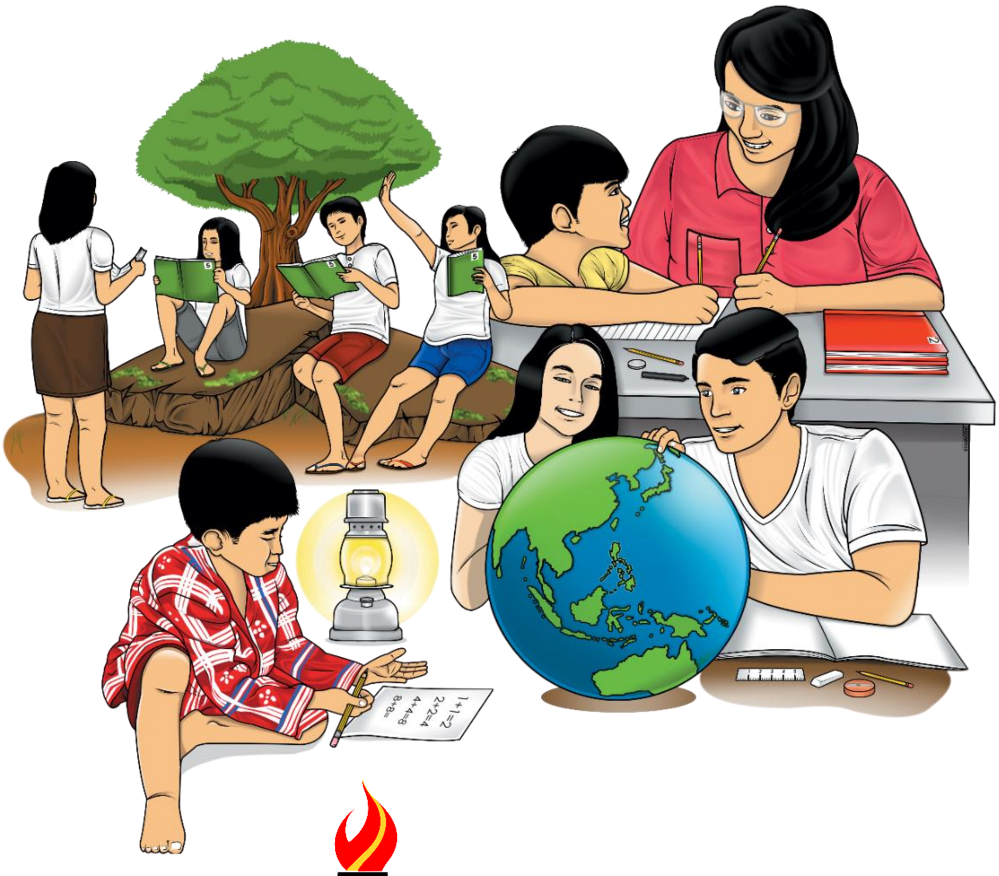
8

Mathematics

Quarter 4 – Module 2

Applying Theorems on Triangle Inequalities



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Mathematics

Quarter 4 – Module 2

Applying Theorems on Triangle Inequalities

DEPARTMENT OF EDUCATION

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

What I Need to Know

This module was designed and written with you in mind. It is here to help you master the skills of applying theorems on triangle inequalities. You are provided with varied activities to process the knowledge and skills learned and to deepen and transfer your understanding of the lesson. The scope of this module enables you to use it in many different learning situations. The lessons are arranged to follow the standard sequence of the course. But the order in which you read them can be changed to correspond with the textbook you are now using.

This module contains lesson on applying theorems on triangle inequalities (M8GE-IVb-1).

After going through this module, you are expected to:

1. demonstrate understanding about the theorems on triangle inequalities;
2. apply theorems on triangle inequalities to find measures of angles and sides of a triangle; and
3. manifest appreciation in applying theorems on triangle inequalities to real-life situations.

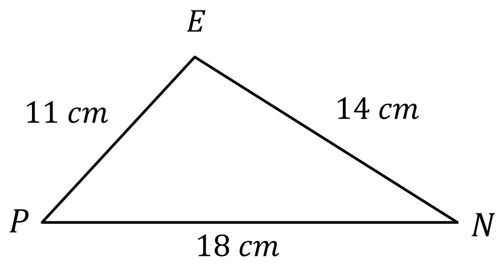
What I Know

Directions: Read and answer each item carefully. Choose the letter of the correct answer. Write your answers on a separate sheet of paper.

1. Carlos concluded that the longest side in ΔMAP is $\overline{AP}$ after finding that the angle opposite to $\overline{AP}$ is the largest angle. What theorem did Carlos use to make such conclusion?
   A. Triangle Inequality Theorem 1 ($Ss \rightarrow Aa$)
   B. Triangle Inequality Theorem 2 ($Aa \rightarrow Ss$)
   C. Hinge Theorem or SAS Triangle Inequality Theorem
   D. Converse of Hinge Theorem or SSS Triangle Inequality Theorem

2. Liza, Kathryn, and Nadine were each given a 21-inch piece of stick. They were instructed to create a triangle. Each stick was cut in their own chosen lengths as follows: Liza - 6 in, 7 in, 8 in; Kathryn - 4 in, 6 in, 11 in; and Nadine - 3 in, 5 in, 13 in. Who among them was able to make a triangle?
   A. Liza B. Nadine C. Kathryn D. All of them

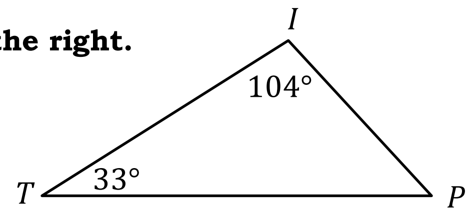
For items 3 – 4, refer to ΔPEN below.



3. What is the largest angle?
   A. $\angle E$ B. $\angle N$ C. $\angle P$ D. Cannot be determined.

4. What is the smallest angle?
   A. $\angle E$ B. $\angle N$ C. $\angle P$ D. Cannot be determined.

For items 5 – 7, consider the triangle ΔTIP at the right.



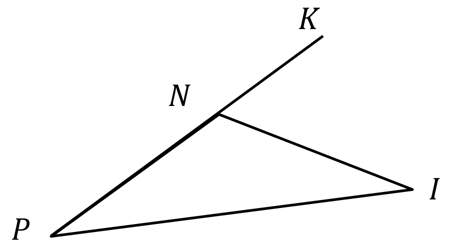
5. Which side of ΔTIP is the longest?
   A. $\overline{TI}$ C. $\overline{TP}$
   B. $\overline{IP}$ D. Cannot be determined.
6. Which side of ΔTIP is the shortest?
   A. $\overline{TI}$ B. $\overline{IP}$ C. $\overline{TP}$ D. Cannot be determined.

7. What theorem did you use in determining the longest and shortest side of ΔTIP?
   A. Triangle Inequality Theorem 1 ($Ss \rightarrow Aa$)
   B. Triangle Inequality Theorem 2 ($Aa \rightarrow Ss$)
   C. Hinge Theorem or SAS Triangle Inequality Theorem
   D. Converse of Hinge Theorem or SSS Triangle Inequality Theorem

8. Which of the following lengths in cm are possible measures of the sides of a triangle?
   A. 1, 2, 3 B. 4, 5, 10 C. 5, 6, 7 D. 5, 9,15

9. Which of the following theorems will support your answer in number 8?
   A. Exterior Angle Inequality Theorem
   B. Triangle Inequality Theorem 1 ($Ss \rightarrow Aa$)
   C. Triangle Inequality Theorem 3 ($S_1 + S_2 > S_3$)
   D. Converse of Hinge Theorem or SSS Triangle Inequality Theorem

For items 10 – 12, consider the figure at the right.



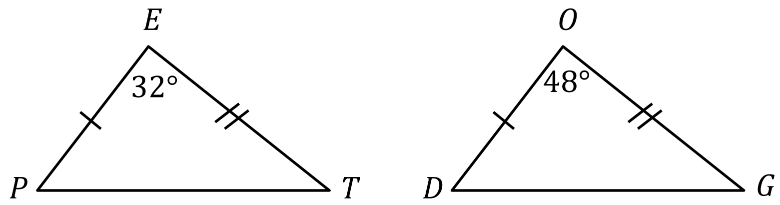
10. Which of the following statements is true?
    A. $m\angle KNI = m\angle PIN.$
    B. $m\angle KNI > m\angle PIN.$
    C. $m\angle KNI < m\angle PIN.$
    D. Cannot be determined.

11. Which of the following statements is true?
    A. $m\angle NPI = m\angle INK.$
    B. $m\angle NPI > m\angle INK.$
    C. $m\angle NPI < m\angle INK.$
    D. Cannot be determined.

12. What theorem did you apply to answer item numbers 10 and 11?
    A. Exterior Angle Inequality Theorem
    B. Triangle Inequality Theorem 1 ($Ss \rightarrow Aa$)
    C. Triangle Inequality Theorem 3 ($S_1 + S_2 > S_3$)
    D. Converse of Hinge Theorem or SSS Triangle Inequality Theorem

13. The measures of the angles of ΔDEF are as follows: $m\angle D = 2x + 3^0$; $m\angle E = 2x + 5^0$; and $m\angle F = x - 8^0$. Arrange the sides in increasing order of length.
    A. $\overline{DE}, \overline{EF}, \overline{DF}$ B. $\overline{EF}, \overline{DE}, \overline{DF}$ C. $\overline{DE}, \overline{DF}, \overline{EF}$ D. $\overline{DF}, \overline{DE}, \overline{EF}$

For items 14 – 15, consider the given ΔPET and ΔDOG below.

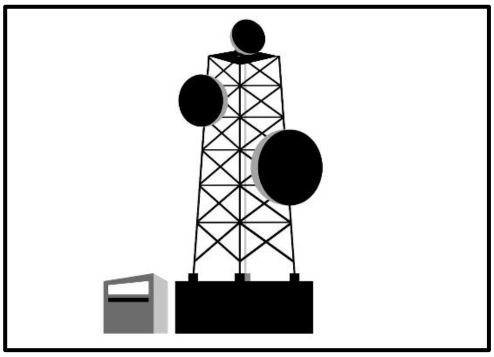
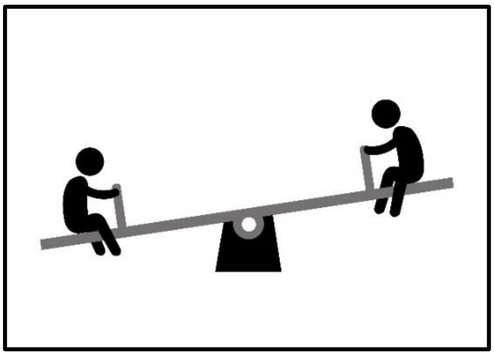
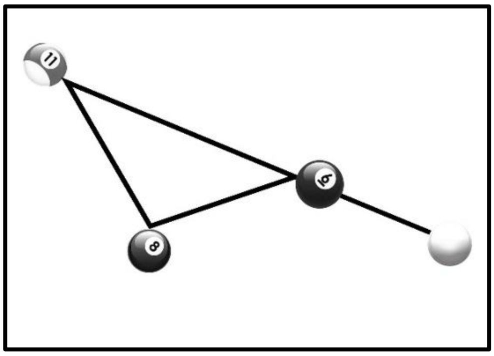
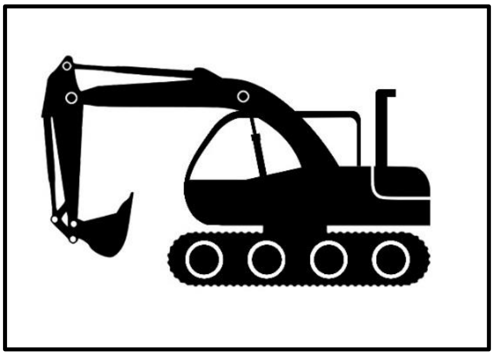
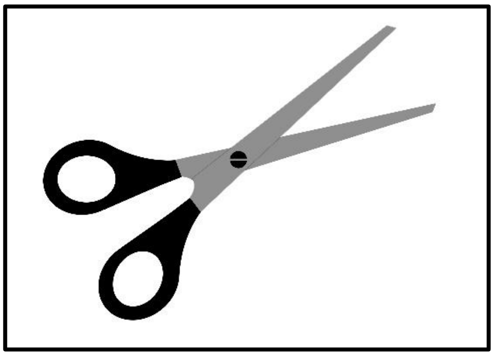
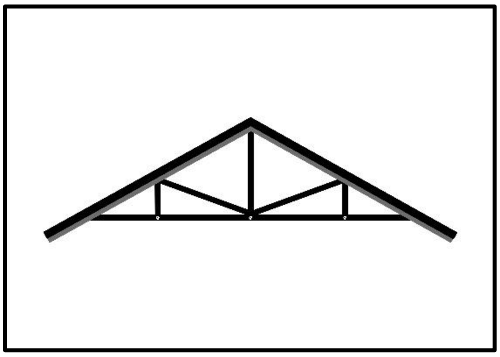


14. What can you conclude in the given figures?
    A. $|PT| = |DG|$ B. $|PT| > |DG|$ C. $|DG| < |PT|$ D. $|DG| > |PT|$

15. What theorem did you use to answer item number 14?
    A. Exterior Angle Inequality Theorem
    B. Triangle Inequality Theorem 1 ($Ss \rightarrow Aa$)
    C. Hinge Theorem or SAS Triangle Inequality Theorem
    D. Converse of Hinge Theorem or SSS Triangle Inequality Theorem

Lesson 1
Applying Theorems on Triangle Inequalities

Have you ever thought of how artists use triangles in their artworks? Have you ever wondered how builders, architects, and engineers use triangular features in their designs? This module reveals the answers to these questions.

What's In

Activity: Find Our Connection

Directions: Connect the if-clause in Column A with the then-clause in Column B to create a statement describing the theorem of triangle inequalities listed in each item below. The first number is done for you. Write your answer on a separate sheet of paper.

THEOREM	COLUMN A	COLUMN B
1. Hinge Theorem or SAS Inequality Theorem	If one side of a triangle is longer than a second side,	then the angle opposite the longer side is larger than the angle opposite the shorter side.
2. Triangle Inequality Theorem 1 (Ss→Aa)	If two sides of one triangle are congruent to two sides of another triangle, but the included angle of the first triangle is greater than the included angle of the second,	then the third side of the first triangle is longer than the third side of the second.
3. Exterior Angle Theorem	If one angle of a triangle is larger than a second angle,	then the side opposite the larger angle is longer than the side opposite the smaller angle.
4. Triangle Inequality Theorem 2 (Aa→Ss)	If the angle is exterior,	then the included angle of the first triangle is larger than the included angle of the second.
5. Converse of Hinge Theorem or SSS Inequality Theorem	If the lengths of two sides of a triangle are added,	then its measure is greater than the measure of either remote interior angle of a triangle.
6. Triangle Inequality Theorem 3 ($S_1 + S_2 > S_3$)	If two sides of one triangle are congruent to two sides of another triangle, but the third side of the first triangle is longer than the third side of the second,	then the sum of their measures is greater than the measure of the third side.

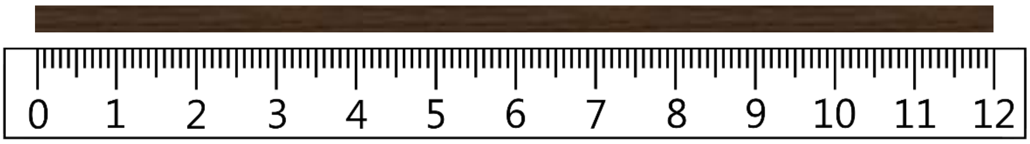
What's New

Activity: Follow My Steps

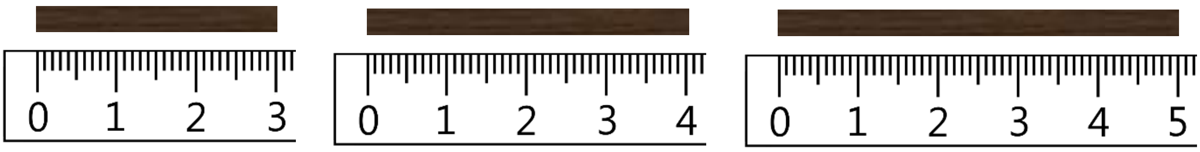
Materials: broom stick, ruler, pen/pencil, cutter

Directions: Do as directed, then answer the questions that follow. Write your answer on a separate sheet of paper.

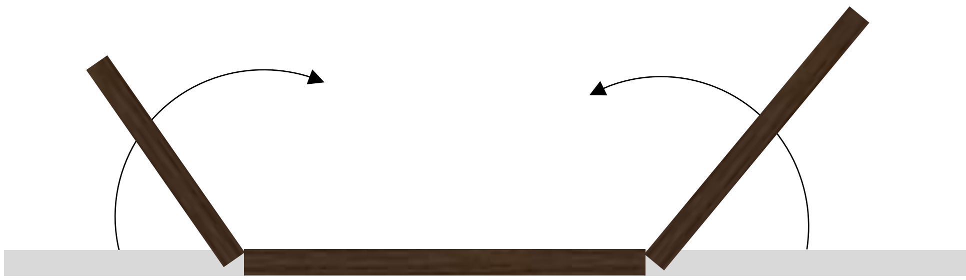
1. Prepare a stick measuring 12 inches. (*Note: The figure is not drawn to scale.*)



2. Using a cutter, cut the stick into parts measuring 3 inches, 4 inches, and 5 inches respectively.



3. Position the sticks as shown in the illustration. Simultaneously move the two shorter lengths up then down.



Questions

1. What figure were you able to form?
2. Was there a chance that the shorter lengths (3 inches and 4 inches) met as you moved them down
3. Can you still form the same figure if the sticks measure 2 inches, 3 inches, and 7 inches, respectively? Explain your answer.

What is It

Theorems on triangle inequalities are categorized into two. These are the inequalities in one triangle and inequalities in two triangles. For inequalities in one triangle, the following theorems apply:

1. Triangle Inequality Theorem 1 ($Ss \rightarrow Aa$)
2. Triangle Inequality Theorem 2 ($Aa \rightarrow Ss$)
3. Triangle Inequality Theorem 3 ($S_1 + S_2 > S_3$)
4. Exterior Angle Inequality Theorem.

On the other hand, inequalities in two triangles use either of these theorems:

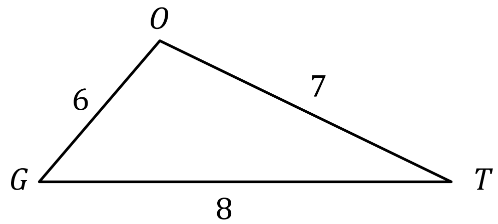
1. Hinge Theorem or SAS Inequality Theorem
2. Converse of Hinge Theorem or SSS Inequality Theorem.

Triangle Inequality Theorem 1 (Ss→Aa)

If one side of a triangle is longer than a second side, then the angle opposite the longer side is larger than the angle opposite the shorter side.

Example 1

List down the angles of ΔGOT from greatest to least measure. *(Note: The figure is not drawn to scale.)*



Solution

In the figure, $\overline{GT}$ is the longest side, thus, the angle opposite it which is $\angle O$,has the greatest measure. Since $\overline{OG}$ is the shortest side, hence the angle opposite it which is $\angle T$, has the least measure.

Listing the angles of ΔGOT from greatest to least measure results to

$$\angle O, \angle G, \angle T.$$

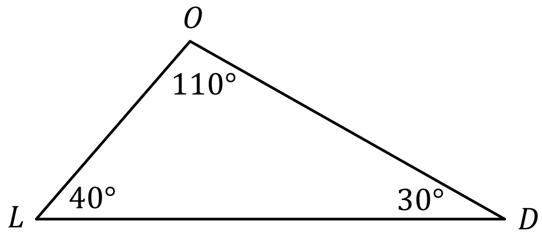
Triangle Inequality Theorem 2 (Aa→Ss)

If one angle of a triangle is larger than a second angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.

[

Example 1

List down the sides of ΔLOD from longest to shortest. *(Note: The figure is not drawn to scale.)*

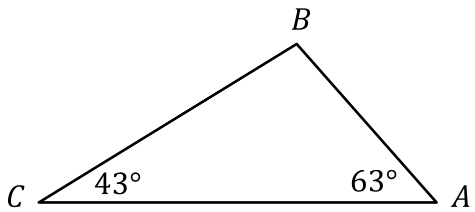


Solution

Writing the angles from the largest to the smallest measure gives $\angle O$, $\angle L$, $\angle D$. Applying Triangle Inequality Theorem 2, it can be concluded that the longest side is $\overline{LD}$, $\overline{OD}$ is the second longest, and $\overline{LO}$ is the shortest side.

Example 2

List down the sides of ΔBCA from the shortest to the longest measure. *(Note: The figure is not drawn to scale.)*



Solution

The first thing to do is to find the unknown measure of $\angle B$. Note that the sum of the measures of the interior angles of a triangle is 180^0. To solve for the measure of $\angle B$, we use the equation $m\angle B + m\angle C + m\angle A = 180°$.

$m\angle B + 43° + 63° = 180°$	*Use Substitution*
$m\angle B + 106° = 180°$	*Simplify*
$m\angle B + 106° - 106° = 180° - 106°$	*Use Subtraction Property of Equality*
$m\angle B + 0 = 180° - 106°$	*Use Identity Property for Addition*
$m\,\angle B = 74°$	*Simplify*

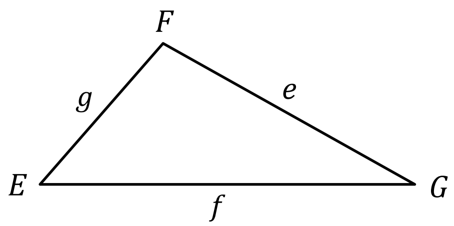
Since $\angle B$ has the greatest measure of 74°, it can be concluded that the side opposite it which is $\overline{AC}$ is the longest , followed by $\overline{BC}$ since $\angle A$ is the second largest, and $\overline{AB}$ is the shortest side since $\angle C$ has the least measure. Listing down the sides of ΔBCA from shortest to longest measure gives $\overline{AB}$, $\overline{BC}$, $\overline{AC}$.

Triangle Inequality Theorem 3 ($S_1 + S_2 > S_3$)

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Illustration

Consider ΔEFG as shown below, with e, f, and g as the side lengths.



The triangle inequality theorem 3 states that :

i) $f + g > e$ ii) $e + g > f$ iii) $e + f > g$

To be able to form a triangle, each of the three inequalities must be true. If one of these inequalities is false, then it is not possible for these lengths to form a triangle. Consider the following examples.

Example 1

Check whether it is possible to form a triangle with lengths 13 cm, 14 cm, and 22 cm.

Solution:

For 13 cm, 14 cm and 22 cm to be considered as side lengths of a triangle, these should satisfy the inequality theorem ($S_1 + S_2 > S_3$).

Hence,

$13 + 14 > 22$	$13 + 22 > 14$	$14 + 22 > 13$
$27 > 22$ True	**$35 > 14$ True**	**$36 > 13$ True**

All the three conditions are satisfied, therefore it is possible to form a triangle with lenghts 13 cm, 14 cm, and 22 cm.

Example 2

Tell whether it is possible to form a triangle with lengths 2 cm, 8 cm, and 12 cm.

Solution

For 2 cm, 8 cm and 12 cm to be considered as side lengths of a triangle, these should satisfy the inequality theorem ($S_1 + S_2 > S_3$).

Thus,

$2 + 8 > 12$	$2 + 12 > 8$	$8 + 12 > 2$
$\mathbf{10 > 12}$ **False**	$\mathbf{14 > 8}$ **True**	$\mathbf{20 > 2}$ **True**

One of the inequalities is false. Therefore, it is not possible to form a triangle with lengths 2 cm, 8 cm, and 12 cm.

Example 3

If two of the sides of a triangular frame measure 8 inches and 15 inches, find the possible lengths of the third side x.

Solution

To find the possible lengths of the third side of the triangular frame, you can use the formula:
$Difference\ of\ the\ lengths\ of\ two\ sides < Unknown\ length\ of\ the\ third\ side < Sum\ of\ the\ lengths\ of\ the\ two\ sides.$

In the situation, the length of the third side is represented by the variable x.

$$Difference\ of\ the\ lengths\ of\ the\ two\ sides < x < Sum\ of\ the\ lengths\ of\ the\ two\ sides$$

$$15 - 8 < x < 15 + 8 \qquad \textit{Use Substitution}$$

$$7 < x < 23 \qquad \textit{Simplify}$$

The third side can have length between 7 inches and 23 inches.

To check whether the length of the third side x could be any value between 7 inches and 23 inches, choose one value in the interval and apply theTriangle Inequality Theorem 3 ($S_1 + S_2 > S_3$). Consider, $x = 10$ inches.

$8 + 15 > 10$	$8 + 10 > 15$	$15 + 10 > 8$
$\mathbf{23 > 10}$ **True**	$\mathbf{18 > 15}$ **True**	$\mathbf{25 > 8}$ **True**

Therefore, the length of the third side can be any value between 7 inches and 23 inches.

Exterior Angle Inequality Theorem

The measure of an exterior angle of a triangle is greater than the measure of either remote interior angles.

Remember

Exterior Angle – an angle that forms a linear pair with one of the interior angles of a triangle.

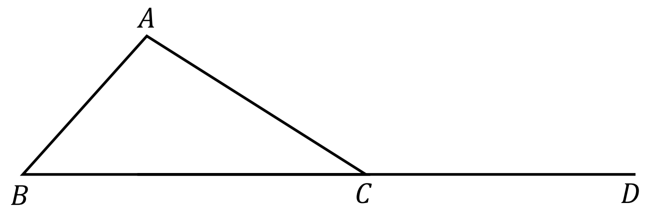
Remote Interior Angle - an angle of a triangle that is not adjacent to a specified exterior angle.

Linear Pair Theorem – if two angles form a linear pair, then the two angles are supplementary and adjacent.

Example 1

Given the figure below, name the following:

a. linear pair
b. exterior angle
c. remote interior angles of $\angle ACD$

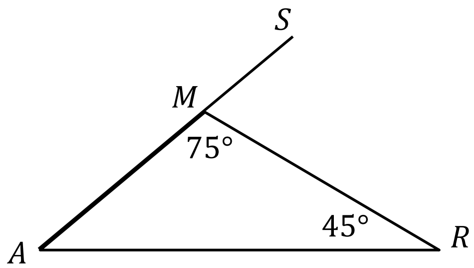


Solution:

a. $\angle ACD$ and $\angle ACB$ form a linear pair.
b. $\angle ACD$ is an exterior angle.
c. $\angle A$ and $\angle B$ are the remote interior angles of $\angle ACD$.

Example 2

Consider the figure below. *(Note: The figure is not drawn to scale.)*



a. Find the measure of $\angle SMR$.
b. Compare the measure of $\angle SMR$ to the measure of each remote interior angle of ΔAMR .

Solution:

a. In the figure, notice that $\angle SMR$ and $\angle AMR$ form a linear pair. This means that the sum of the measures of these angles is 180°. To find the $m\angle SMR$,

$m\angle SMR + m\angle AMR = 180°$	*Apply Linear Pair Theorem*
$m\angle SMR + 75° = 180°$	*Use Substitution*
$m\angle SMR + 75° - 75° = 180° - 75°$	*Use Subtraction Property of Equality*
$m\angle SMR + 0 = 180° - 75°$	*Use Identity Property for Addition*
$m\angle SMR = 105°$	*Simplify*

Hence, $m\angle SMR = 105°$.

b. Find the measure of $\angle A$ of ΔAMR.

$m\angle A + m\angle M + m\angle R = 180°$	
$m\angle A + 75° + 45° = 180°$	*Use Substitution*
$m\angle A + 120° = 180°$	*Simplify*
$m\angle A + 120° - 120° = 180° - 120°$	*Use Subtraction Property of Equality*
$m\angle A + 0 = 180° - 120°$	*Use Identity Property for Addition*
$m\angle A = 60°$	*Simplify*

In the figure, $\angle SMR$ is an exterior angle which has a measure that is greater than either measure of its remote interior angle, $\angle A$ or $\angle R$. In symbols,

$m\angle SMR > m\angle A$
$\mathbf{105°} > \mathbf{60°}$ *True*

and

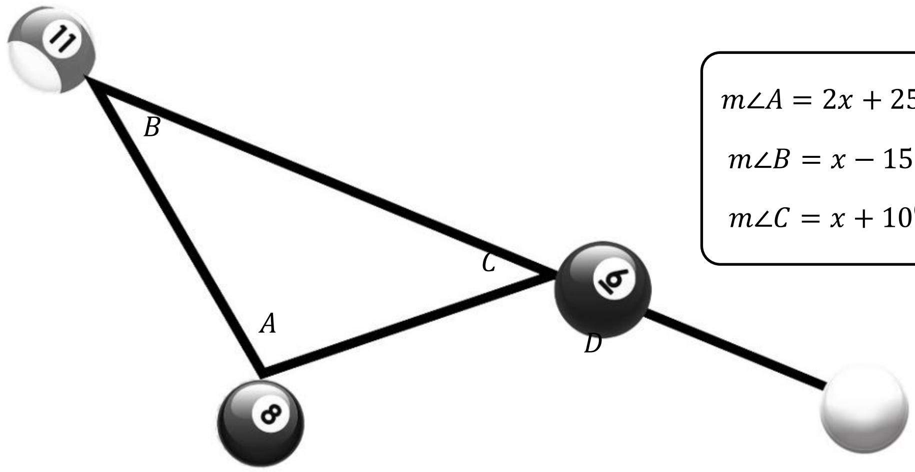
$m\angle SMR > m\angle R$
$\mathbf{105°} > \mathbf{45°}$ *True*

Further, adding the measures of these two remote interior angles will give the measure of the exterior angle. In symbols,

$$m\angle A + m\angle R = m\angle SMR$$
$$60° + 45° = 105°$$

Example 3

Situation: Four billiard balls are left on the table as shown below. Use the expressions to determine the measure of the exterior angle. *(Note: The figure is not drawn to scale.)*



$m\angle A = 2x + 25^0$

$m\angle B = x - 15^0$

$m\angle C = x + 10^0$

Solution:

$(2x + 25^0) + (x - 15^0) + (x + 10^0) = 180^0$ *Apply Sum of the Interior Angles of a Triangle is 180°*

$(2x + x + x) + (25^0 - 15^0 + 10^0) = 180^0$ *Combine like terms*

$4x + 20 = 180°$ *Simplify*

$4x + 20^0 - 20^0 = 180° - 20^0$ *Use Subtraction Property of Equality*

$4x + 0 = 160°$ *Use Identity Property for Addition*

$4x = 160°$ *Simplify*

$\left(\frac{1}{4}\right) 4x = \left(\frac{1}{4}\right) 160°$ *Use Multiplication Property of Equality*

$x = 40°$ *Simplify*

Now, substitute x by its value in the given expressions.

$m\angle A = 2x + 25^0 = 2(40^0) + 25^0 = 80^0 + 25^0 = \mathbf{1}05^0$

$m\angle B = x - 15^0 = 40^0 - 15^0 = 25^0$

$m\angle C = x + 10 = 40^0 + 10^0 = 50^0$

The measures of $\angle A, \angle B$ and $\angle C$ are $105°, 25°$ and $50°$, respectively.

To find the measure of the exterior angle, you can either do the following solutions:

a. Sum of the measures of the remote interior angles
   $25° + 105° = 130°$

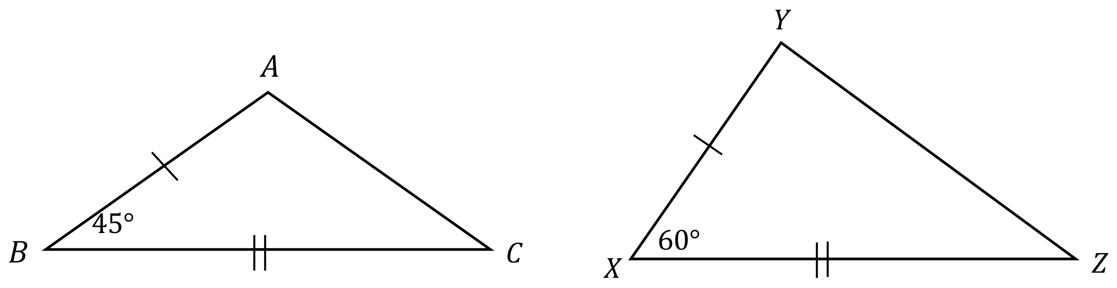
b. Linear Pair Theorem
   $180° - 50° = 130°$

Hinge Theorem or SAS Inequality

If two sides of one triangle are congruent to two sides of a second triangle, and the included angle in the first triangle is greater than the included angle in the second, then the third side of the first triangle is longer than the third side of the second.

Example 1

Consider ΔABC and ΔXYZ. Describe the lengths of sides $\overline{AC}$ and $\overline{YZ}$.
(Note: The figure is not drawn to scale.)



Solution

As shown in the figure,sides $\overline{AB}$ and $\overline{BC}$ of ΔABC are congruent respectively to sides $\overline{YX}$ and $\overline{XZ}$ of ΔXYZ. Applying the Hinge Theorem, $\overline{AC}$ is shorter than $\overline{YZ}$ since the opposite angle to $\overline{AC}$ which is $\angle B$ has a smaller measure than $\angle X$ which is opposite to $\overline{YZ}$.

Example 2

Situation: The figures below show two pairs of scissors of the same size in two different positions. In which figure is the distance between the tips of the two blades greater? Use the Hinge Theorem to justify your answer. *(Note: The figure is not drawn to scale.)*

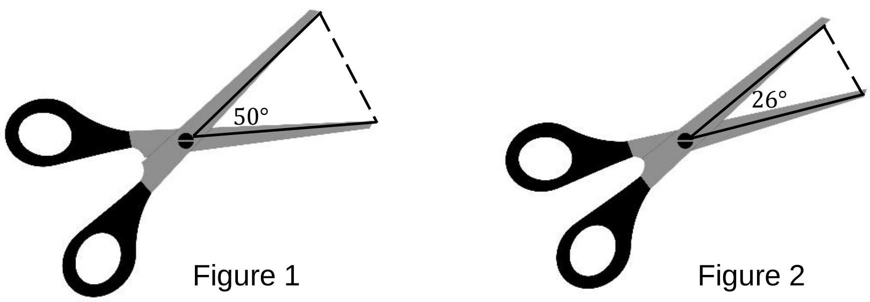


Figure 1 Figure 2

Solution:

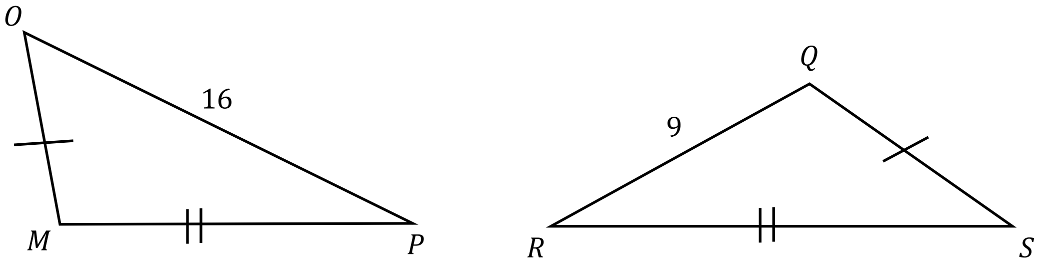
The distance between the tips of the two blades in Figure 1 is greater since the angle opposite this side has a greater measure which is 50° as compared to that in Figure 2 which is only 26°.

Converse of Hinge Theorem or SSS Inequality Theorem

If two sides of one triangle are congruent to two sides of a second triangle, and the third side of the first triangle is longer than the third side of the second, then the included angle in the first triangle is larger than the included angle in the second triangle.

Example 1

Consider ΔMOP and ΔQRS. Which angle is larger, $\angle M$ or $\angle S$?

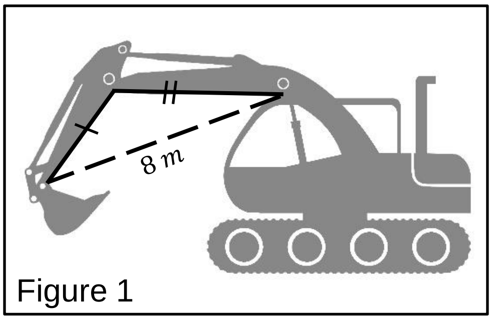


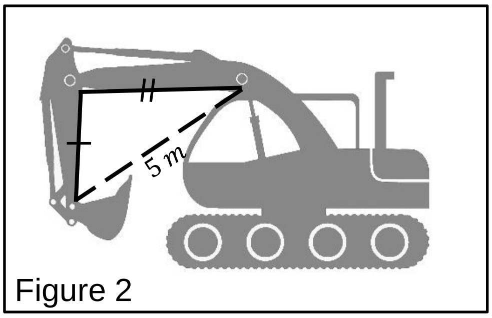
Solution:

In the figure $|MO| = |SQ|$, $|MP| = |SR|$ and $|OP| > |QR|$ or $16 > 9$. Thus, by the use of SSS Inequality Theorem, the measure of $\angle M$ is greater than the measure of $\angle S$.

Example 2

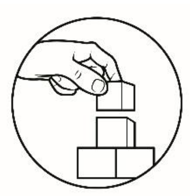
Situation: Apply the SSS Inequality Theorem to determine which figure has a greater measure of the included angle. *(Note: The figure is not drawn to scale.)*





Solution:

Applying the Converse of Hinge Theorem or SSS Inequality Theorem makes the angle opposite the 8-meter side length larger than the angle opposite the 5-meter side length of a triangle. Thus, the included angle in Figure 1 has a greater measure than the included angle in Figure 2.



What's More

Let us deepen your understanding on triangle inequality theorems by performing the following activities. Write your answers on a separate sheet of paper. *(Note: All figures are not drawn to scale.)*

Activity 1

Directions: Arrange the angles of ΔABC from greatest to least measure given the lengths of its sides.

1. $|AB| = 5\ cm,\ |BC| = 10\ cm,\ |AC| = 12\ cm$
2. $|BA| = 10\ cm,\ |CB| = 19\ cm,\ |CA| = 11\ cm$
3. $|AC| = 7\ cm,\ |AB| = 3\ cm,\ |BC| = 5\ cm$

Activity 2

A. Use the Triangle Inequality Theorem 3 ($S_1 + S_2 > S_3$) in determining whether the following numbers in cm can be side lengths of a triangle. Put a check mark (√) to indicate your answer.

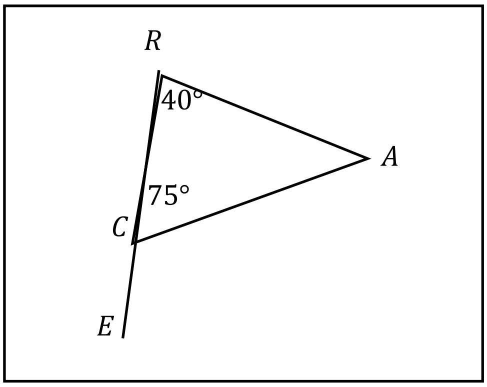
	Yes	No
1) 9, 13, 20	☐	☐
2) 3, 4, 8	☐	☐
3) 4, 25, 30	☐	☐

B. Find the possible values for the length of the third side x using the Triangle Inequality Theorem 3 ($S_1 + S_2 > S_3$).

1) 14, 36 2) 8, 21 3) 13, 40

Activity 3: tri-G L E (Greater than, Less than, or Equal)

Directions: Study the figure below and use >, <, or = to compare the measures of angles and sides.

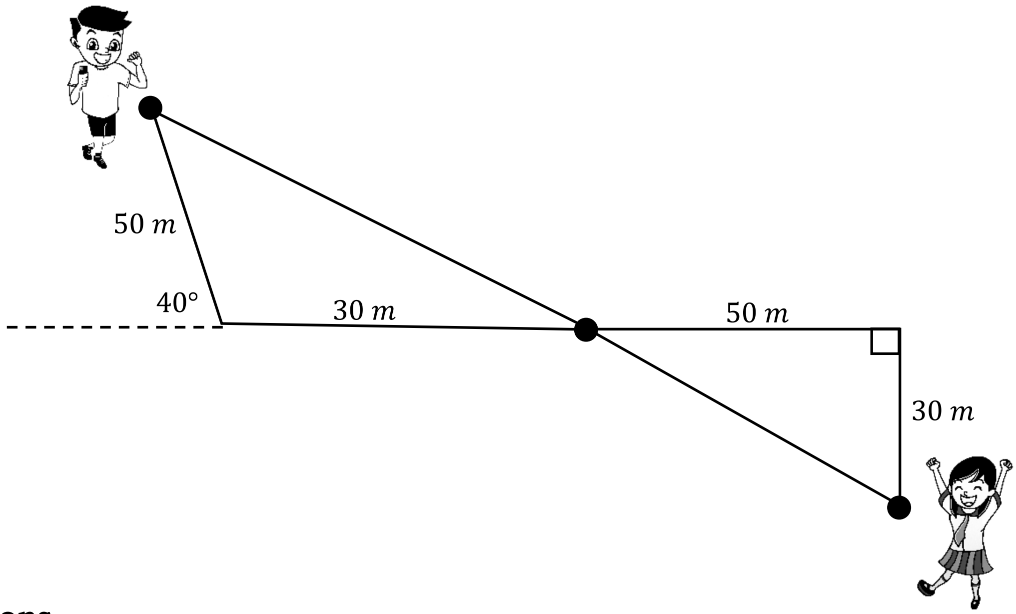


1. $m\angle R$ _____ $m\angle A$
2. $m\angle RCA$ _____ $m\angle RAC$
3. $m\angle ECA$ _____ $m\angle CRA$
4. $m\angle CAR$ _____ $m\angle ECA$
5. $m\angle R + m\angle A$ ______ $m\angle ACE$
6. $|RA|$ _____ $|CR|$
7. $|AC|$ _____ $|CR|$

Activity 4:

Directions: Consider the situation below and answer the questions that follow.

Situation: Anna and Leo start from the same point and decided to walk in opposite directions. Anna walks 50 meters East then took another 30 meters South. Leo walks 30 meters West. He then takes a right turn of 40° and walks another 50 meters as shown in the figure below.

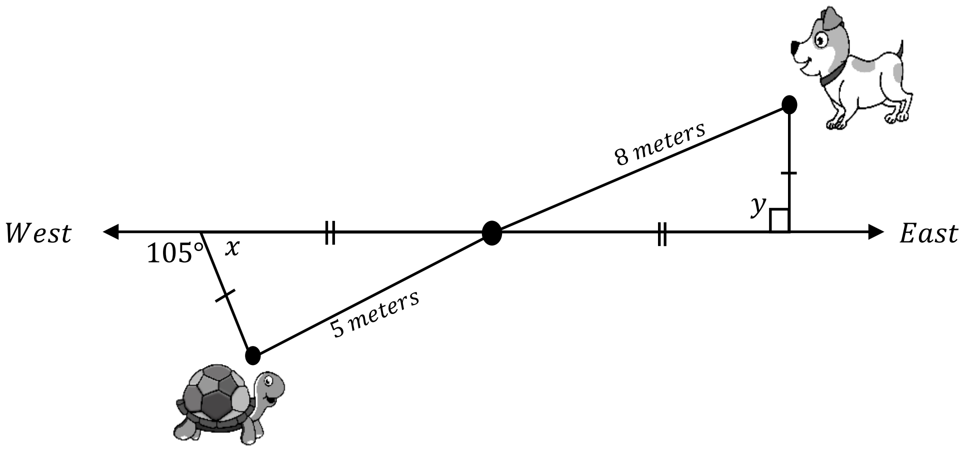


Questions

1. Who is farther from the point where they started? Support your answer.
2. What theorem/s is/are applied in the situation?

Activity 5:

Directions: Study the illustration and answer the questions that follow.



Questions

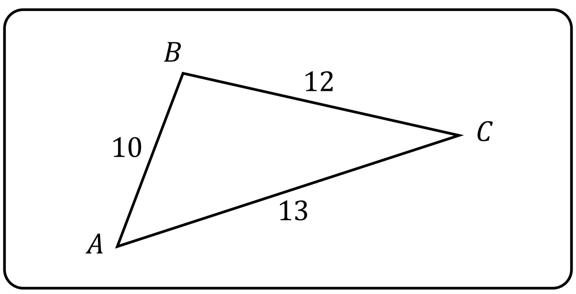
1. In the illustration, which angle between x and y has a greater measure? Support your answer.
2. What theorem/s is/are applied in the situation?

What I Have Learned

Let us synthesize the things that you've learned in this module by performing the next activity. Write your answers on a separate sheet of paper. *(Note: All figures are not drawn to scale.)*

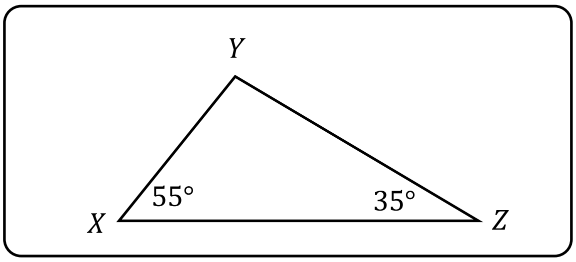
Directions: Give what is asked in each of the following figures.

A.



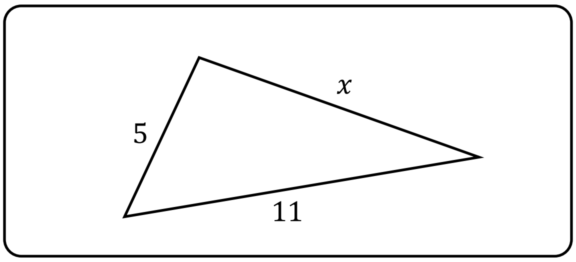
1. Which angle has the greatest measure?
2. What theorem supports your answer?
3. Arrange the angles of ΔABC from least to greatest measure.

B.



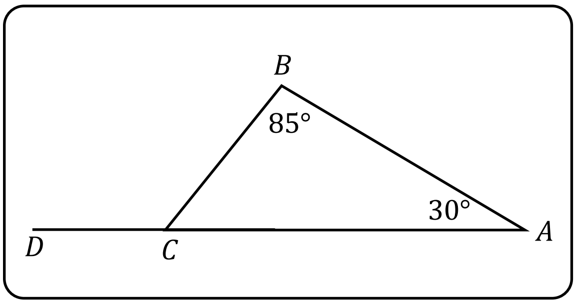
1. What is the measure of $\angle Y$?
2. Which segment is shortest? longest?
3. What theorem justifies your answer in question 2?

C.



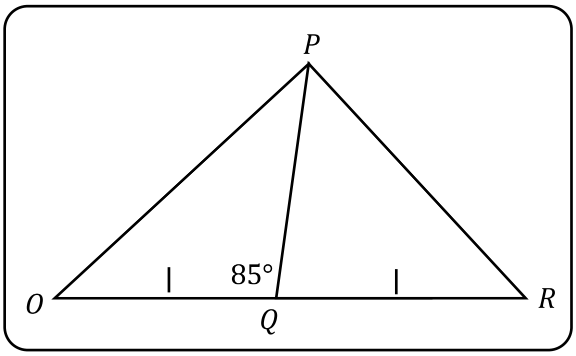
1. What is the range of possible measures of the third side x?
2. What theorem is applied in the problem?

D.



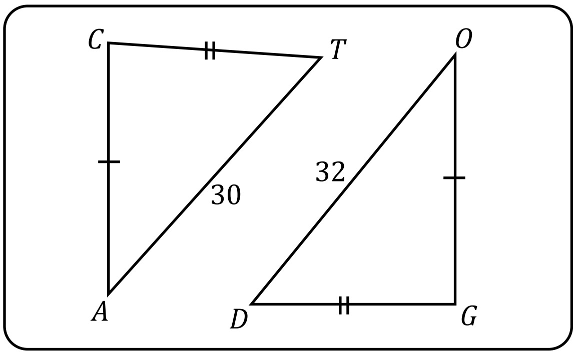
1. Name the exterior angle.
2. What is the measure of $\angle BCD$?
3. Compare the measure of the exterior angle a) to the sum of the measures of the remote interior angles, b) to the measure of either remote interior angle.

E.



1. What is the measure of $\angle RQP$?
2. Between $\overline{OP}$ and $\overline{PR}$, which is longer?
3. What theorem justifies your answer?

F.



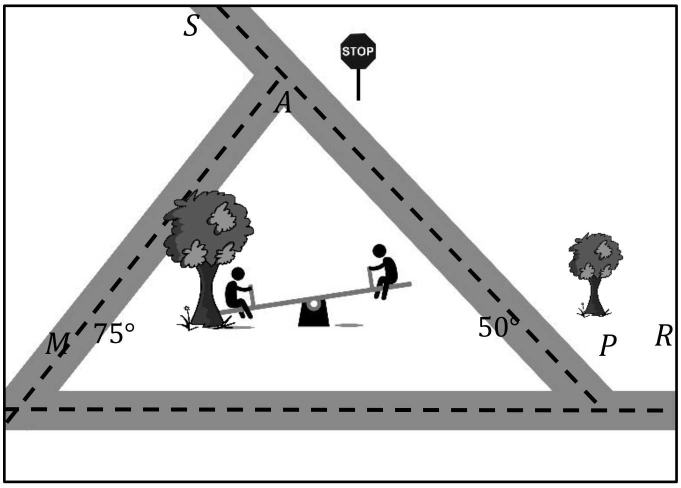
1. Between $\angle C$ and $\angle G$, which is larger?
2. What theorem supports your answer?

What I Can Do

Directions: Read and analyze the given situation and answer the following questions. Write your answers on a separate sheet of paper.

Activity 1: Intersection Ahead!

Situation: Three roads intersect at certain points as illustrated in the figure below.



Questions

1. What is the measure of $\angle A$?
2. List down the angles of ΔMAP from least to greatest measure.
3. Arrange the sides of ΔMAP in ascending order.
4. Between M and A, which intersection is closer to P?
5. List down the exterior angles found in the illustration.
6. Which of these exterior angles has a measure of 125°?

Activity 2: Time's Up!

One of the real-life applications of Hinge Theorem is seen in clocks. In the illustration below, each clock displays different time.

Directions: Observe each clock and answer the questions that follow.

Questions

1. Do the lengths of the hands of the clock change as the time (hour) changes?
2. What do you observe about the measures of the angles formed by the hands of the clock at different hours?
3. What affects the measure of the distance between the tips of the hands of the clock? Explain.

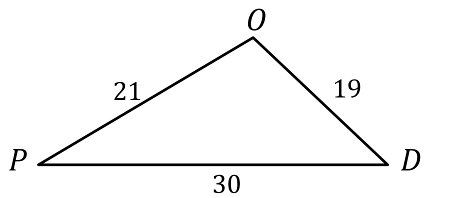
Assessment

Directions: Read and answer each of the questions carefully. Write the letter that corresponds to the correct answer on a separate sheet of paper. *(Note: The figures are not drawn to scale.)*

1. Which is always true about the measure of an exterior angle of a triangle?
   A. It is less than the measure of its adjacent interior angle.
   B. It is less than the measure of either remote interior angle.
   C. It is greater than the measure of its adjacent interior angle.
   D. It is greater than the measure of either remote interior angle.
2. Which of the following numbers of the same unit could represent the side lengths of a triangle?
   A. 1, 3, 5
   B. 2, 4, 5
   C. 3, 6, 10
   D. 4, 5, 11

For items 3 – 5, refer to the △POD at the right.

3. Which is the largest angle?
   A. $\angle D$
   B. $\angle O$
   C. $\angle P$
   D. Cannot be determined

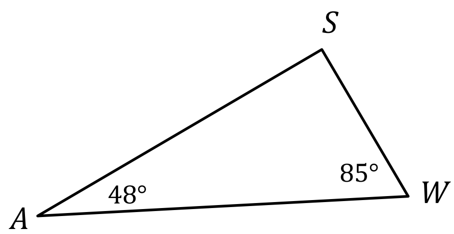


4. Which is the smallest angle?
   A. $\angle D$
   B. $\angle O$
   C. $\angle P$
   D. Cannot be determined.

5. What theorem is applicable in determining the smallest and largest angles of ΔPOD?

   A. Triangle Inequality Theorem 1 (Ss→Aa)
   B. Triangle Inequality Theorem 2 (Aa→Ss)
   C. Hinge Theorem or SAS Triangle Inequality Theorem
   D. Converse of Hinge Theorem or SSS Triangle Inequality Theorem

For items 6 – 8, refer to △SAW at the right.



6. Which side of ΔSAW is the shortest?
   A. $\overline{AS}$
   B. $\overline{SW}$
   C. $\overline{WA}$
   D. Cannot be determined

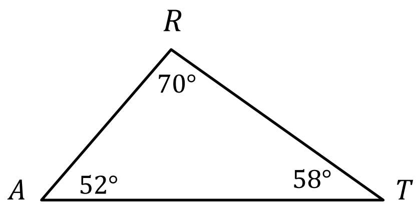
7. Which side of ΔSAW is the longest?
   A. $\overline{AS}$ B. $\overline{SW}$ C. $\overline{WA}$ D. Cannot be determined.

8. What theorem is applicable in determining the shortest and longest sides of ΔSAW?
   A. Triangle Inequality Theorem 1 (Ss→Aa)
   B. Triangle Inequality Theorem 2 (Aa→Ss)
   C. Hinge Theorem or SAS Triangle Inequality Theorem
   D. Converse of Hinge Theorem or SSS Triangle Inequality Theorem

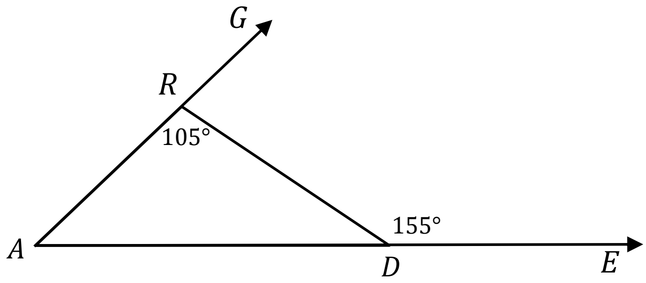
9. Which of the following could be a possible measure of the third side of the triangle if the two sides measure 12 meters and 25 meters?
   A. 11 meters B. 13 meters C. 28 meters D. 37 meters

10. Given below is ΔART. List the sides from the longest to the shortest.

   A. $\overline{AT}, \overline{AR}, \overline{RT}$
   B. $\overline{AR}, \overline{TR}, \overline{TA}$
   C. $\overline{RT}, \overline{RA}, \overline{TA}$
   D. $\overline{TA}, \overline{TR}, \overline{AR}$



For items 11 – 13, consider the figure below.



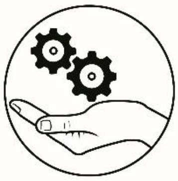
11. What is the measure of $\angle A$?
    A. 25° B. 50° C. 105° D. 155°

12. What is the measure of $\angle GRD$?
    A. 25° B. 75° C. 105° D. 155°

13. Which is the longest side of ΔRAD?
    A. $\overline{AR}$ B. $\overline{DA}$ C. $\overline{RD}$ D. Cannot be determined.

14. What theorem supports this statement, "In an obtuse triangle, the longest side is opposite the obtuse angle."
    A. Triangle Inequality Theorem 1 ($Ss \rightarrow Aa$)
    B. Triangle Inequality Theorem 2 ($Aa \rightarrow Ss$)
    C. Hinge Theorem or SAS Triangle Inequality Theorem
    D. Converse of Hinge Theorem or SSS Triangle Inequality Theorem

15. You are asked to fence a triangular lot. Two sides of the lot have lengths 20 meters and 14 meters. What is the maximum whole number of meters of fence do you possibly need?
    A. 34 metersB. 35 meters C. 67 meters D. 68 meters

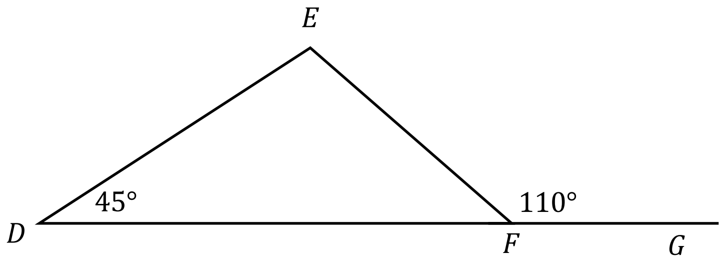


Additional Activities

Directions: Answer the following activities as indicated. Write your answers on a separate sheet of paper.

Activity 1

In ΔDEF, $m\angle D = 45°$and the exterior angle $\angle EFG$ measures 110°.



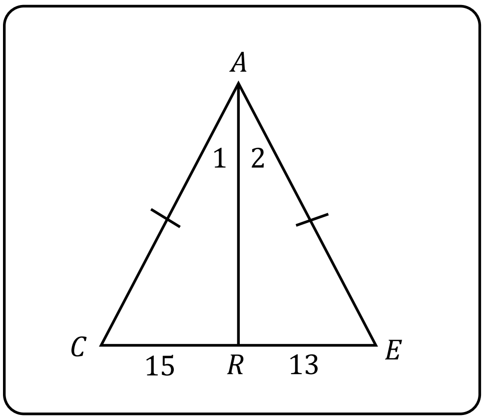
Questions:

1. What is the measure of $\angle DFE$?
2. Arrange the sides of ΔDEF from shortest to longest.
3. Suppose $|DE| = 8$ cm, $|EF| = 4$ cm, $|DF| = 11cm$, arrange the angles of ΔDEF from largest to smallest.
4. What are the remote interior angles in the figure?

Activity 2

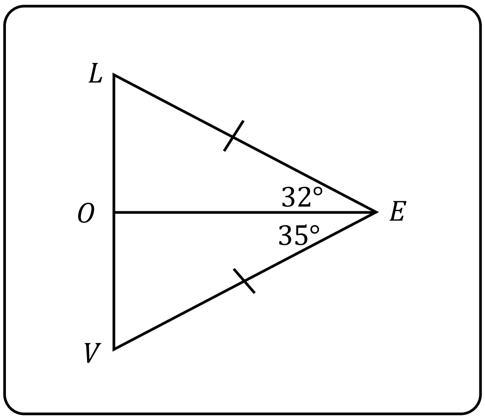
Refer to each figure below. Write an inequality relating the given pair of angle measures or pair of segment measures.

a.



$m\angle 1$ $\square$ $m\angle 2$

b.



LO $\square$ VO

Answer Key

What I Know

1. B
2. A
3. A
4. B
5. C
6. B
7. B
8. C
9. C
10. B
11. C
12. A
13. A
14. D
15. C

What's In

2. **Triangle Inequality Theorem 1.** If one side of a triangle is longer than a second side, then the angle opposite the longer side is larger than the angle opposite the shorter side.
3. **Exterior Angle Inequality Theorem**. The measure of an exterior angle of a triangle is greater than the measure of either remote interior angles.
4. **Triangle Inequality Theorem 2.** If one angle of a triangle is larger than a second angle, then the side opposite the larger angle is longer than the side opposite the shorter angle.
5. **Converse of Hinge Theorem.** If two sides of one triangle are congruent to two sides of another triangle, but the third side of the first triangle is longer than the third side of the second, then the included angle of the first triangle is larger than the included angle of the second.
6. **Triangle Inequality Theorem 3.** The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

What's New

Expected Answers:

1. Triangle
2. Yes
3. No, the shorter legs will not meet.

What's More

Activity 1

1. $\angle B, \angle A, \angle C$
2. $\angle A, \angle B, \angle C$
3. $\angle B, \angle A, \angle C$

Activity 2

A.
1. Yes
2. No
3. No

B.
1. $22 < x < 50$
2. $13 < x < 29$
3. $27 < x < 53$

Activity 3

1. $<$
2. $>$
3. $>$
4. $<$
5. $=$
6. $>$
7. $<$

Activity 4

1. Leo
2. Hinge Theorem

Activity 5

1. y
2. Converse of Hinge Theorem

What I Have learned

A. 1. $\angle B$
   2. Triangle Inequality Theorem 1
   3. $\angle C, \angle A, \angle B$

B. 1. 90°
   2. shortest $\overline{XY}$; Longest $\overline{XZ}$
   3. Triangle Inequality Theorem 2

c. 1. $6 < x < 16$
   2. Triangle Inequality Theorem 3

D. 1. $\angle BCD$
   2. 115°
   3. a. $=$, b. $>$

E. 1. 95°
   2. $\overline{PR}$
   3. Hinge Theorem

F. 1. < 6
   2. Converse of Hinge Theorem

What I Can Do

Activity 1

1. 55°
2. $\angle P, \angle A, \angle M$
3. $\overline{AM}$, $\overline{MP}$, $\overline{AP}$
4. M
5. $\angle MAS$ and $\angle SPR$
6. $\angle MAS$

Activity 2

1. No
2. Change
3. angle, the smaller the angle the shorter its opposite side

Assessment

1.	D
2.	B
3.	B
4.	C
5.	A
6.	C
7.	A
8.	B
9.	C
10.	A
11.	B
12.	B
13.	B
14.	B
15.	C

Additional Activities

Activity 1

1. 70°
2. $\overline{EF}$, $\overline{DF}$, $\overline{DE}$
3. $\angle E, \angle F, \angle D$
4. $\angle D, \angle E$

Activity 2

a. $>$
b. $<$

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