difficulty/ies in performing the task given to you? Please write them.
4. How would you describe the lines formed?
5. Do these lines meet or intersect?
6. Are these lines parallel? Why or why not?
7. How many angles are formed after the transversal cut the parallel lines?

Activity 2: Try Me

Study the situation below and answer the questions that follow:

Mario is an owner of Molave Farm found in San Agustin, Surigao del Sur in which one of his offered rides to the public is a zip line. A zip line is a very strong cable between two points with a pulley attached to it. The zip line as illustrated in the figure below goes from a 25-foot tall tower, represented by ***c*** *to a 20-foot tower as* ***d*** *and it is 200 meters apart in a slightly inclined ground.*



Questions:

1. If the two posts which are represented by lines $\boldsymbol{c}$ and $\boldsymbol{d}$, respectively, are parallel, what do you call the strong cable which is represented by line $\boldsymbol{AM}$?
2. If lines $\boldsymbol{c}$ and $\boldsymbol{d}$ are extended to form other angles, name the following:
   a. two pairs of alternate-interior angles;
   b. two pairs of alternate-exterior angles;
   c. four pairs of corresponding angles;
   d. two pairs of same-side interior angles;
   e. two pairs of same-side exterior angles; and
   f. four pairs of vertical angles.
3. Do you think $\angle A$ and $\angle q$ are congruent angles?
4. What about $\angle M$ and $\angle r$, are they congruent?
5. What about $\angle x$ and $\angle s$, are they congruent?
6. Using the information in the figure above, find the measures of the following angles:
   a. $\angle A$ c. $\angle F$
   b. $\angle M$ d. $\angle E$

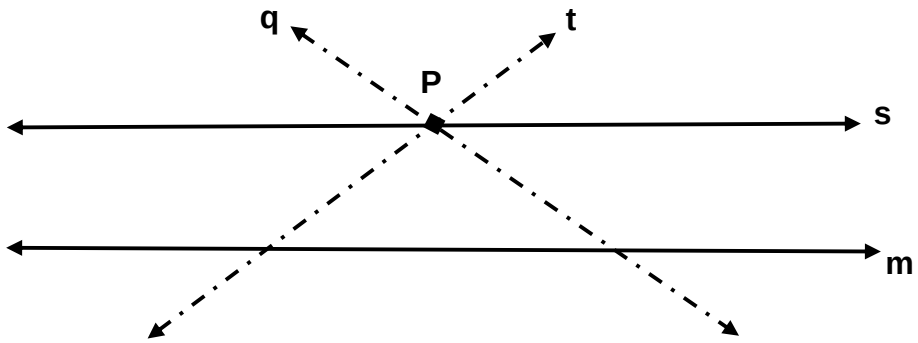
   (***Hint:*** To get the measure of those angles, first, find the values of $\boldsymbol{a}$ and $\boldsymbol{b}$, then use them in the equation).
7. If you add m$\angle A$ and m$\angle M$, what is their sum?
8. If you add m$\angle F$ and m$\angle E$, what is their sum?

What is It

The results of the activity above lead you to the following properties of parallel lines cut by a transversal, as well as theorems and postulates on parallel lines.

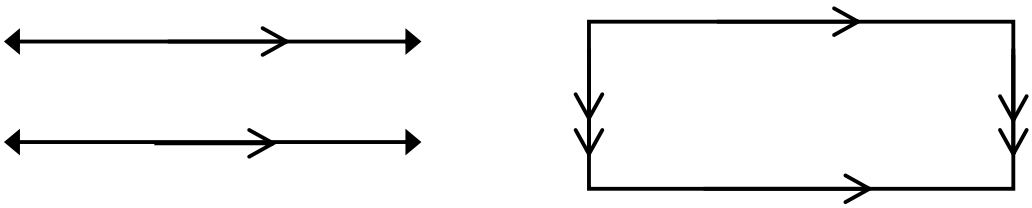
Properties, Theorems and Postulates on Parallel lines cut by a Transversal:

A. **Parallel Postulates**: *If there is a line and a point not on the line, then there is exactly one line through the point that is parallel to the given line.*



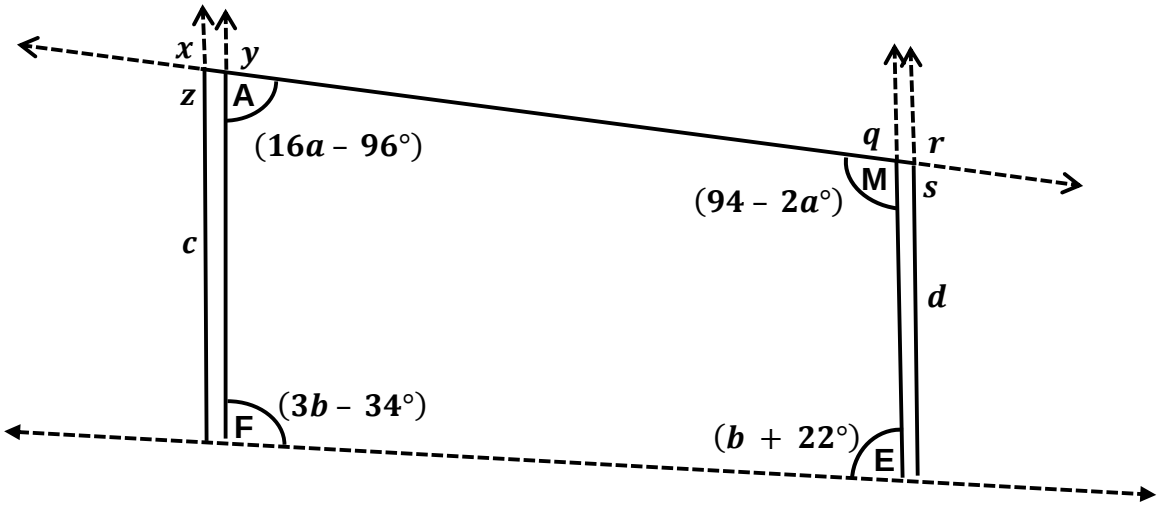
Given a line $\boldsymbol{m}$ and a point $\boldsymbol{P}$ not in $\boldsymbol{m}$, there is only one line ($\boldsymbol{line\ s}$) that is parallel to line $\boldsymbol{m}$ passing through point P. In symbol, $\boldsymbol{s} \parallel \boldsymbol{m}$ (read as, line $\boldsymbol{s}$ is parallel to line $\boldsymbol{m}$). Lines $\boldsymbol{t}$ and $\boldsymbol{q}$ are not parallel to line $\boldsymbol{m}$ through $\boldsymbol{P}$.

Also, two or more lines are parallel if you find the symbols like the ones illustrated below.



$\boldsymbol{s} \parallel \boldsymbol{m}$ is read as "line $\boldsymbol{s}$ is parallel to line $\boldsymbol{m}$." In geometry, the term parallel is denoted by $\parallel$ which means "is parallel to". Also, an arrow is used in diagrams to indicate that lines are parallel. Moreover, the term parallel is used for lines, segments, rays, and planes.

Going back to the situation about the zip line, the two posts where the cable is connected, represented by line $\boldsymbol{c}$ and line $\boldsymbol{d}$, are the two parallel lines ($\boldsymbol{c} \parallel \boldsymbol{d}$), and the cable which is represented by line $\boldsymbol{AM}$ is called the ***transversal.*** The imaginary line $\boldsymbol{FE}$ that is drawn with the ground is also considered transversal line since it cuts the two posts at distinct points.



A ***transversal*** is a line that intersects two or more coplanar lines at two or more distinct points.

B. **Angles Formed by Parallel Lines Cut by a Transversal**

Definition: Alternate Interior Angles – *are two nonadjacent interior angles on opposite sides of the transversal.*

Parallel-Alternate Interior Angle Postulate

If two parallel lines are cut by a transversal, then any pair of alternate interior angles are congruent.

In the figure above, the pairs of alternate interior angles are $\angle A$ & $\angle q$ and $\angle y$ & $\angle M$. By the postulate, $\angle A \cong \angle q$ and $\angle y \cong \angle M$. Since they are congruent, their measures are equal ($m\angle A = m\angle q$ and $m\angle y = m\angle M$).

Definition: Alternate Exterior Angles – *are two nonadjacent exterior angles on opposite sides of a transversal.*

Parallel-Alternate Exterior Angle Theorem

If two parallel lines are cut by a transversal, then any pair of alternate exterior angles are congruent.

In the same figure, the pairs of alternate exterior angles are $\angle z$ & $\angle r$ and $\angle x$ & $\angle s$. By the theorem, $\angle z \cong \angle r$ and $\angle x \cong \angle s$. Since they are congruent, their measures are equal ($m\angle z = m\angle r$ and $m\angle x = m\angle s$).

Definition: Corresponding Angles – *are two nonadjacent angles, one interior, and one exterior on the same side of the transversal.*

Parallel-Corresponding Angles Theorem

If two parallel lines are cut by a transversal, then the corresponding angles are congruent.

In the same figure, the four pairs of corresponding angles are $\angle z$ & $\angle M$, $\angle A$ & $\angle s$, $\angle x$ & $\angle q$, and $\angle y$ & $\angle r$. By the theorem, $\angle z \cong \angle M$, $\angle A \cong \angle s$, $\angle x \cong \angle q$, and $\angle y \cong \angle r$. Since they are congruent, their measures are equal ($m\angle z = m\angle M$, $m\angle A = m\angle S$, $m\angle x = m\angle q$, and $m\angle y = m\angle r$).

Definition: Vertical Angles – *Two angles are vertical angles if and only if they are nonadjacent angles formed by two intersecting lines.*

Vertical Angle Theorem

Vertical angles are congruent.

In the same figure, the four pairs of vertical angles are $\angle x$ & $\angle A$, $\angle z$ & $\angle y$, $\angle q$ & $\angle s$, and $\angle M$ & $\angle r$. By the theorem, $\angle x \cong \angle A$, $\angle z \cong \angle y$, $\angle q \cong \angle s$, and $\angle M \cong \angle r$. Since they are congruent, their measures are equal ($m\angle x = m\angle A$, $m\angle z = m\angle y$, $m\angle q = m\angle s$, and $m\angle M = m\angle r$).

Parallel-Interior Angle-Same Side Theorem

If two parallel lines are cut by a transversal, then the interior angles on the same side of the transversal are supplementary.

In the same figure, the pairs of interior angles on the same side of the transversal are $\angle A$ & $\angle M$ and $\angle y$ & $\angle q$. Along the ground, the pair of interior angles on the same side of the transversal is $\angle F$ & $\angle E$. By the theorem, these pairs of angles are supplementary, hence, $m\angle A + m\angle M = 180°$, **and** $m\angle y + m\angle q = 180°$. Also, $m\angle F + m\angle E = 180°$.

Parallel-Exterior Angle-Same Side Theorem

If two parallel lines are cut by a transversal, then the exterior angles on the same side of the transversal are supplementary.

In the same figure, the pairs of exterior angles on the same side of the transversal are $\angle z$ & $\angle s$ and $\angle x$ & $\angle r$. By the theorem, these pairs of angles are supplementary, hence, $m\angle z + m\angle s = 180°$, **and** $m\angle x + m\angle r = 180°$.

The table below sums up the properties, postulates and theorems of the angles formed by parallel lines c and d cut by a transversal line AM.

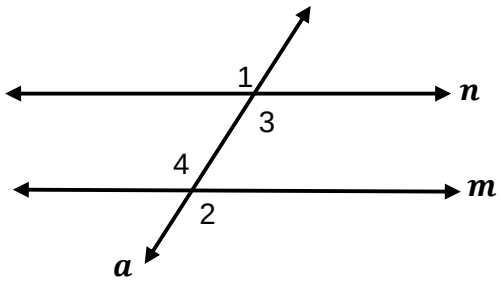
Angles Formed	Properties, Postulate, Theorems		
Alternate Interior Angles	***Pair of Angles:*** $\angle A$ and $\angle q$ $\angle y$ and $\angle M$	***Congruent angles:*** $\angle A \cong \angle q$ $\angle y \cong \angle M$	***Equal angles:*** $m\angle A = m\angle q$ $m\angle y = m\angle M$
Alternate Exterior Angles	***Pair of Angles:*** $\angle z$ and $\angle r$ $\angle x$ and $\angle s$	***Congruent angles:*** $\angle z \cong \angle r$ $\angle x \cong \angle s$	***Equal angles:*** $m\angle z = m\angle r$ $m\angle x = m\angle s$
Corresponding Angles	***Pair of Angles:*** $\angle z$ and $\angle M$ $\angle A$ and $\angle s$ $\angle x$ and $\angle q$ $\angle y$ and $\angle r$	***Congruent angles:*** $\angle z \cong \angle M$ $\angle A \cong \angle s$ $\angle x \cong \angle q$ $\angle y \cong \angle r$	***Equal angles:*** $m\angle z = m\angle M$ $m\angle A = m\angle s$ $m\angle x = m\angle q$ $m\angle y = m\angle r$
Vertical Angles	***Pair of Angles:*** $\angle x$ and $\angle A$ $\angle z$ and $\angle y$ $\angle q$ and $\angle s$ $\angle M$ and $\angle r$	***Congruent angles:*** $\angle x \cong \angle A$ $\angle z \cong \angle y$ $\angle q \cong \angle s$ $\angle M \cong \angle r$	***Equal angles:*** $m\angle x = m\angle A$ $m\angle z = m\angle y$ $m\angle q = m\angle s$ $m\angle M = m\angle r$

Interior Angles on the Same Side of the Transversal	***Pair of Angles:*** $\angle A$ *and* $\angle M$ $\angle y$ *and* $\angle q$ $\angle F$ *and* $\angle E$	***Supplementary angles:*** $m\angle A + m\angle M = 180°$ $m\angle y + m\angle q = 180°$ $m\angle F + m\angle E = 180°$
Exterior Angles on the Same Side of the Transversal	***Pair of Angles:*** $\angle z$ *and* $\angle s$ $\angle x$ *and* $\angle r$	***Supplementary angles:*** $m\angle z + m\angle s = 180°$ $m\angle x + m\angle r = 180°$

Now that we know these postulates and theorems, let us take a look at the following examples:

Example 1

Given: $\boldsymbol{n} \parallel \boldsymbol{m}$, transversal $\boldsymbol{a}$
Prove: $\angle 1 \cong \angle 2$.



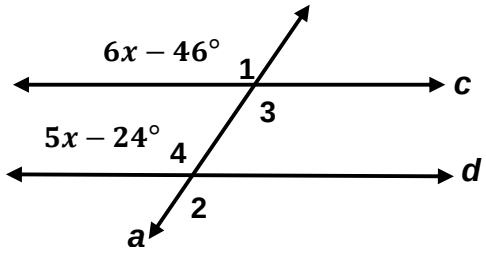
Proof:

Statements	Reasons
1. $n \parallel m$, transversal a	1. Given
2. $\angle 1 \cong \angle 3$	2. Vertical angles are congruent.
3. $\angle 3 \cong \angle 4$	3. Parallel-Alternate Interior Angle Postulate
4. $\angle 1 \cong \angle 4$	4. Transitive Property of Congruence
5. $\angle 4 \cong \angle 2$	5. Vertical angles are congruent
6. $\angle 1 \cong \angle 2$	6. Transitive property of congruence

Example 2

Given: $\boldsymbol{c} \parallel \boldsymbol{d}$**,** transversal $\boldsymbol{a}$ and if $\boldsymbol{m\angle 1} = (\boldsymbol{6x - 46°})$ and $\boldsymbol{m\angle 4} = (\boldsymbol{5x - 24°})$**,** find the value of $\boldsymbol{x}$, then solve for the measure of each angle below:

1. $m\angle 1$
2. $m\angle 4$
3. $m\angle 2$
4. $m\angle 3$



Solutions:

1. To find the measure of ∠1, we must find first the value of $\boldsymbol{x}$. We know that ∠1 *and* ∠4 are corresponding angles, and corresponding angles are congruent, hence, their measures are equal. From then, we can now formulate an equation:

$m\angle 1 = m\angle 4$	*Corresponding angles are congruent*
$(6x - 46^0) = (5x - 24^0)$	*Substitution*
$6x - 5x - 46^0 = 5x - 5x - 24^0$	*Addition Property of Equality, add (-5x) to both sides of the equation*
$x - 46^0 = -24^0$	*Simplify*
$x - 46^0 + 46^0 = -24^0 + 46^0$	*Addition Property of Equality, add (46) to both sides of the equation*
$x = 22^0$	*Simplifiy*

So, $x = 22^0$.

Substitute $\boldsymbol{x}$ **by its value** in the equation, $m\angle 1 = (6x - 46°)$, we have:

$m\angle 1 = (6x - 46°)$	*Equation for* $m\angle 1$
$m\angle 1 = [6(22°) - 46°]$	*Substitute* $\boldsymbol{x}$ *by its value*
$m\angle 1 = (132° - 46°)$	*Simplify*
$m\angle 1 = 86°$	*Result*

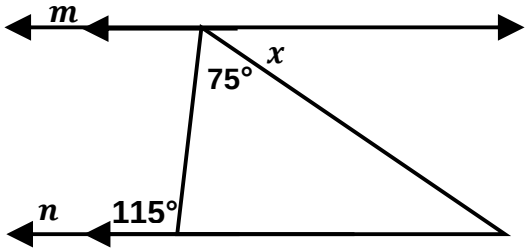
2. | | |
|---|---|
| $m\angle 4 = (5x - 24°)$ | *Equation for* $\angle 4$ |
| $m\angle 4 = [5(22°) - 24°]$ | *Substitute* $\boldsymbol{x}$ *by its value* |
| $m\angle 4 = (110° - 24°)$ | *Simplify* |
| $m\angle 4 = 86°$ | *Result* |

3. $m\angle 2$ =?
   Answer: $\boldsymbol{m\angle 2 = 86°}$
   Reason: $\boldsymbol{\angle 2\ and\ \angle 4}$ are vertical angles, and vertical angles are congruent

4. $m\angle 3$ =?
   Answer: $\boldsymbol{m\angle 3 = 86°}$
   Reason: $\boldsymbol{\angle 2\ and\ \angle 3}$ are corresponding angles, and corresponding angles are congruent, $\boldsymbol{\angle 3\ and\ \angle 4}$ are alternate-interior angles and they are congruent, and $\boldsymbol{\angle 1\ and\ \angle 3}$ are vertical angles and they are also congruent.

Example 3

In the figure at *the right,* $\boldsymbol{m \parallel n}$. *What is the value of* $\boldsymbol{x}$**?**



Solution:

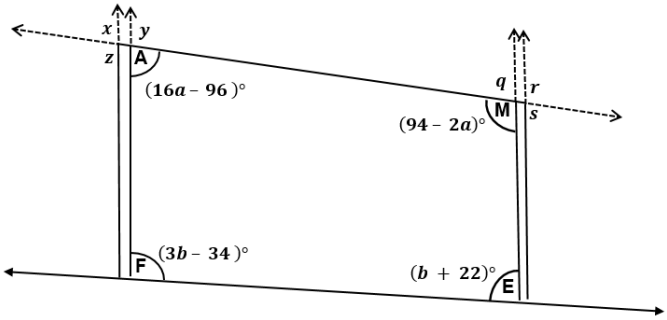
Given $\boldsymbol{m \parallel n}$, we get

$\mathbf{115° = 75° + x}$	*Alternate interior angles are congruent*
$\mathbf{115° - 75° = 75° - 75° + x}$	*Addition property*
$\mathbf{40° = x}$	*Simplify*
$\mathbf{x = 40°}$	*Symmetric property*

Example 4

Now, let us revisit our answers to the measures of the angles in Activity 2.

a. $m\angle A = 16a - 96°$
b. $m\angle M = 94° - 2a$
c. $m\angle F = 3b - 34°$
d. $m\angle E = b + 22°$



Solution:

We know that $\angle A$ and $\angle M$ are interior angles on the same side of the transversal, therefore, the sum of their measures is 180°. Solving for the value of $\boldsymbol{a}$, we have:

$$m\angle A + m\angle M = 180°$$
$$16a - 96° + 94° - 2a = 180°$$
$$14a - 2° + 2° = 180° + 2°$$
$$14a = 182°$$
$$a = 13°$$

a. $m\angle A = 16a - 96°$
$= 16(13°) - 96°$
$= 208° - 96°$
$= 112°$

b. $m\angle M = 94° - 2a$
$= 94° - 2(13°)$
$= 94° - 26°$
$= 68°$

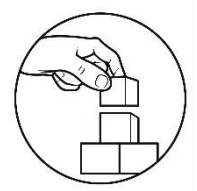
We know that $\angle F$ and $\angle E$ are interior angles on the same side of the transversal, therefore, the sum of their measures is 180°. Solving for the value of $\boldsymbol{b}$, we have:

$$m\angle F + m\angle E = 180°$$
$$3b - 34° + b + 22° = 180°$$
$$4b - 12° + 12° = 180° + 12°$$
$$4b = 192°$$
$$b = 48°$$

c. $m\angle F = 3b - 34°$
$= 3(48°) - 34°$
$= 144° - 34°$
$= 110°$

d. $m\angle E = b + 22°$
$= 48° + 22°$
$= 70°$

Now that you have learned the properties, postulate, and theorems on parallel lines cut by a transversal, pack them with you on your journey to the next activity.

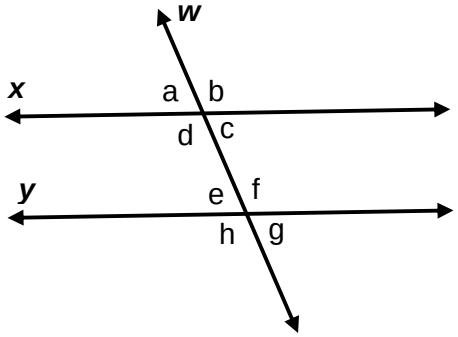


What's More

Activity 1: Fill Me!

Given the figure at the right, $x \parallel y$, and transversal w

A. Name the following:



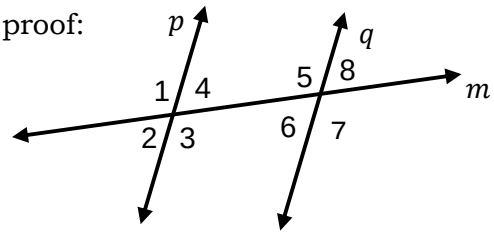
1. two pairs of alternate interior angles
2. four pairs of corresponding angles
3. two pairs of same side interior angles
4. four pairs of vertical angles
5. two pairs of alternate exterior angles
6. two pairs of same side exterior angles

B. Complete each statement.

7. Any pair of alternate interior angles are _____.
8. Any pair of corresponding angles are _____.
9. Same side exterior angles are _____.
10. Any pair of vertical angles are _____.

Activity 2: Prove It

Given the figure at the right, complete each proof:



1. Given: $\boldsymbol{p} \parallel \boldsymbol{q}$, transversal $\boldsymbol{m}$
   Prove: $\angle 3 \cong \angle 5$

Proof:

Statements	Reasons
1. $p \parallel q$, transversal m	1. _____
2. $\angle 3 \cong \angle 7$	2. _____
3. _____	3. Vertical Angles are congruent
4. $\angle 3 \cong \angle 5$	4. Transitive Property of Congruence

2. Given: $\boldsymbol{p} \parallel \boldsymbol{q}$, transversal $\boldsymbol{m}$
   Prove: $\angle 1 \cong \angle 7$

Proof:

Statements	Reasons
1. _____	1. Given
2. $\angle 1 \cong \angle 3$	2. _____
3. _____	3. Alternate interior angles are congruent
4. $\angle 1 \cong \angle 5$	4. _____
5. $\angle 5 \cong \angle 7$	5. Vertical angles are congruent
6. _____	6. _____

3. Given: $\boldsymbol{p} \parallel \boldsymbol{q}$, transversal $\boldsymbol{m}$
   Prove: $\angle 2 \cong \angle 6$

Proof:

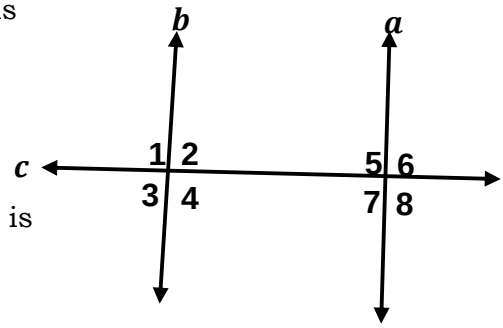
Statements	Reasons
1. _____	1. Given
2. _____	2. Vertical Angles are congruent
3. _____	3. _____
4. $\angle 2 \cong \angle 6$	4. _____

Activity 3: Find It!

Given the figure at the right such that $\boldsymbol{a} \parallel \boldsymbol{b}$, and transversal c

A. Find the value of $\boldsymbol{x}$ and the measure of each specified angle**.**

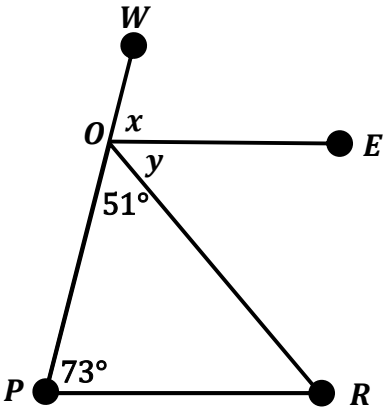
1. If $m\angle 2 = 3x + 14°$ and $m\angle 4 = 2x + 6°$, what is
   a. $m\angle 2$
   b. $m\angle 4$
   c. $m\angle 8$
   d. $m\angle 6$



2. If $m\angle 6 = 2x + 24°$ and $m\angle 7 = 3x + 12°$, what is
   a. $m\angle 6$
   b. $m\angle 7$
   c. $m\angle 4$
   d. $m\angle 3$

B. Given the figure at the right, $\overline{\boldsymbol{OE}} \parallel \overline{\boldsymbol{PR}}$. Answer the following questions and justify your answer.

1. What is the value of x?
2. What is the value of y?
3. What is the measure of $\angle PRO$?



What I Have Learned

Remember Me*:* Is it True or Not?

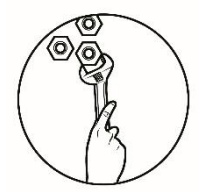
A. Directions: Write ***AT*** if the statement is always true, ***ST*** if the statement is sometimes true, or ***NT*** if the statement is never true.

If two parallel lines are cut by a transversal, then

1. alternate interior angles are congruent.
2. corresponding angles are congruent.
3. interior angles on the same side of the transversal are supplementary.
4. exterior angles on the same side of the transversal form a linear pair.
5. vertical angles are supplementary.
6. vertical angles are congruent.
7. linear pair are congruent.
8. alternate interior angles are supplementary.
9. same side exterior angles are congruent.
10. same side interior angles are supplementary.

B. Directions: Determine whether the statement is True or False. Write ***T*** if the statement is True and ***F*** if it is False.

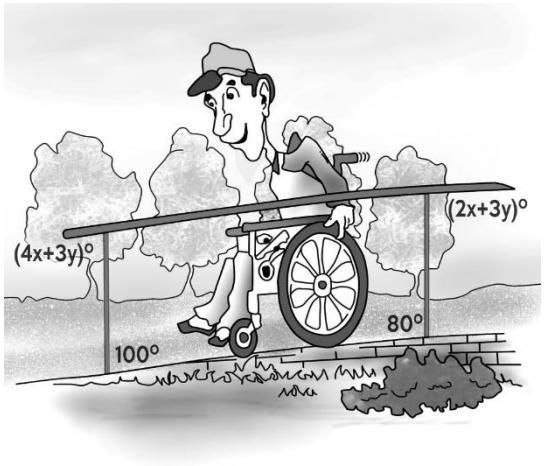
1. Through a point not on a line, you can sometimes construct a parallel line.
2. Two lines that are parallel to a third line are never parallel.
3. Two lines are parallel if they are coplanar and do not intersect.
4. Through a point outside a line, there is one and only one line parallel to the given line.
5. The shelves of a bookcase are parallel to each other.
6. The ceiling and a wall of a room form a linear pair.
7. The opposite edges of a book are sometimes parallel.
8. The service lines on a tennis court are parallel to each other.
9. A line that intersects two skew lines is a transversal.
10. The opposite sides of the base of the Great Pyramid of Giza are always parallel with each other.



What I Can Do

Study the situation below and solve for the value of the variables:

Reynante was a polio victim and was using his wheelchair for almost 5 years now. One day, he visits his doctor for his quarterly check-up and he passes by the wheelchair ramp which has a railing.



As shown in the illustration at the right, the railing of the wheelchair ramp is parallel to the ramp and is supported by braces which formed angles given below:

brace A : $(4x + 3y)°$, and

brace B : $(2x + 3y)°$

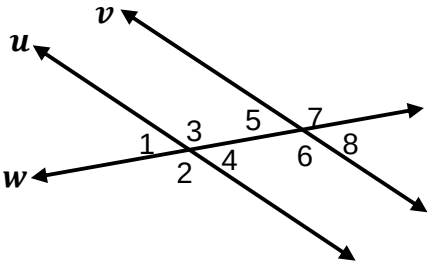
Find the value of x and the value of y.

Solution:	Answers:
	x=____
	y= _____

Assessment

Directions: Answer each of the following items. Write the letter of the correct answer on a separate sheet of paper.

For numbers 1 – 5, refer to the figure at the right.



Given: $\boldsymbol{u} \parallel \boldsymbol{v}$

1. What do you call the line w?
   A. Bisector
   B. Base
   C. Side
   D. Transversal

2. $\angle 1$ *and* $\angle 8$ called _____.
   A. Vertical angles
   B. Corresponding angles
   C. Alternate exterior angles
   D. Alternate interior angles

3. $\angle 6$ *and* $\angle 2$ are called _____.
   A. Alternate Exterior Angles
   B. Alternate Interior Angles
   C. Corresponding angles
   D. Vertical Angles

4. If the $m\angle 1 = 40°$, what is the measure of $\angle 7$?
   A. $40°$
   B. $50°$
   C. $130°$
   D. $140°$

5. Which pair of angles are supplementary?
   A. $\angle 1$ *and* $\angle 4$
   B. $\angle 3$ *and* $\angle 5$
   C. $\angle 4$ *and* $\angle 8$
   D. $\angle 6$ *and* $\angle 7$

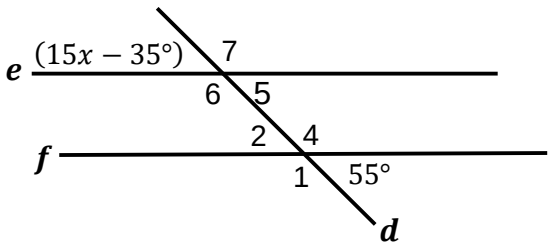
6. Which of the following statements is ALWAYS TRUE when parallel lines are cut by a transversal?
   A. Corresponding angles are congruent.
   B. The vertical angles are always acute.
   C. The sum of the measures of alternate interior angles is 180°.
   D. Interior angles on the same side of the transversal are congruent.

7. Given two parallel lines cut by a transversal, then a pair of alternate interior angles are _____.
   A. complementary
   B. congruent
   C. parallel
   D. perpendicular

8. A commercial airplane begins to land at Davao International Airport at an elevation of 36,000 feet and at a distance of 110 miles. What concept is being applied?
   A. Skew line
   B. Parallel line
   C. Perpendicular line
   D. none of the above

For items 9 – 11, refer to the figure at the right. $e \parallel f$ and d as a transversal.

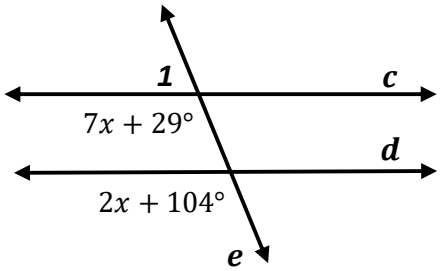
9. What is the value of x?
   A. 6
   B. 5
   C. 4
   D. 3



10. All of the following statements are true **EXCEPT** ?
    A. $\angle 6$ and $\angle 7$ are congruent.
    B. $\angle 1$ and $\angle 7$ form a linear pair.
    C. $\angle 5$ and $\angle 2$ are congruent angles.
    D. $\angle 5$ and $\angle 6$ are supplementary angles.

11. What could be the measure of $\angle 6$?
    A. 55°
    B. 75°
    C. 115°
    D. 125°

For items 12 – 14, refer to the figure at the right. $c \parallel d$ **and** e **as a transversal.**



12. Alma was asked by her teacher to find the measure of $\angle 1$. How will she find it?
    i. Subtract the value of $7x + 29°$ or $2x + 104°$ from 180°.
    ii. Solve for the value of x using the equation $(7x + 29°) + (2x + 104°) = 180°$.
    iii. Substitute x by its value in $7x + 29°$ or $2x + 104°$.
    iv. Solve for the value of x using the equation $(7x + 29°) = (2x + 104°)$.

    A. iv, ii, iii
    B. iii, iv, i
    C. ii, iii, iv
    D. iv, iii, i

13. What pair of angles are represented by $(7x + 29°)$ and $(2x + 104°)$?
    A. Supplementary angles
    B. Corresponding angles
    C. Linear pair
    D. Vertical angles

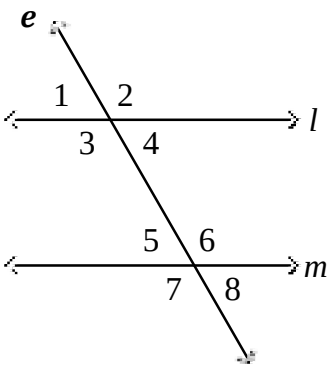
14. To complete her task, Alma solved for the measure of $\angle 1$ as follows:

$$(7x + 29°) = (2x + 104°)$$
$$7x - 2x + 29° - 29° = 2x - 2x + 104° - 29°$$
$$5x = 75°$$
$$x = 15°$$
$$m\angle 1 = 15°$$

Is she correct?

    A. Yes, because she used corresponding angles.
    B. Yes, because she followed the steps correctly.
    C. No, because the two angles are supplementary.
    D. No, because the value of x is not the measure of $\angle 1$.

15. Refer to the figure at the right.



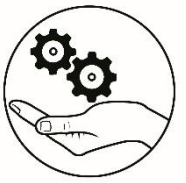
Given: $l \parallel m$ **and** e **is a transversal line.**

Prove: $\angle 3 \cong \angle 7$

Choose from the following statements to complete the table of proof.

A. $\angle 6 \cong \angle 7$
B. $m\angle 3 + m\angle 5 = 180°$
C. $m\angle 5 + m\angle 7 = 180°$
D. $\angle 2 \cong \angle 7$

Statements	Reasons
1. $l \parallel m$	1. Given
2. $\angle 3 \cong \angle 6$	2. Alternate interior angles are congruent
3. _____(15)_____	3. Vertical angles are congruent
4. $\angle 3 \cong \angle 7$	4. Transitivity

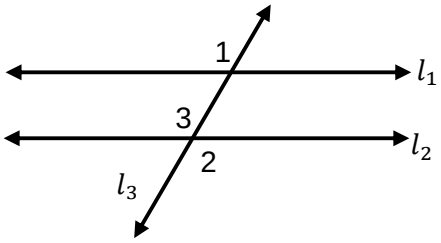


Additional Activities

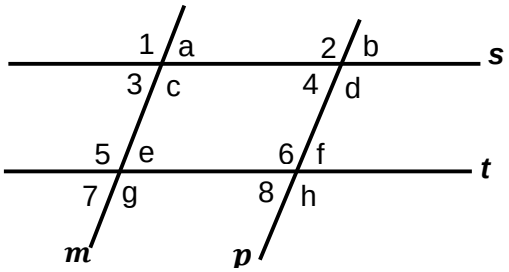
Prove It!

Directions: Construct a two-column proof to prove the statement below.

1. Given: $l_1 \parallel l_2$ and l_3 is a transversal line
   Prove: $\angle 1 \cong \angle 3$.



2. Refer to the given figure below for which $s \parallel t$, $m \parallel p$, and $m\angle 4 = 57°$.



a. What is $m\angle a$?
b. What is $m\angle c$?
c. What is $\angle 2$?
d. What is $\angle 7$?
e. What is $\angle h$?
f. What is $\angle 5$?
g. What is $\angle 8$?
h. What is $\angle f$?

Answer Key

What I Know

1. B 9. A
2. B 10. B
3. D 11. D
4. B 12. A
5. A 13. D
6. C 14. D
7. C 15. C
8. B

What's In

A.
1. INTERSECTING
2. PARALLEL
3. COPLANAR
4. PERPENDICULAR
5. SKEW
6. TRANSVERSAL

B.
1. Mabini St. & Mac Arthur Highway; Andres Ave. & Mac Arthur Highway; Bonifacio Intersection & Mac Arthur Highway
2. Andres Ave. & Bonifacio Intersection; Mabini St. & Bonifacio Intersection
3. Mabini St. & Mac Arthur Highway; Andres Ave. & Mac Arthur Highway; Bonifacio Intersection & Mac Arthur Highway

What I have learned

A.	B.
1. AT	1. F
2. AT	2. F
3. AT	3. T
4. NT	4. T
5. ST	5. T
6. AT	6. F
7. ST	7. F
8. ST	8. T
9. ST	9. F
10. AT	10. T

What I can Do

$x = 10\ ;\ y = 20$

What's New

Activity 1: Uncover me!
1. Answer varies
2. Parallel lines cut by a transversal
3. Answer varies
4. The 8 & 10 inches stick/straw represent parallel lines while the remaining stick represents the transversal line.
5. Yes
6. Yes, 8 & 10 sticks are parallel to each other.
7. 8 angles formed

Activity 2: Try me!
1. Transversal
2.
   a. $\angle q$ & $\angle A$; $\angle y$ & $\angle M$
   b. $\angle x$ & $\angle s$; $\angle z$ & $\angle r$
   c. $\angle z$ & $\angle M$; $\angle A$ & $\angle s$; $\angle x$ & $\angle q$; $\angle y$ & $\angle r$
   d. $\angle A$ & $\angle M$; $\angle y$ & $\angle q$
   e. $\angle x$ & $\angle r$; $\angle z$ & $\angle s$

   f. $\angle x$ & $\angle A$; $\angle z$ & $\angle y$; $\angle q$ & $\angle s$; $\angle M$ & $\angle r$
3. Yes, Alternate interior angles are congruent
4. Yes, vertical angles are congruent
5. Yes, alternate exterior angles are congruent
6.
   a. 112°
   b. 68°
   c. 110°
   d. 70°
7. 180°
8. 180°

Assessment

1. D
2. C
3. C
4. D
5. B
6. A
7. B
8. B
9. A
10. B
11. D
12. D
13. B
14. D
15. A

What's More

Activity 1: Fill me!

A.
1. $\angle d$ & $\angle f$; $\angle e$ & $\angle c$
2. $\angle a$ & $\angle e$; $\angle d$ & $\angle h$; $\angle b$ & $\angle f$; $\angle c$ & $\angle g$
3. $\angle d$ & $\angle e$; $\angle c$ & $\angle f$
4. $\angle a$ & $\angle c$; $\angle b$ & $\angle d$; $\angle e$ & $\angle g$; $\angle f$ & $\angle h$
5. $\angle a$ & $\angle g$; $\angle b$ & $\angle h$
6. $\angle a$ & $\angle h$; $\angle b$ & $\angle g$

B.

7. Congruent
8. Congruent
9. Supplementary
10. Congruent

Activity 2: Prove It!

1.
   1.1. Given
   1.2. Corresponding angles are congruent
   1.3. $\angle 7 \cong \angle 5$
2.
   2.1. $p \parallel q$, transversal m
   2.2. Vertical angles are congruent
   2.3. $\angle 3 \cong \angle 5$
   2.4. Corresponding angles are congruent
   2.6. Statement: $\angle 1 \cong \angle 7$
   2.6. Reason: Transitive Property of Congruence
3.
   3.1. $p \parallel q$, transversal m
   3.2. $\angle 2 \cong \angle 4$
   3.3. Statement: $\angle 4 \cong \angle 6$; Reason: Alternate interior angles are congruent
   3.4. Transitive Property of Congruence

Activity 3

A1. x = 32^0	A2. x = 12^0	B.
a. 112^0 b. 68^0 c. 68^0 d. 112^0	a. 48^0 b. 48^0 c. 132^0 d. 48^0	1. x = 73^0 2. y = 56^0 3. m∠PRO= 56^0

Additional Activities

1. Proof

STATEMENT	REASON
1. $L_1 \parallel L_2$	1. Given
2. $\angle 1 \cong \angle 2$	2. Alternate exterior angles are congruent
3. $\angle 2 \cong \angle 3$	3. Vertical angles are congruent
4. $\angle 1 \cong \angle 3$	4. Transitive property of congruence

2.

a. 57°	e. 123°
b. 123°	f. 123°
c. 123°	g. 57°
d. 57°	h. 57°

References

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