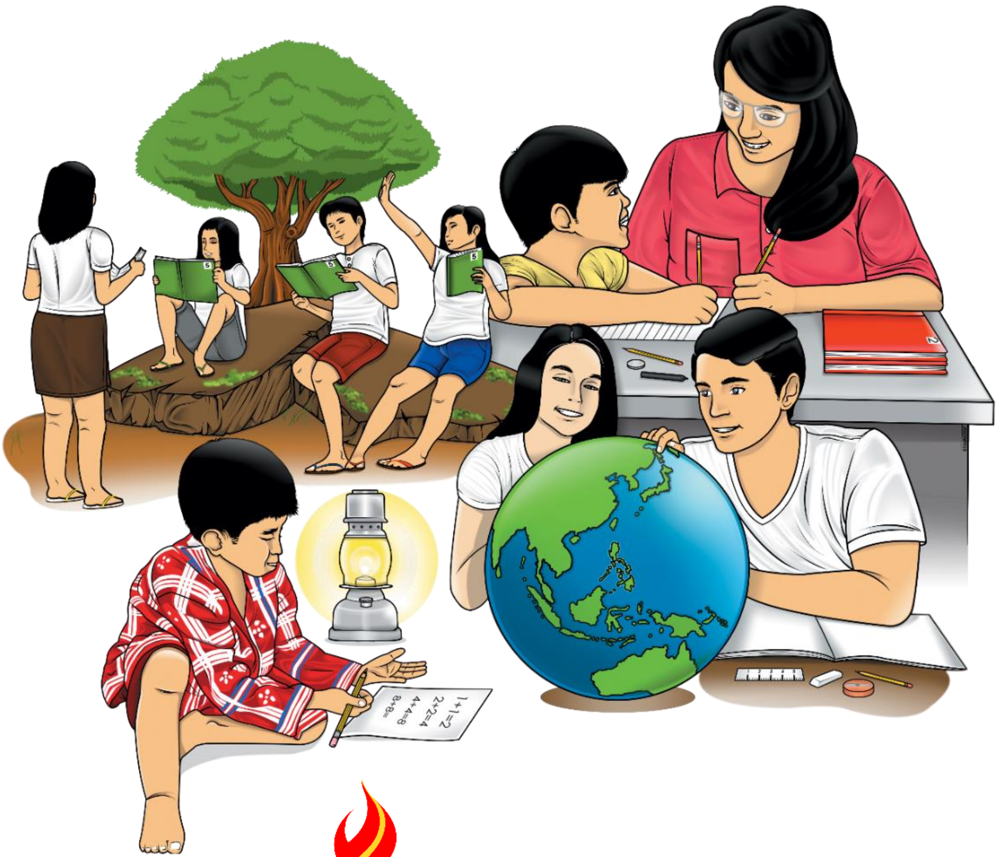
8

Mathematics

Quarter 4 – Module 3

Proving Inequalities in a Triangle



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Mathematics

Quarter 4 – Module 3

Proving Inequalities in a Triangle

DEPARTMENT OF EDUCATION

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

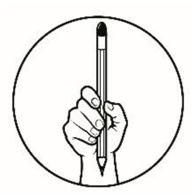
What I Need to Know

This module was designed and written with you in mind. It is here to help you master the skills of proving inequalities in a triangle. You are provided with varied activities to process the knowledge and skills learned and to deepen and transfer your understanding of the lesson. The scope of this module enables you to use it in many different learning situations. The lessons are arranged to follow the standard sequence of the course. But the order in which you read them can be changed to correspond with the textbook you are now using.

This module contains lesson on proving inequalities in a triangle (M8GE-IVc-1).

After going through this module, you are expected to:

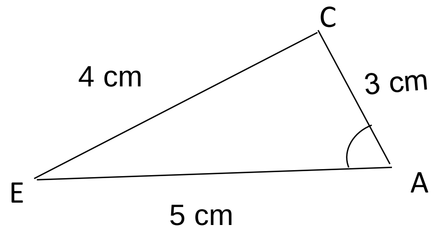
1. state the theorems on exterior angle inequality, triangle inequality, Hinge and its converse,
2. prove statements involving triangle inequalities using theorems on inequalities in triangles, and
3. justify the importance of proving triangle inequalities in real life situation.



What I Know

Directions: Read and answer each of the questions carefully and choose the letter of the correct answer. Write your answer in a separate sheet of paper.

1. Which of the following is the correct ascending arrangement of the measures of the angles of ΔCEA?
   A. $\angle E,\ \angle A,\ \angle C$
   B. $\angle A,\ \angle C,\ \angle E$
   C. $\angle E,\ \angle C,\ \angle A$
   D. cannot be determined



2. Which property of equality is defined by the statement "For all real numbers $p,\ p = p$"?
   A. Reflexive Property
   B. Transitive property
   C. Symmetric Property
   D. Substitution Property

3. Which statement below defines Addition Property of Equality?
   A. For all real numbers, p, q and r, if $p = q$, then $p + r = q + r$.
   B. For all real numbers, p, q and r, if $q = r$, then $p + r = q + r$.
   C. For all real numbers, p, q and r, if $p = q$, then $p + q = q + r$.
   D. For all real numbers, p, q and r, if $p = r$, then $p + r = q + r$.

4. Which of the following theorems states that "base angles of isosceles triangles are congruent"?
   A. Linear Pair Theorem
   B. Exterior Angle Theorem
   C. Vertical Angles Theorem
   D. Isosceles Triangle Theorem

5. What triangle inequality theorem states that "If one side of a triangle is longer than a second side, then the angle opposite the longer side is larger than the angle opposite the second side"?
   A. Exterior Angle Inequality Theorem
   B. Triangle Inequality Theorem 1 (Ss→Aa)
   C. Triangle Inequality Theorem 2 (Aa→Ss)
   D. Triangle Inequality Theorem 3 (S1 + S2 > S3)

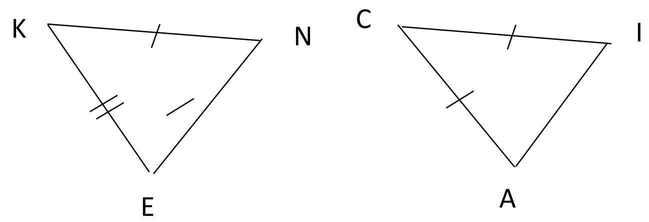
6. Which statement defines Exterior Angle Inequality Theorem?
   A. The sum of the lengths of any two sides of a triangle is greater than the length of the third side.
   B. The measure of an exterior angle of a triangle is greater than the measure of either remote interior angle.
   C. If one side of a triangle is longer than a second side, then the angle opposite the longer side is larger than the angle opposite the second side.

D. If one angle of a triangle is larger than a second angle, then the side opposite the larger angle is longer than the side opposite the second angle.

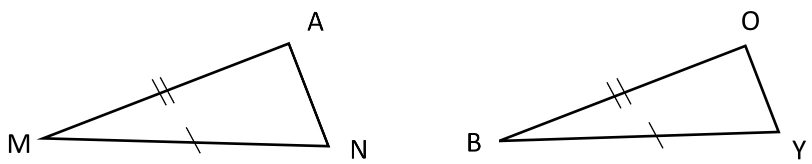
7. Which theorem below does not belong to the inequality theorems in one triangle?
   A. Hinge Theorem
   B. Exterior Angle Inequality Theorem
   C. Triangle Inequality Theorem 1 (Ss→Aa)
   D. Triangle Inequality Theorem 2 (Aa→Ss)

8. Which of the following statements is NOT true about writing proofs?
   A. It does not make use of hints to aid one's thinking.
   B. Guide questions are provided to help one succeed in the next activities.
   C. It will develop observation skills, deductive thinking and logical reasoning.
   D. There is a need to determine the appropriate statements and give reasons behind these statements.

9. Given ΔNKE and ΔICA below, if $m\angle K > m\angle C$, using the Hinge Theorem, which could be the conclusion about the given figures?
   A. $|EN| > |AI|$
   B. $|EN| < |AI|$
   C. $m\angle E < m\angle A$
   D. $m\angle N < m\angle I$



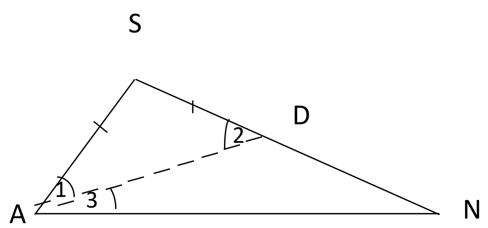
10. With the use of the Converse of the Hinge Theorem, which of the statements below is true about the sides and angles of ΔMAN and ΔBOY?



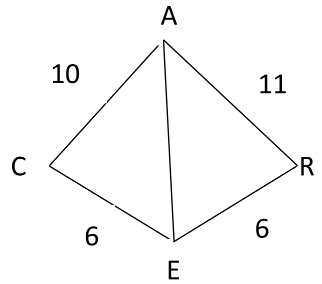
   A. If $\overline{MA} \cong \overline{BO}$, $\overline{MN} \cong \overline{BY}$ and $|AN| > |OY|$, then $m\angle M > m\angle B$
   B. If $\overline{MA} \cong \overline{BO}$, $\overline{MN} \cong \overline{BY}$ and $|AN| > |OY|$, then $m\angle M < m\angle B$
   C. If $\overline{MA} \cong \overline{BO}$, $\overline{MN} \cong \overline{BY}$ and $|AN| > |OY|$, then $m\angle A > m\angle O$
   D. If $\overline{MA} \cong \overline{BO}$, $\overline{MN} \cong \overline{BY}$ and $|AN| > |OY|$, then $m\angle N > m\angle Y$

11. Given ΔSAN at the right, which one supports the statement $m\angle SAN = m\angle 1 + m\angle 3$?

   A. Linear Pair Postulate
   B. Substitution Property
   C. Property of Inequality
   D. Angle Addition Postulate



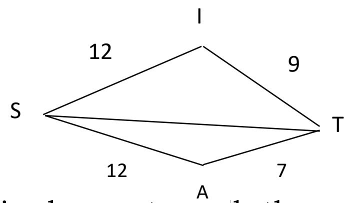
12. Ryan is analyzing the pair of triangles in the figure below. Which of the following is the correct observation?



A. $\overline{RA} \cong \overline{CE}$
B. $|CE| > |RE|$
C. $m\angle REA < m\angle CEA$
D. $m\angle REA > m\angle CEA$

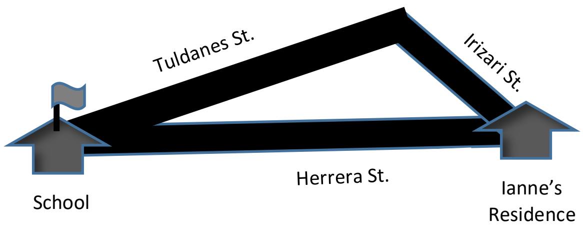
13. Which of the following would justify Ryan's correct observation in item no. 12?
A. Hinge Theorem
B. Converse of Hinge Theorem
C. Triangle Inequality Theorem 1 (Ss→Aa)
D. Triangle Inequality Theorem 3 (S1 + S2 > S3)

14. From the triangles shown below, a conclusion can be reached using the converse of hinge theorem. Which of the following is the conclusion?



A. $\overline{SI} \cong \overline{ST}$
B. $|SI| < |SA|$
C. $m\angle IST > m\angle AST$
D. $m\angle AST > m\angle IST$

15. In going to school, Ianne concluded that it will take him longer to walk through Irizari and Tuldanes Streets. He chooses to walk instead through Herrera St. daily in order to arrive early. Which triangle inequality theorem justifies his choice?



A. Exterior Angle Inequality Theorem
B. Triangle Inequality Theorem 1 (Ss→Aa)
C. Triangle Inequality Theorem 2 (Aa→Ss)
D. Triangle Inequality Theorem 3 (S1 + S2 > S3)

Lesson 1

Proving Inequalities in a Triangle

In your previous lessons, you have investigated and discovered different inequality theorems in a triangle or pair of triangles which consist of the side-angle, angle-side inequality theorems, exterior angle inequality theorem, and the Hinge theorem and its converse. These inequality theorems were discussed, illustrated and their applications were explained.

As we go through this module, think about how those theorems can be proven and how important writing proofs in learning geometry is. The skills you will gain in writing proofs will enable you to justify inequalities in triangles and in triangular designs and structures that are found in the things around us.

What's In

Activating Prior Knowledge!

I. Directions: Identify the postulate or property defined in each of the following statements. Choose your answer from the given options A to F. Write only the letter of the correct answer in a separate sheet of paper.

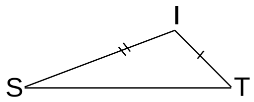
A. Angle Addition Postulate
B. Segment Addition Postulate
C. Addition Property of Equality
D. Reflexive Property of Equality
E. Transitive Property of Equality
F. Substitution Property of Equality

1. For all real numbers p, q and r, if $p = q$ and $q = r$, then $p = r$.
2. If point S lies in the interior of $\angle PQR$, then $m\angle PQS + m\angle SQR = m\angle PQR$
3. If points P, Q ad R are collinear $(P - Q - R)$ and Q is between points P and R, then $|PQ| + |QR| = |PR|$
4. For all real numbers p and q, if $p = q$, then q can be used to substitute p in any expression.
5. For all real numbers p, q and r if $p = q$, then $p + r = q + r$.

II. Directions: Match each figure in Column A to the theorem it illustrates in column B. Write the letter that corresponds to the correct answer in a separate sheet of paper.

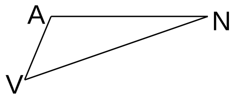
Column A

__________1.



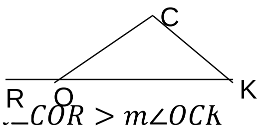
$m\angle T > m\angle S$

__________2.



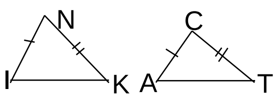
$|VA| + |AN| > |VN|$

__________3.



$m\angle COR > m\angle OCK$

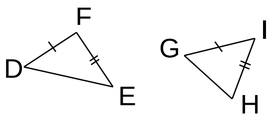
__________4.



Given: $m\angle C > m\angle N$

$|AT| > |IK|$

__________5.



Given: $|DE| > |GH|$

$m\angle F > m\angle I$

Column B

A. Hinge Theorem

B. Converse of Hinge Theorem

C. Triangle Inequality Theorem 1 (Ss→Aa)

D. Triangle Inequality Theorem 2 (Aa→Ss)

E. Triangle Inequality Theorem 3 (S1 + S2 > S3)

F. Exterior Angle Inequality Theorem

What's New

Discovering!

This part consists of proving activities for the 6 theorems on triangle inequalities. Analyze and use the hint provided in each activity to complete the proof. Note that inequalities in triangles, even without actual measurements, can be justified deductively using theorems on inequalities in triangles.

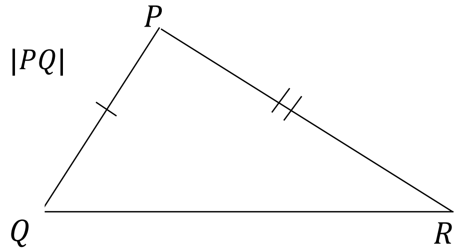
Activity 1

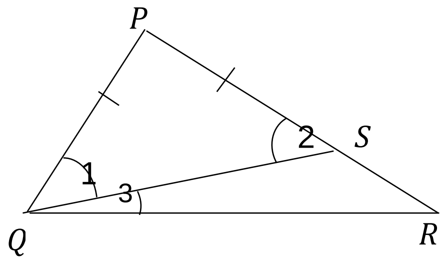
Directions: Complete the proof of Triangle Inequality Theorem 1(Ss→Aa) in two-column form by choosing the right statement written in the box below. Write your answer in a separate sheet of paper.

$\angle 1 \cong \angle 2$	$\overline{PQ} \cong \overline{PS}$	Angle Addition Postulate
$m\angle 2 > m\angle QRP$	$m\angle PQR > m\angle PRQ$	Property of Inequality
$m\angle QSR + m\angle PRQ + m\angle 3 = 180^0$		$m\angle 2 = m\angle PRQ + m\angle 3$
ΔPQS is isosceles triangle	Substitution Property	

Given: ΔPQR; $|PR| > |PQ|$

Prove: $m\angle Q > m\angle R$





Proof:

We cannot directly prove that $m\angle Q > m\angle R$, thus, there is a need to make additional constructions (see the second figure). Locate S on $\overline{PR}$ such that $|PS| = |PQ|$, and connect S to Q with a segment to form a triangle PQS.

Statements	Reasons
1. ________________ (How do you describe the relationship between $\overline{PQ}$ and $\overline{PS}$?)	1. By construction
2. ________________ (Based on statement 1, what kind of a triangle is ΔPQS?)	2. Definition of isosceles triangle
3. ______________ (Based on statement 1, how do you describe the relationship between $\angle 1$ and $\angle 2$?)	3. Base angles of isosceles triangles are congruent.
4. $m\angle Q = m\angle 1 + m\angle 3$	4. ________________ (What postulate supports the statement that the sum of the measures of $\angle 1$ and $\angle 3$ is equal to measure of $\angle PQR$?)
5. $m\angle Q > m\angle 1$	5. ________________

Statements	Reasons
	(What property supports the inequality statement focusing on $\angle 1$ based on statement 4?)
6. $m\angle Q > m\angle 2$	6. ______________ (What property supports the step where the right side of the inequality in statement 5 is replaced with its equivalent in statement 3?)
7. ____________ (Based on the illustration, write an operation statement involving measures of $\angle QSR, \angle R, and \angle 3$.	7. The sum of the measures of the interior angles of a triangle is 180^0
8. $m\angle 2 + m\angle QSR = 180^0$	8. Linear pair theorem
9. $m\angle 2 + m\angle QSR = m\angle QSR + m\angle R + m\angle 3$	9. Substitution/ Transitive Property
10. ____________ (What will be the result if $m\angle QSR$ is subtracted from both sides of statement 9?)	10. Subtraction Property
11. ____________ (Based on statement number 10, write an inequality statement focusing on $\angle R$.)	11. Property of Inequality
12. ___________ (Based on statements 6 and 11: If $m\angle PQR > m\angle 2$ and $m\angle 2 > m\angle R$, then __)	12. Transitive Property

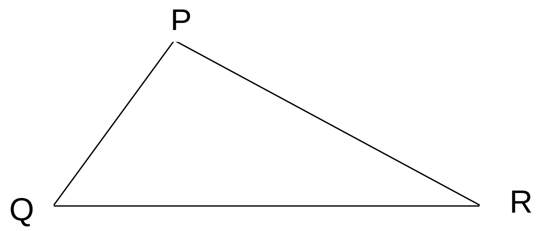
Activity 2

Directions: Complete the proof of Triangle Inequality Theorem 2 (Aa→Ss) in two-column form by choosing the right statement or words written in the box below. Write your answer on a separate sheet of paper.

$\angle P \cong \angle R$	Equal	True
$m\angle P > m\angle R$	Larger	False
$m\angle P > m\angle Q$	$\angle P \cong \angle Q$	Smaller
$m\angle P < m\angle R$	Triangle Inequality Theorem 1	Hinge Theorem

Given: ΔPQR; $m\angle P > m\angle R$

Prove: $|QR| > |PQ|$



To prove that $|QR| > |PQ|$, we will use indirect proof. Thus, we need to assume that $|QR| \ngtr |PQ|$.

Proof:

Statements	Reasons
1. $\lvert QR\rvert = \lvert PQ\rvert \text{ or } \lvert QR\rvert < \lvert PQ\rvert$	1. Assumption that $\lvert QR\rvert \ngtr \lvert PQ\rvert$
2. Consider $\lvert QR\rvert = \lvert PQ\rvert$. If $\lvert QR\rvert = \lvert PQ\rvert$, then ΔPQR is an isosceles triangle. (If $\lvert QR\rvert = \lvert PQ\rvert$, then what kind of triangle is ΔPQR?)	2. Definition of isosceles triangle
3. ____________ (Consequently, what can you say about $\angle P$ and $\angle R$?)	3. Base angles of isosceles triangles are congruent.
4. The assumption that $\lvert QR\rvert = \lvert PQ\rvert$ is false.	4. The conclusion that $\angle P \cong \angle R$ contradicts the given that $m\angle P > m\angle R$
5. Consider $\lvert QR\rvert < \lvert PQ\rvert$. If $\lvert QR\rvert < \lvert PQ\rvert$, then ____________.	5. ____________ (By what theorem of triangle inequality?)
6. ____________ (The assumption that $QR < PQ$ is true or false?)	6. The conclusion that $m\angle P < m\angle R$ contradicts the given that $m\angle P > m\angle R$
7. ____________ (Therefore, $QR > PQ$ must be true or false?)	7. The assumption that $\lvert QR\rvert \ngtr \lvert PQ\rvert$ contradicts the known fact that $m\angle P > m\angle R$

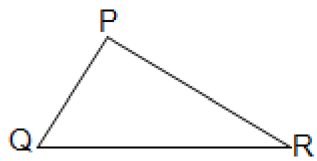
Activity 3

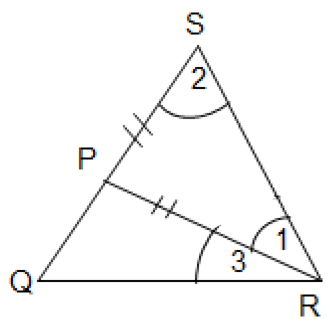
Direction: Complete the proof of Triangle Inequality Theorem 3 (S1 + S2 > S3) in two-column form by choosing the right statement written in the box below. Use the previous theorem to prove the next theorem. Write your answer on a separate sheet of paper. Enjoy!

$\angle PRS \cong \angle PSR$	$\overline{PS} \cong \overline{PR}$	Transitive Property
$PQ + PS > QR$	$\angle PRS \cong \angle QSR$	Angle Addition Postulate
Substitution Property	$\lvert PQ\rvert + \lvert PR\rvert > \lvert QR\rvert$	ΔPRS is equilateral triangle
ΔPRS is isosceles triangle	ΔPRS is scalene triangle	$m\angle SRQ = m\angle QRP + m\angle SRP$
Triangle Inequality Theorem 1 (Ss→ Aa)	Reflexive Property of Equality	Triangle Inequality Theorem 2 (Aa→Ss)

Given: ΔPQR where $\lvert PQ\rvert < \lvert PR\rvert < \lvert QR\rvert$

Prove: $\lvert PQ\rvert + \lvert PR\rvert > \lvert QR\rvert$
$\lvert PQ\rvert + \lvert QR\rvert > \lvert PR\rvert$
$\lvert QR\rvert + \lvert PR\rvert > \lvert PQ\rvert$





Proof:

Since $\lvert QR\rvert > \lvert PR\rvert$ and $\lvert QR\rvert > \lvert PQ\rvert$, then $\lvert QR\rvert + \lvert PQ\rvert > \lvert PR\rvert$ and $\lvert QR\rvert + \lvert PR\rvert > \lvert PQ\rvert$ are true. Hence, what remains to be proven is the last statement, $\lvert PQ\rvert + \lvert PR\rvert > \lvert QR\rvert$.

To prove $|PQ| + |PR| > |QR|$, let us construct $\overline{PS}$ as an extension of $\overline{PQ}$ such that P is between Q and S, $\overline{PS} \cong \overline{PR}$ and ΔPRS is formed.

Statements	Reasons
1. ____________ (What is the relationship between $\overline{PS}$ and $\overline{PR}$?)	1. By construction
2. ____________ (Based on statement 1, what kind of a triangle is ΔPRS?)	2. Definition of isosceles triangle
3. ____________ (Based on statement 1, what is the relationship between $\angle PRS$ and $\angle PSR$?)	3. Base angles of isosceles triangles are congruent.
4. Based on the illustration, $\angle PSR \cong \angle QSR$	4. ____________ (By what property of equality?)
If $\angle PRS \cong \angle PSR$ (statement 3) and $\angle PSR \cong \angle QSR$(statement 4), then 5. ____________	5. Transitive Property of Equality
6. From the illustration, $m\angle QRS = m\angle PRQ + m\angle PRS$	6. ____________ (By what postulate?)
7. Based on statements 5 and 6, $m\angle QRS = m\angle PRQ + m\angle QSR$	7. ____________ (By what property?)
8. Based on statement 7, $m\angle QRS > m\angle QSR$	8. Property of Inequality
9. $|QS| > |QR|$	9. ____________ (By what theorem of triangle inequality?)
10. $|PQ| + |PS| = |QS|$	10. Segment Addition Postulate
11. ____________ (Write a statement using statements 9 and 10.)	11. Substitution Property
12. ____________ (Write a statement using statements 1 and 11.)	12. Substitution Property

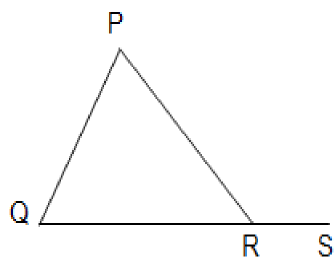
Activity 4

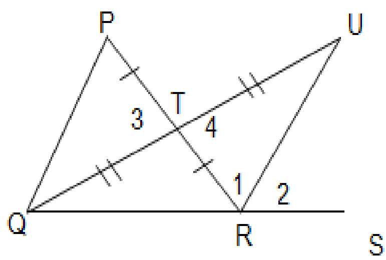
Directions: Complete the proof of Exterior Angle Inequality Theorem 1 in two-column form by choosing the right statement written in the box below. Write your answer in a separate sheet of paper.

$m\angle 1 + m\angle 2$	$\angle 1 \cong \angle 2$	$\angle 3 \cong \angle 4$
$\angle PRS \cong \angle QPR$	$\angle QPR \cong \angle 1$	$\Delta PTQ \cong \Delta RTU$
$m\angle PRS > m\angle QPR$		

Given: ΔPQR with exterior angle $\angle PRS$

Prove: $m\angle PRS > m\angle QPR$





Proof:

Let us prove that $m\angle PRS > m\angle QPR$ by constructing the following:

- midpoint T on $\overline{PR}$ such that $\overline{PT} \cong \overline{RT}$
- $\overline{QU}$ through T such that $\overline{QT} \cong \overline{TU}$

Statements	Reasons
1. $\overline{PT} \cong \overline{RT}$; $\overline{QT} \cong \overline{TU}$	1. By construction
2. ______________ (What is the relationship between $\angle 3$ and $\angle 4$?)	2. Vertical Angles are congruent
3. ______________ (Based on statements 1 and 2, relate two triangles in the illustration)	3. SAS Triangle Congruence Postulate
4. $\angle QPR$ $\cong$ ________. (Based on question number 2)	4. Corresponding parts of congruent triangles are congruent
5. $m\angle PRS =$ ________. (Based on the illustration)	5. Angle Addition Postulate
6. $m\angle PRS > m\angle 1$	6. Property of Inequality
7. _______________ (Using statement in question 3 and $m\angle PRS > m\angle 1$)	7. Substitution Property

Activity 5

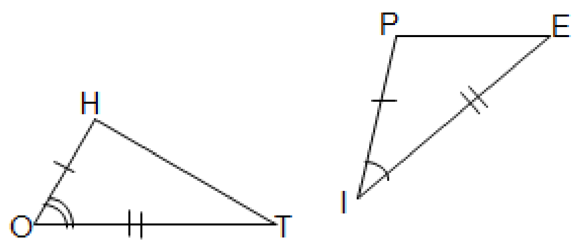
Directions: Complete the proof of Hinge Theorem or SAS triangle inequality theorem 1 in two-column form by choosing the right statement written in the box below. Write your answer/result on a separate sheet of paper. Enjoy!

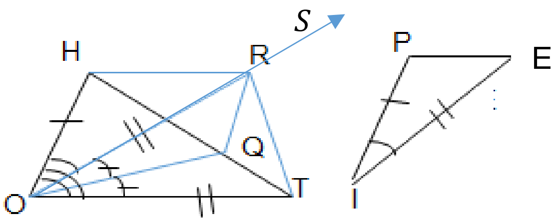
$\lvert HQ\rvert + \lvert RQ\rvert > \lvert HR\rvert$	Property of Inequality
$\lvert HQ\rvert + \lvert RQ\rvert = \lvert HR\rvert$	Segment Addition Postulate
$\lvert HQ\rvert + \lvert RQ\rvert < \lvert HR\rvert$	Triangle Inequality Theorem 1
Triangle Inequality Theorem 2	Definition of Congruent Triangles
Substitution Property of Equality	

Given: ΔHOT and ΔPIE;

$\overline{HO} \cong \overline{PI}$, $\overline{OT} \cong \overline{IE}$,
$m\angle HOT > m\angle PIE$

Prove: $|HT| > |PE|$





Proof:

1. Locate S on the same side of $\overleftrightarrow{OT}$ as H such that $\angle HOS \cong \angle PIE$.
   - Consider point R on $\overleftrightarrow{OS}$ so that $\overline{OR} \cong \overline{OT} \cong \overline{IE}$
   - Consequently, $\Delta HOR \cong \Delta PIE$ by SAS Triangle Congruence Postulate.

2. Construct the bisector $\overline{OQ}$ of $\angle TOR$ such that Q is on $\overline{HT}$
   - $\angle TOQ \cong \angle ROQ$
   - Consequently, $\Delta TOQ \cong \Delta ROQ$ by SAS Triangle Congruence Postulate because $\overline{OQ} \cong \overline{OQ}$ by reflexive property of equality and $\overline{OR} \cong \overline{OT}$ from construction no.1. So, $\overline{RQ} \cong \overline{QT}$ because corresponding parts of congruent triangles are congruent (CPCTC).

Statements	Reasons
1. $\lvert HT\rvert = \lvert HQ\rvert + \lvert QT\rvert$ (Based on the illustration)	1. ______________ (By what postulate?)
2. $\lvert HT\rvert = \lvert HQ\rvert + \lvert RQ\rvert$	2. ______________ (By what property of equality?)
3. In ΔHQR, ______________	3. Triangle Inequality Theorem 3
4. $\lvert HT\rvert > \lvert HR\rvert$ (Using statements 2 and 3)	4. ______________ (By what property of inequality)
5. $\lvert HR\rvert = \lvert PE\rvert$	5. ______________ (Consider ΔHOR and ΔPIE)
6. $\lvert HT\rvert > \lvert PE\rvert$	6. ______________ (______________________)

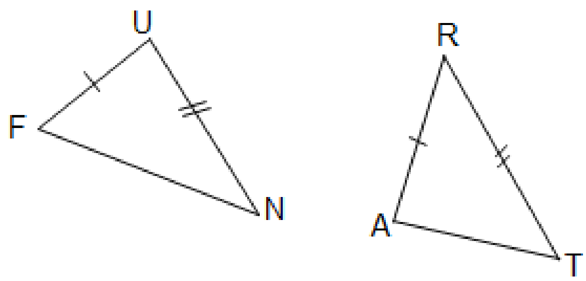
Activity 6

Directions: Complete the proof of the Converse of Hinge Theorem or SSS Triangle Inequality Theorem in two-column form by choosing the right statement written in the box below. Write your answer/result on a separate sheet of paper. Enjoy!

$m\angle FUN \ngtr m\angle ART$	$m\angle FUN > m\angle ART$	$m\angle FUN = m\angle ART$
AAS	SAS	ASA
Hinge Theorem	True	False

Given: ΔFUN and ΔART
$\overline{FU} \cong \overline{AR}$, $\overline{UN} \cong \overline{RT}$,
$\lvert FN\rvert > \lvert AT\rvert$

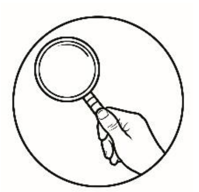
Prove: $m\angle U > m\angle R$



Proof:

Let us use indirect proof to show that $m\angle U > m\angle R$. Thus, we will assume that
$m\angle U \ngtr m\angle R$.

Statements	Reasons
1. ΔFUN and ΔART $\overline{FU} \cong \overline{AR}$, $\overline{UN} \cong \overline{RT}$, $\lvert FN\rvert > \lvert AT\rvert$	1. Given
2. $m\angle U = m\angle R$ or $m\angle U < m\angle R$	2. Assumption that 1. ________________
3. Consider: $\angle U \cong \angle R$. If $\angle U \cong \angle ART$, then $\Delta FUN \cong \Delta ART$.	3. _______________ (By what triangle congruence postulate?)
4. $\overline{FN} \cong \overline{AT}$	4. CPCTC
5. The assumption that $\angle U \cong \angle R$ is false	5. $\overline{FN} \cong \overline{AT}$ contradicts the given that $\lvert FN\rvert > \lvert AT\rvert$.
6. Consider $m\angle U < m\angle R$. If $m\angle U < m\angle R$, then $\lvert FN\rvert < \lvert AT\rvert$	6. _______________ (By what theorem of triangle inequality?)
The assumption that $m\angle U < m\angle R$ is 7. ______________.	7. $\lvert FN\rvert < \lvert AT\rvert$ contradicts the given that $\lvert FN\rvert > \lvert AT\rvert$.
Therefore, $\angle FUN > \angle ART$ must be 8. ______________	8. Assumption that $m\angle FUN \ngtr m\angle ART$ is proven to be false.



What is It

In writing proofs for inequalities in a triangle or in two triangles, it is important to justify each logical step with a reason. You can use symbols and abbreviations, but they must be clear enough so that they can be understood easily. Here, you can use paragraph form, two-column form, or flow chart form. But in this module, we will be using the two-column form to prove the theorems on inequalities in triangles.

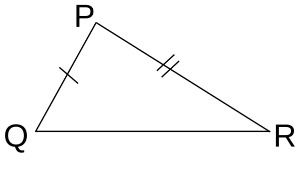
To check your answers in the previous activities, study and understand the proof for each theorem on inequalities in triangles.

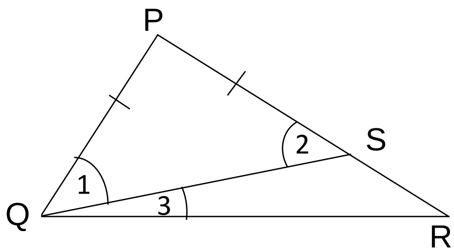
Triangle Inequality Theorem 1 (Ss → Aa)

If one side of a triangle is longer than a second side, then the angle opposite the longer side is larger than the angle opposite the second side.

Given: ΔPQR; $\lvert PR\rvert > \lvert PQ\rvert$

Prove: $m\angle PQR > m\angle PRQ$





Proof:

We cannot directly prove that $m\angle PQR > m\angle PRQ$, thus, there is a need to make additional constructions (see the second figure). With the compass point on P and with radius $\overline{PQ}$, mark a point S on $\overline{PR}$ and connect Q and S with a segment to form a triangle.

Statements	Reasons
1. $\overline{\boldsymbol{PQ}} \cong \overline{\boldsymbol{PS}}$	1. By construction
2. $\Delta \boldsymbol{PQS}$ **is isosceles triangle.**	2. Definition of isosceles triangle
3. $\angle\mathbf{1} \cong \angle\mathbf{2}$	3. Base angles of isosceles triangles are congruent.
4. $m\angle PQR = m\angle 1 + m\angle 3$	4. **Angle Addition Postulate**
5. $m\angle PQR > m\angle 1$	5. **Property of Inequality**
6. $m\angle PQR > m\angle 2$	6. **Substitution Property**
7. $m\angle\boldsymbol{QSR} + \boldsymbol{m\angle PRQ} + \boldsymbol{m\angle 3} = \mathbf{180^0}$**.**	7. The sum of the measures of the interior angles of a triangle is 180^0.
8. $m\angle 2 + m\angle QSR = \mathbf{180^0}$	8. Linear pair theorem
9. $m\angle 2 + m\angle QSR = m\angle QSR + m\angle R + m\angle 3$	9. Substitution/ Transitive Property
10. $m\angle\mathbf{2} = \boldsymbol{m\angle R} + \boldsymbol{m\angle 3}$	10. Subtraction Property
11. $m\angle\mathbf{2} > \boldsymbol{m\angle R}$	11. Property of Inequality
12. $m\angle\boldsymbol{Q} > \boldsymbol{m\angle R}$	12. Property of Inequality

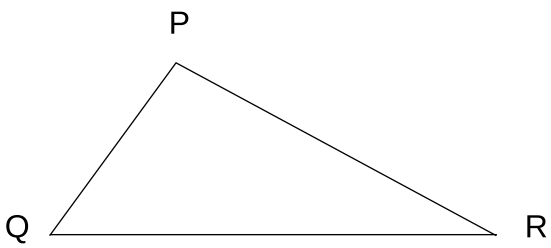
Triangle Inequality Theorem 1**(Ss → Aa)** is now proven. Let us proceed to prove Triangle Inequality Theorem 2 **(Aa→Ss)**.

Triangle Inequality Theorem 2 (Aa→Ss)

If one angle of a triangle is larger than a second angle, then the side opposite the larger angle is longer than the side opposite the second angle.

Given: ΔPQR; $m\angle P > m\angle R$

Prove: $|QR| > |PQ|$



Proof:

To prove that $|QR| > |PQ|$, we will use indirect proof. Thus, we need to assume that $|QR| \ngtr |PQ|$.

Statements	Reasons
1. $|QR| = |PQ|$ or $|QR| < |PQ|$	1. Assumption: $|QR| \ngtr |PQ|$
2. Consider $|QR| = |PQ|$. ΔPQR is an isosceles triangle. If $\overline{QR} \cong \overline{PQ}$, then what kind of triangle is ΔPQR?)	2. Definition of isosceles triangle

3. $\angle \boldsymbol{QPR} \cong \angle \boldsymbol{QRP}$	3. Base angles of isosceles triangles are congruent.
4. The assumption that $\lvert QR \rvert = \lvert PQ \rvert$ is false.	4. The conclusion that $\angle QPR$ and $\angle QRP$ are congruent contradicts the given that $m\angle QPR > m\angle QRP$.
5. Consider $\lvert QR \rvert < \lvert PQ \rvert$. If $\lvert QR \rvert < \lvert PQ \rvert$, then $m\angle QPR < m\angle QRP$.	5. Triangle Inequality Theorem 1
6. The assumption that $\lvert QR \rvert < \lvert PQ \rvert$ is **false**.	6. The conclusion that $m\angle QPR < m\angle QRP$ contradicts the given that $m\angle QPR > m\angle QRP$.
7.Therefore, $\lvert QR \rvert > \lvert PQ \rvert$ must be **true**.	7. The assumption that $QR \ngtr PQ$ contradicts the known fact that $m\angle QPR > m\angle QRP$.

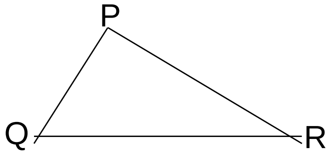
Triangle inequality Theorem 2 **(Aa→Ss)** is already proven. You will observe that this theorem will be used as reason in proving the next theorem.

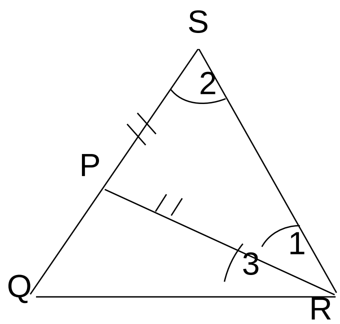
Triangle Inequality Theorem 3 (S_1+S_2 > S_3)

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Given: ΔPQR where $\lvert PQ \rvert < \lvert PR \rvert < \lvert QR \rvert$

Prove: $\lvert PQ \rvert + \lvert PR \rvert > \lvert QR \rvert$
$\lvert PQ \rvert + \lvert QR \rvert > \lvert PR \rvert$
$\lvert QR \rvert + \lvert PR \rvert > \lvert PQ \rvert$





Proof:

Since $\lvert QR \rvert > \lvert PR \rvert$ and $\lvert QR \rvert > \lvert PQ \rvert$, then $\lvert QR \rvert + \lvert PQ \rvert > \lvert PR \rvert$ and $\lvert QR \rvert + \lvert PR \rvert > \lvert PQ \rvert$ are true. Hence, what remains to be proved is the statement $\lvert PQ \rvert + \lvert PR \rvert > \lvert QR \rvert$.

To prove $\lvert PQ \rvert + \lvert PR \rvert > \lvert QR \rvert$, let us construct $\overline{PS}$ as an extension of $\overline{PQ}$ such that P is between Q and S, $\overline{PS} \cong \overline{PR}$ and ΔPRS is formed.

Statements	Reasons
1. $\lvert \boldsymbol{PS} \rvert = \lvert \boldsymbol{PR} \rvert$**.**	1. By construction
2. $\Delta \boldsymbol{PRS}$ **is isosceles triangle.**	2. Definition of isosceles triangle
3. $\angle \boldsymbol{PRS} \cong \angle \boldsymbol{PSR}$**.**	3. Base angles of isosceles triangle are congruent.
4. The illustration shows that $\angle PSR \cong \angle QSR$	4. **Reflexive property of equality**
5. $\angle \boldsymbol{PRS} \cong \angle \boldsymbol{QSR}$.	5. Transitive Property of Equality

6. From the illustration, $m\angle QRS = m\angle PRQ + m\angle PRS$	6. **Angle Addition Postulate**
7. $m\angle QRS = m\angle PRQ + m\angle QSR$ (Based from the two previous statements above)	7. **Substitution property**
8. $m\angle QRS > m\angle QSR$ (Based from the previous statement.)	8. Property of Inequality
9. $\lvert QS\rvert > \lvert QR\rvert$	9. **Triangle Inequality Theorem 2 (Aa → Ss)**
10. $\lvert PQ\rvert + \lvert PS\rvert = \lvert QS\rvert$.	10. Segment Addition Postulate
11. $\boldsymbol{\lvert PQ\rvert + \lvert PS\rvert > \lvert QR\rvert}$	11. Substitution Property
12. $\boldsymbol{\lvert PQ\rvert + \lvert PR\rvert > \lvert QR\rvert}$	12. Substitution Property

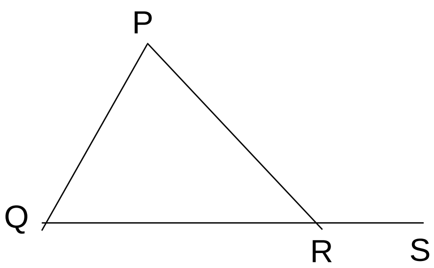
The proof of Triangle Inequality Theorem 3 ($S1 + S2 > S3$) is now done. Let us proceed to writing the proof of Exterior Angle Inequality Theorem.

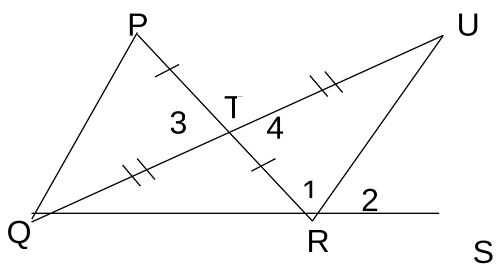
Exterior Angle Inequality Theorem

The measure of an exterior angle of a triangle is greater than the measure of either remote interior angle.

Given: ΔPQR with exterior angle $\angle PRS$

Prove: $m\angle PRS > m\angle QPR$





Proof:

Let us prove that $m\angle PRS > m\angle QPR$ by constructing the following:

- midpoint T on $\overline{PR}$ such that $\overline{PT} \cong \overline{RT}$
- $\overline{QU}$ through T such that $\overline{QT} \cong \overline{TU}$

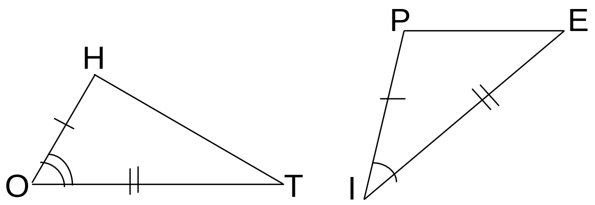
Statements	Reasons
1. $\overline{PT} \cong \overline{RT}$; $\overline{QT} \cong \overline{TU}$	1. By construction
2. $\angle\mathbf{3} \cong \angle\mathbf{4}$	2. Vertical Angles are congruent
3. $\Delta\boldsymbol{PTQ} \cong \Delta RTU$	3. SAS Triangle Congruence Postulate
4. $\angle\boldsymbol{QPR} \cong \angle\mathbf{1}$	4. Corresponding parts of congruent triangles are congruent
5. $\boldsymbol{m\angle PRS = m\angle 1 + m\angle 2}$	5. Angle Addition Postulate
6. $m\angle PRS > m\angle 1$	6. Property of Inequality
7. $m\angle\boldsymbol{PRS} > \boldsymbol{m\angle QPR}$	7. Substitution Property

We have just proven the Exterior Angle Inequality Theorem. In the next discussion, we will prove two theorems on inequalities in two triangles.

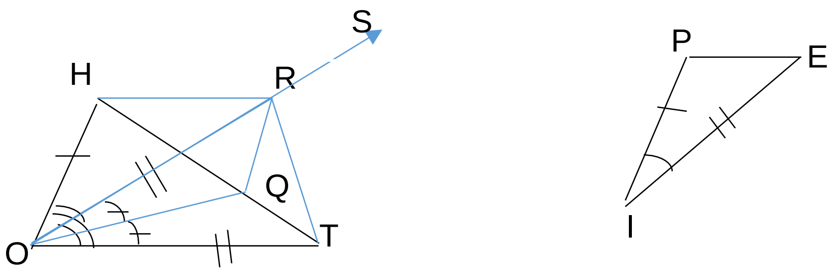
Hinge Theorem or SAS Inequality Theorem

If two sides of one triangle are congruent to two sides of another triangle, but the included angle in the first triangle is greater than the included angle in the second, then the third side of the first triangle is longer than the third side of the second.

Given: ΔHOT and ΔPIE;
$\overline{HO} \cong \overline{PI}$, $\overline{OT} \cong \overline{IE}$,
$m\angle HOT > m\angle PIE$



Prove: $|HT| > |PE|$



Proof:

1. Locate S on the same side of $\overleftrightarrow{OT}$ as H such that $\angle HOS \cong \angle PIE$.
   - Consider point R on $\overleftrightarrow{OS}$ so that $\overline{OR} \cong \overline{OT} \cong \overline{IE}$
   - Consequently, $\Delta HOR \cong \Delta PIE$ by SAS Triangle Congruence Postulate.

2. Construct the bisector $\overline{OQ}$ of $\angle TOR$ such that Q is on $\overline{HT}$ and $\angle TOQ \cong \angle ROQ$.

   - Consequently, $\Delta TOQ \cong \Delta ROQ$ by SAS Triangle Congruence Postulate because $\overline{OQ} \cong \overline{OQ}$ by reflexive property of equality and $\overline{OR} \cong \overline{OT}$ from construction no.1. So, $\overline{RQ} \cong \overline{QT}$ because corresponding parts of congruent triangles are congruent (CPCTC).

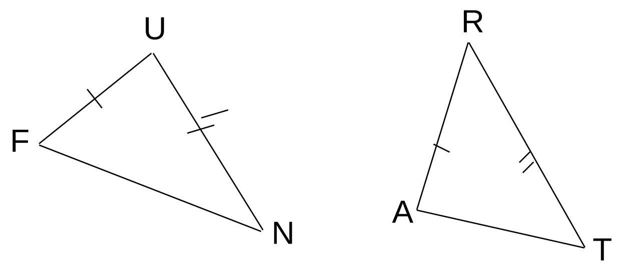
Statements	Reasons
1. $|HT| = |HQ| + |QT|$.	1.Segment Addition Postulate
2. $|HT| = |HQ| + |RQ|$.	2.Substitution Property of Equality
3. In ΔHQR, $|HQ| + |RQ| > |HR|$.	3.Triangle Inequality Theorem 3 ($S1 + S2 > S3$)
4. $|HT| > |HR|$ (Using statements 2 and 3)	4. Substitution Property of Inequality
5. $|HR| = |PE|$	5. Definition of congruent triangles.
6. $|HT| > |PE|$ (Using statement in construction 1 and question number 3.)	6. Substitution Property

We are almost finished proving the theorems on inequalities in triangles. You just proved the Hinge theorem. Let us see how the Converse of Hinge Theorem is done.

Converse of Hinge Theorem or SSS Inequality Theorem

If two sides of one triangle are congruent to two sides of another triangle, but the third side of the first triangle is longer than the third side of the second, then the included angle in the first triangle is larger than the included angle in the second triangle.

Given: ΔFUN and ΔART
$\overline{FU} \cong \overline{AR}$, $\overline{UN} \cong \overline{RT}$,
$|FN| > |AT|$
Prove: $m\angle FUN > m\angle ART$



Proof:

Let us use indirect proof to show that $m\angle FUN > m\angle ART$. Thus, we will assume that $m\angle FUN \ngtr m\angle ART$.

Statements	Reasons
1. ΔFUN and ΔART $\overline{FU} \cong \overline{AR}$, $\overline{UN} \cong \overline{RT}$, $\lvert FN\rvert > \lvert AT\rvert$	1. Given
2. $\angle FUN \cong \angle ART$ or $m\angle FUN < m\angle ART$.	2. Assumption: $\boldsymbol{m\angle FUN \ngtr m\angle ART}$.
3. Consider: $\angle FUN \cong \angle ART$. If $\angle FUN \cong \angle ART$, then $\Delta FUN \cong \Delta ART$.	3. **SAS**
4. $\overline{FN} \cong \overline{AT}$.	4. CPCTC
5. The assumption that $\angle FUN \cong \angle ART$ is false.	5. $\overline{FN} \cong \overline{AT}$ contradicts the given that $\lvert FN\rvert > \lvert AT\rvert$.
6. Consider $m\angle FUN < m\angle ART$. If $m\angle FUN < m\angle ART$, then $\lvert FN\rvert < \lvert AT\rvert$.	6. **Hinge Theorem**
7. The assumption that $\angle FUN < \angle ART$ is **False.**	7. $\lvert FN\rvert < \lvert AT\rvert$ contradicts the given that $\lvert FN\rvert > \lvert AT\rvert$.
8. Therefore, $m\angle FUN > m\angle ART$ must be **True.**	8. Assumption that $m\angle FUN \ngtr m\angle ART$ is proven to be false.

Hurray! All theorems on inequalities in triangles have already been proven. You are now equipped with skills on how to write proofs in two-column form. Always remember that in writing proofs, the properties of equality and congruence will be used as bases for reasoning. Moreover, you should be equipped with the needed knowledge such as undefined terms, definitions, postulates, and theorems in geometry since these are essential to support the statement of a proof.

Having proven all the theorems on triangle inequalities, you can now take a closer look on the applications of these theorems in any aspects of real-life situations. You can use these skills in identifying unknown quantities in triangle inequalities and in justifying them. This time, let us apply what you know about inequalities to solve problems about triangles. Learn how to compare the lengths of sides and angle measures without knowing the actual measures. Here is a math problem you might have encountered in real life that will make use of one of the triangle inequality theorems. It will help you gain ideas and hints on how to justify real life problems regarding inequalities in triangles.

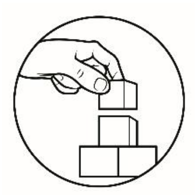
Illustrative Example:

Ellaine is going to the market. She can choose one of the walking routes, either the wide roads or the narrow one as illustrated. Ellaine was told by her mother to go back home as fast as possible.

1. If you were Ellaine, which road will you choose? What is your basis in choosing which road to take when the measures of the roads are not given in the illustration? Which among the theorems will support your answer?

2. If Ellaine were not told by her mother to go back home as fast as possible, do you think Ellaine would have taken the wide roads? Justify your answer.

In this problem, we do not know the lengths of the roads, but we can see that the wide and the narrow roads together form a triangle. Here, let us represent the wide roads as the first two sides of the triangle and the narrow one represents the third side. If you were Ellaine, it would be better to take the narrow road. It can be justified by Triangle Inequality Theorem 3 ($S1 + S2 > S3$), which states that the sum of the lengths of any two sides of a triangle is greater than the length of the third side. If Ellaine was not told to go back home as fast as possible, she could have taken the wider roads for more walking exercise.



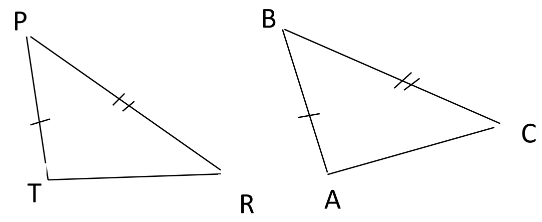
What's More

Activity 1: Complete Me!

Directions: Write the statement or reason in the two-column proof. Use the different definitions, postulates, theorems in geometry to support the given statement.

Given: $\overline{TP} \cong \overline{BA}$, $\overline{PR} \cong \overline{BC}$, $|TR| > |AC|$

Prove: $m\angle TPR > m\angle ABC$



Proof:

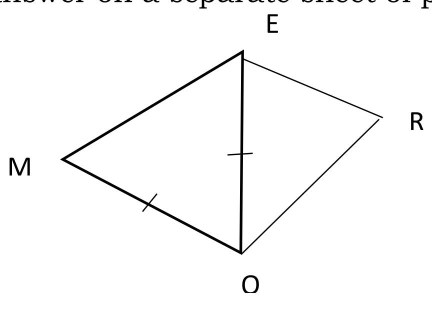
Statements	Reasons
1. $\overline{TP} \cong \overline{BA}$, $\overline{PR} \cong \overline{BC}$, $\lvert TR\rvert > \lvert AC\rvert$	1. Given
2. $\angle TPR \cong \angle ABC$ or $m\angle TPR < m\angle ABC$.	2. Assumption: $m\angle TPR \ngtr m\angle ABC$
3. Consider: $\angle TPR \cong \angle ABC$. If $\angle TPR \cong \angle ABC$, then $\Delta TPR \cong \Delta ABC$.	3. _______________
4. $\overline{TR} \cong \overline{AC}$.	4. _______________
5. The assumption that $\angle TPR \cong \angle ABC$ is false.	5. $\overline{TR} \cong \overline{AC}$ contradicts the given that $\lvert TR\rvert > \lvert AC\rvert$.
6. Consider $m\angle TPR < m\angle ABC$. If $m\angle TPR < m\angle BAC$, then $\lvert TR\rvert < \lvert AC\rvert$.	6. ____________
7. The assumption that $m\angle TPR < m\angle ABC$ is ___________.	7. $\lvert TR\rvert < \lvert AC\rvert$ contradicts the given that $\lvert TR\rvert > \lvert AC\rvert$.
8. Therefore, $m\angle TPR > m\angle ABC$ must be ___________.	8. Assumption that $m\angle TPR \ngtr m\angle ABC$ is proven to be false.

Activity 2: Proving

Directions: Write the statements supported by the reasons on the right side of the two-column proof. Write your answer on a separate sheet of paper.

Given: $\overline{MO} \cong \overline{OE}$

Prove: $|ER| + |RO| > |MO|$



Proof:

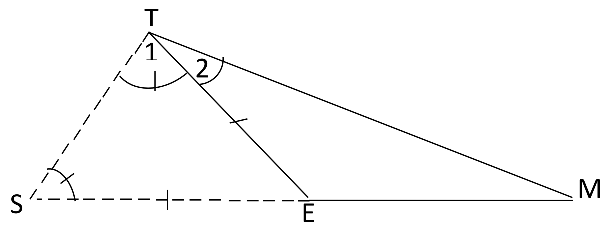
Statement	Reasons
1.	1. Given
2.	2. Definition of congruent segments
3. $\lvert ER\rvert + \lvert RO\rvert > \lvert OE\rvert$.	3.
4. $\lvert ER\rvert + \lvert RO\rvert > \lvert MO\rvert$.	4.

Activity 3: Reasoning

Directions: Observe and notice the congruent sides and congruent angles shown in the illustration below. Write the reasons for the statements in the two-column proof.

Given: ΔTEM

Prove: $ET + EM > TM$



Proof:

Statements	Reasons
1. $\lvert ES \rvert = \lvert ET \rvert$.	1.
2. ΔTSE is isosceles triangle.	2.
3. $\angle ETS \cong \angle TSE$.	3.
4. $\angle TSE \cong \angle TSM$.	4.
5. $\angle ETS \cong \angle TSM$.	5.
6. $m\angle STM = m\angle STE + m\angle ETM$.	6.
7. $m\angle STM = m\angle TSM + m\angle ETM$.	7.
8. $m\angle STM > m\angle TSM$.	8.
9. $\lvert SM \rvert > \lvert TM \rvert$.	9.
10. $\lvert ES \rvert + \lvert EM \rvert = \lvert SM \rvert$.	10.
11. $\lvert ES \rvert + \lvert EM \rvert > \lvert TM \rvert$.	11.
12. $\lvert ET \rvert + \lvert EM \rvert > \lvert TM \rvert$.	12.

Activity 4: Justifying

Directions: Read and understand the problem and answer the questions that follow.

Two bikers Chris and Rey have uniform biking speed and are headed in opposite directions from their school. Chris is headed to the east while Rey is to the west. After biking 5 kilometers each, both of them took right turns at different angles. Chris turned at 30^0 while Rey at 40^0. Both continued biking and covered another 6 kilometers before taking a rest.

Questions:

1.Who is farther from the school, Chris or Rey? Justify your answer.
2. Which inequality theorem justifies your answer? Explain.

What I Have Learned

Directions: In your own words, complete each statement below based on the concepts you have learned in the topic.

1. I have learned that the different theorems of triangle inequalities are ____.

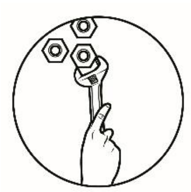
2. I have learned that the theorems on inequalities in triangle or triangles can be justified through ____.

3. I need to be equipped with the knowledge of ____ to support the statement of my proofs.

4. I learned that proofs on theorems on inequalities can be written in many ways, but a two-column proof is _______________________________________.

5. In writing proofs, I developed my skills in

   ____.



What I Can Do

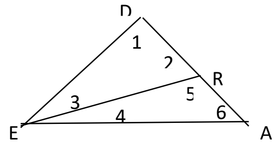
Directions: Observe your surroundings and make a design where triangle inequalities are applied. Make justifications that the triangle inequality exists in the created design.

Criteria	**Points**				**Rating**
	4	**3**	**2**	**1**	
Appropriate-ness	The output contains a design which shows concrete example of triangle inequalities in real life.	The output contains a design which shows an example of triangle inequalities in real life.	The output contains a design which show an example of triangle inequalities but does not show its application in real life	The output contains a design which shows an example not connected to triangle inequalities.	
Clarity/ precision	All parts of the design are clear and precise	Some parts of the design are unclear and not precise	Most parts of the design are unclear and not precise	The drawing is unclear and not precise	
Justification	Insight and depth of content understanding are evident	Some depth of content understanding are evident	Lacks content understanding	Does not give a clear understanding of the content	

Assessment

Directions: Answer each of the following items accurately. Write the letter of the correct answer on a separate sheet of paper.

1. Which of the following statements is true about the figure?
    - A. $m\angle 6 = m\angle 3$
    - B. $m\angle 5 < m\angle 3$
    - C. $m\angle 5 > m\angle 1$
    - D. $m\angle 4 > m\angle 2$



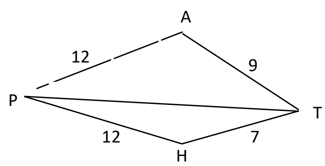
2. Study the figure in item 1. Notice that $m\angle 5 > m\angle 3$ and $m\angle 5 > m\angle 1$. Which theorem justifies these observations?
    - A. Exterior Angle Inequality Theorem
    - B. Triangle Inequality Theorem 1 (Ss→Aa)
    - C. Triangle Inequality Theorem 2 (Aa→Ss)
    - D. Triangle Inequality Theorem 3 (S1 + S2 > S3)

3. In ΔPAY, if $|PA| = 4\ cm$, $|AY| = 5\ cm$, and $|PY| = 3\ cm$, which statement is true?
    - A. $m\angle A > m\angle Y$.
    - C. $m\angle A > m\angle P$.
    - B. $m\angle Y > m\angle P$.
    - D. $m\angle P > m\angle A$.

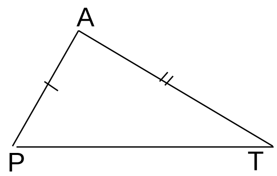
4. Which theorem justifies the statement/answer in item 3?
    - A. Exterior Angle Inequality Theorem
    - B. Triangle Inequality Theorem 1 (Ss→Aa)
    - C. Triangle Inequality Theorem 2 (Aa→Ss)
    - D. Triangle Inequality Theorem 3 (S1 + S2 > S3)

5. In ΔGOD, $|GO| = |DO|$ and $|GD| > |DO|$. Which of the following statements is NOT true?
    - A. $m\angle GOD > m\angle ODG$
    - C. $m\angle ODG = m\angle DGO$
    - B. $m\angle GOD > m\angle DGO$
    - D. $m\angle GOD < m\angle ODG$

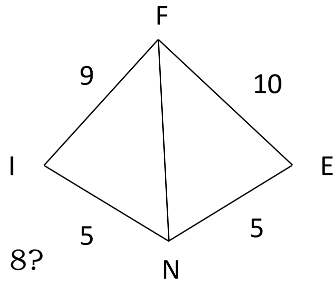
6. Using the lengths of the sides shown in the triangles below, what conclusion can be formed using the Converse of Hinge Theorem?
    - A. $|PT| = |PT|$.
    - B. $|PA| = |PH|$.
    - C. m∠APT > m∠HPT.
    - D. m∠HPT > m∠APT.



7. In the figure below, Khen concluded that $m\angle TPA > m\angle PTA$. Which theorem on inequalities in triangle justifies Khen's conclusion?
   A. Exterior Angle Inequality Theorem
   B. Triangle Inequality Theorem 1 (Ss→Aa)
   C. Triangle Inequality Theorem 2 (Aa→Ss)
   D. Triangle Inequality Theorem 3 (S_1 + S_2 >S_3)



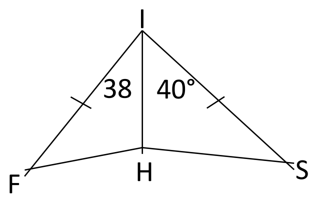
8. Joan analyzed the triangles in the given figure at the right. Which of the following is a correct observation?
   A. $|EF| = |IF|$.
   B. $|IN| > |EN|$.
   C. $|FN| < |FN|$.
   D. $m\angle ENF > m\angle INF$



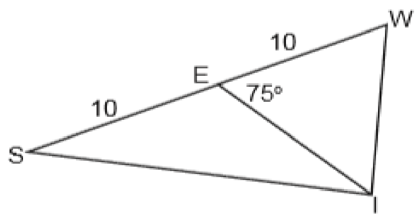
9. Which theorem justifies Joan's conclusion in item no. 8?
   A. Hinge Theorem
   B. Converse of Hinge Theorem
   C. Triangle Inequality Theorem 1 (Ss→Aa)
   D. Triangle Inequality Theorem 3 (S1 + S2 > S3)

10. Using the given figure below, can you conclude that $|HS| > |HF|$ if one of the following statements is not established: $\overline{IH} \cong \overline{IH}$, $\overline{IF} \cong \overline{IS}$, $m\angle SIH > m\angle FIH$?

   A. Yes, I will.
   B. No, I will not.
   C. It is impossible to decide.
   D. It depends on which statement is left out.



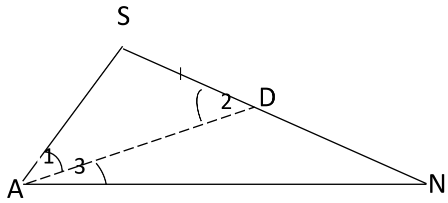
11. In the figure below, Chris has proved that $|IS| > |IW|$. Which of the following statements is NOT part of his proof?
   A. $\overline{ES} \cong \overline{EW}$.
   B. $\overline{EI} \cong \overline{EI}$.
   C. $m\angle W < m\angle S$.
   D. $m\angle WEI + m\angle SEI = 180^0$.



12. What theorem did Chris use to justify his statement/answer in item 11?
   A. Hinge Theorem
   B. Converse of Hinge Theorem
   C. Triangle Inequality Theorem 1 (Ss→Aa)
   D. Triangle Inequality Theorem 3 (S_1 + S_2 > S_3)

13. Complete the statement: Even without actual measurements, inequalities in triangles can be justified __________ using theorems on inequalities in triangles.
   A. inductively   C. specifically
   B. deductively   D. generally

14. Based on the figure below, what should be the reason if the statement is $m\angle SAN = m\angle 1 + m\angle 3$?



A. Linear Pair Postulate
B. Substitution Property
C. Property of Inequality
D. Angle Addition Postulate

15. After a home study session, Khen and Khai went home heading different directions from Chris' house. Khen to the east and Khai to the west. After both have walked 30 meters, Khen turned right at 70^0 and Khai to her right at 50^0. After another 40-meter walk, they both arrived at their respective homes. Who is farther from Chris' house? What theorem supports your answer?

A. Khai, Hinge Theorem
B. Khen, Converse of Hinge Theorem
C. Khen, Triangle Inequality Theorem 1 (Ss→Aa)
D. Khai, Triangle Inequality Theorem 3 ($S_1 + S_2 > S_3$)

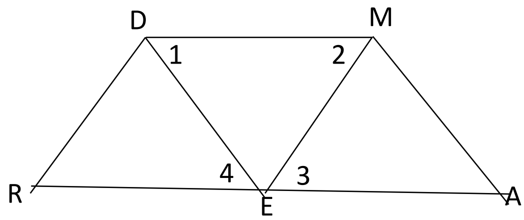
Additional Activities

Directions: Observe and notice the congruent sides and angles shown in the illustration below. Write the missing statement or reason in the two-column proof.

Given: E is the midpoint of $\overline{RA}$,

$\angle 1 \cong \angle 2$, $m\angle 3 > m\angle 4$

Prove: $|MA| > |DR|$



Proof:

Statements	Reasons
1. $\angle 1 \cong \angle 2$	1. Given
2. _____ is an isosceles triangle.	2. Definition of isosceles triangle
3. $\overline{DE} \cong \overline{ME}$.	3. Legs of isosceles triangle are congruent.
4. E is the midpoint of $\overline{RA}$.	4.
5.	5. Definition of a midpoint
6. $m\angle 3 > m\angle 4$.	6.
7. $\|MA\| > \|DR\|$.	7.

Answer Key

What I Know

1. B
2. A
3. A
4. D
5. B
6. B
7. A
8. A
9. A
10. A
11. D
12. D
13. B
14. C
15. D

What's In

I.		II.
1.	E	1. C
2.	A	2. E
3.	B	3. F
4.	F	4. A
5.	C	5. B

What's New

Activity 1:
1. $\overline{PQ} \cong \overline{PS}$
2. ΔPQS is isosceles triangle
3. ∠1≅ ∠2
4. Angle Addition Postulate
5. Property of Inequality
6. Substitution Property
7. m∠QSR+∠R+∠3=180°
10. ∠2=∠PRQ+∠3
11. ∠2>∠R
12. ∠PQR>∠PRQ

Activity 2
3. ∠QPR≅∠PRQ
S: $\angle QPR < \angle QRP$
5. R: Triangle Inequality Theorem 1
6. false
7. true

Activity 3
1. $\overline{PS} \cong \overline{PR}$
2. ΔPRS is isosceles
3. ∠PRS≅∠PSR
4. Reflexive Property of Equality
5. ∠PRS≅∠QSR
6. Angle Addition Postulate
7. Substitution Property
9. Triangle Inequality Theorem 2
11. PQ+PS>QR
12. PQ+PR>QR

Activity 4
2. ∠3 ≅∠4
3. $\Delta \boldsymbol{PTQ} \cong \Delta RTU$
4. ∠QPR≅∠1
5. $\angle 1 + \angle 2$
7. $m < PRS > m < QPR$

Activity 5
1. Segment Addition Postulate
2. Substitution Property of Equality
3. HQ+RQ>HR
4. Substitution Property of Equality
5. Property of Inequality

Activity 6
1 $m\angle FUN \ngtr m\angle ART$.
2. SAS
5. Hinge Theorem
6. False
7. True

What's More

Activity 1: Completing
3. SAS
4. CPCTC
6. Hinge Theorem
7. False
8. True

Activity 2: Proving
1. $\overline{MO} \cong \overline{OE}$
2. $MO = DE$
3. Triangle Inequality Theorem 3 (S1+S2>S3)
4. Substitution Property

Activity 3: Reasoning
1. By construction
2. Definition of isosceles triangle
3. Base angles of isosceles triangle are congruent
4. Reflexive Property of Equality
5. Transitive Property of Equality
6. Angle Addition Postulate
7. Substitution Property
8. Property of Inequality
9. Triangle Inequality Theorem 2
10. Segment Addition Postulate
11. Substitution Property
12. Substitution Property

Activity 4: Justifying
1. Chris
2. Hinge Theorem

What I Have Learned

Answer may vary.

Additional activities

2. ΔDEM
4. Given
5. RE = EA
6. Given
7. Hinge Theorem

What I Can Do

Answers may vary

Assessment

1. C
2. A
3. D
4. B
5. D
6. C
7. B
8. D
9. B
10. B
11. C
12. A
13. B
14. D
15. A

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https://study.com/academy/lessons/applying-Inequaliies-to-triangles,html

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