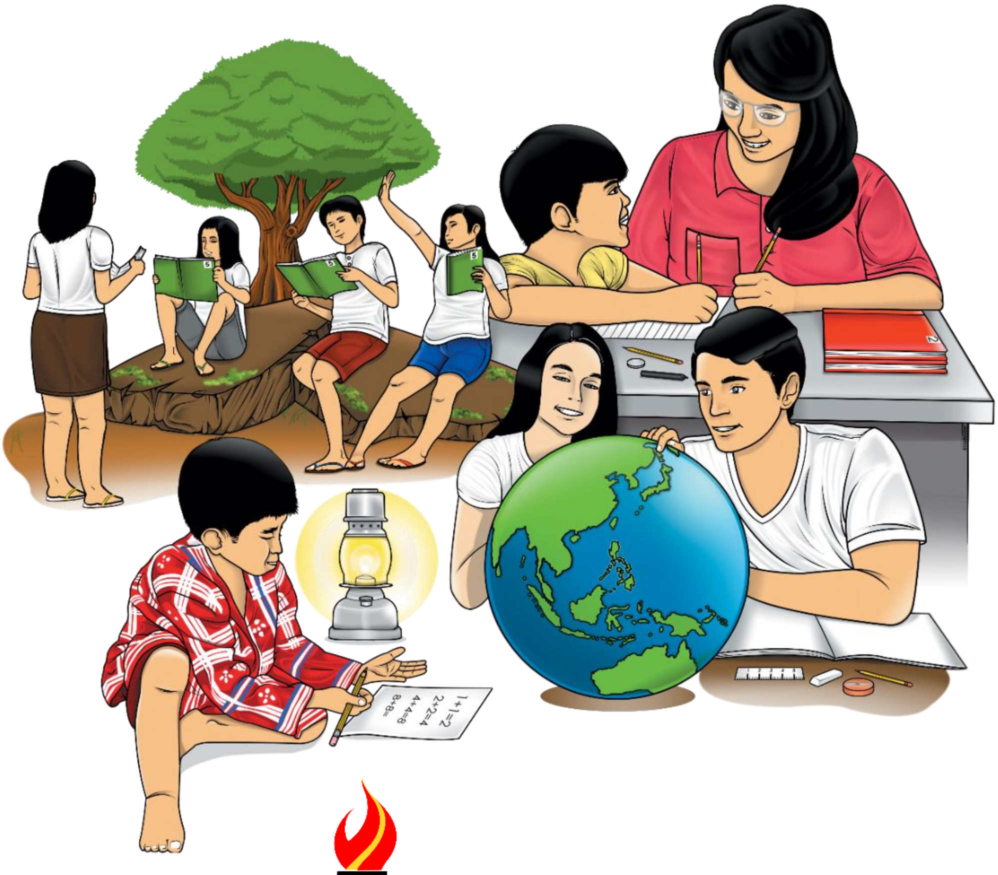
8

Mathematics

Quarter 4 – Module 1
Illustrating Theorems on Triangle Inequalities



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Mathematics

Quarter 4 – Module 1

Illustrating Theorems on Triangle Inequalities

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

What I Need to Know

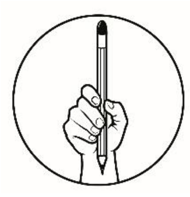
This module will assist you learn the concept of inequalities in triangles. It focuses on how to illustrate and apply theorems on triangle inequalities. You are provided with varied tasks and activities to deepen your understanding of the lesson. This module was designed to be self-sufficient for the current learning situations. The lesson is arranged to follow the standard sequence of the course in the curriculum guide. However, the order in which you read them can be changed to correspond with the textbook you are now using.

This module contains lesson on illustrating theorems on triangle inequalities:

- Exterior Angle Inequality Theorem;
- Triangle Inequality Theorem; and
- Hinge Theorem.

After going through this module, you are expected to:

1. investigate the relationship between the longest side and the largest angle in the triangle and vice versa;
2. investigate the relationship between the sum of any two sides and the remaining sides in a triangle;
3. illustrate theorems on triangle inequalities such as the Exterior Angle Inequality Theorem, Triangle Inequality Theorem, and Hinge Theorem with its converse; and
4. connect theorems in triangle inequalities in real-life setting.



What I Know

Pre- Assessment

Directions**:** Choose the letter of the correct answer. Write your answer on a separate sheet of paper.

1. Which theorem states that if one side of a triangle is longer than a second side, then the angle opposite the first side is larger than the angle opposite the second side?
   A. Triangle Inequality Theorem
   B. Angle-Side Relationship Theorem
   C. Exterior Angle Inequality Theorem
   D. Hinge Theorem or SAS Inequality Theorem

2. In Triangle Inequality Theorem, which of the following describes the relationship of the sum of its two sides to its third length?
   A. greater than
   B. greater than or equal to
   C. less than
   D. less than or equal to

3. Which of the following theorems on triangle inequalities states that the measure of an exterior angle of a triangle is greater than the measure of either remote interior angle?
   A. Triangle Inequality Theorem
   B. Angle-Side Relationship Theorem
   C. Exterior Angle Inequality Theorem
   D. Hinge Theorem or SAS Inequality Theorem

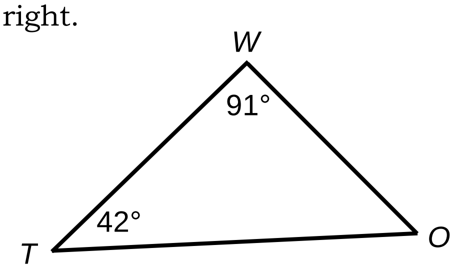
4. Which of the following theorems deals with the two triangles whose two corresponding sides are congruent and whose included angle are unequal?
   A. SAS Inequality Theorem
   B. SSS Inequality Theorem
   C. Angle-Side Relationship Theorem
   D. Exterior Nagle Inequality Theorem

5. Which of the following could NOT be used as the length of the sides of a triangle?
   A. 3, 3, 3 C. 5, 7, 9
   B. 4, 5, 10 D. 10, 20, 20

6. Which of the following statements is FALSE?
   A. Any lengths of the sides can be used to form a triangle.
   B. The largest angle of a triangle is opposite to the longest side.
   C. The smallest side of a triangle is opposite to the smallest angle.
   D. The sum of any two sides of a triangle is greater than the third side.

7. If two sides of one triangle are congruent to two sides of another triangle, but the third side of the first triangle is longer than the third side of the second, then the included angle of the first triangle is larger than the included angle of the second. Which of the following theorems pertains to two triangles whose two corresponding sides are congruent and whose third sides are unequal?
   A. Triangle Inequality Theorem
   B. Exterior Angle Inequality Theorem
   C. Hinge Theorem or SAS Inequality Theorem
   D. Converse of Hinge Theorem or SSS Inequality Theorem

In items 8 to 10, refer to the figure at the right.



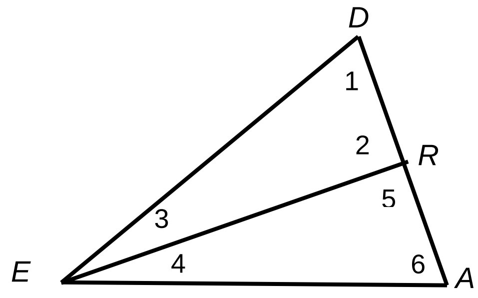
8. Which side of ΔTWO is the longest?
   A. $\overline{WT}$
   B. $\overline{OW}$
   C. $\overline{OT}$
   D. Cannot be determined

9. Which side is the shortest?
   A. $\overline{WT}$
   B. $\overline{OW}$
   C. $\overline{OT}$
   D. Cannot be determined

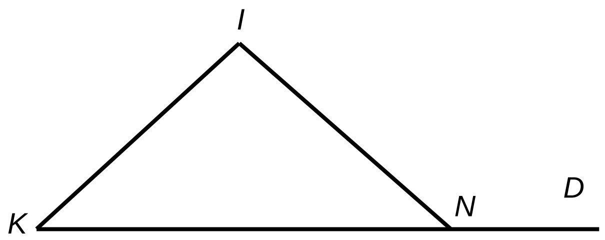
10. What theorem of triangle inequalities is applied?
   A. SSS Inequality Theorem
   B. Triangle Inequality Theorem
   C. Angle-Side Relationship Theorem
   D. Exterior Angle Inequality Theorem

11. In $\Delta\ NET$, $\overline{NE} = 14\ cm$, $\overline{ET} = 15\ cm$, $\overline{NT} = 16\ cm$. What is the correct order of the angles when arranged from least to greatest?
   A. $\angle E, \angle N, \angle T$
   B. $\angle E, \angle T, \angle N$
   C. $\angle T, \angle E, \angle N$
   D. $\angle T, \angle N, \angle E$

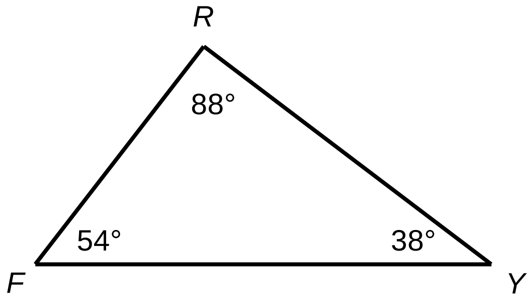
12. Given the figure at the right, which of the following angles is an exterior angle of $\Delta\ DER$?
    A. $\angle 3$
    B. $\angle 4$
    C. $\angle 5$
    D. $\angle 6$



13. Which of the following statement about the given figure is true?
    A. $\angle KIN = \angle DNI$
    B. $\angle KIN > \angle IND$
    C. $\angle DNI > \angle KIN$
    D. $\angle IKN > \angle IND$



14. In $\Delta\ FRY$, which of the following shows the correct arrangement of its sides in descending order?
    A. $\overline{FY}, \overline{FR}, \overline{RY}$
    B. $\overline{FR}, \overline{RY}, \overline{FY}$
    C. $\overline{FY}, \overline{RY}, \overline{FR}$
    D. $\overline{RY}, \overline{FR}, \overline{FY}$



15. Ruel, Geralyn, Reymond and Shiela were given a 24-inch piece of stick each. They were instructed to create a triangle. Each cut the stick in their own chosen lengths as follows: Ruel—*8 in, 8 in, 8 in*; Geralyn—*5 in, 7 in, 12 in*; Reymond—*9 in, 7 in, 8 in*; and Shiela—*6 in, 8 in, 10 in*. Who among them could NOT be able to form a triangle?
    A. Ruel
    B. Geralyn
    C. Shiela
    D. Reymond

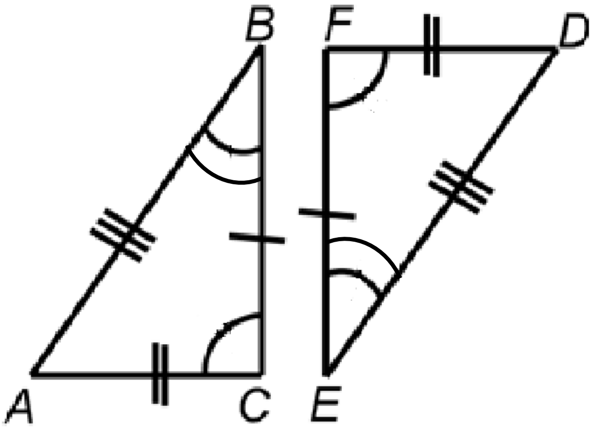
Lesson 1 Illustrate Theorems on Triangle Inequalities

Have you ever thought how the engineers and architects make use of triangular concept in designing a house? What mathematical concept may justify the triangular designs that they use? The answers to these questions are revealed in this module.

What's In

Investigate Me!

Directions: Use the figure below to answer the questions that follow. Write your answer on a separate sheet of paper.



1. What is the included side in ∠B and ∠C? in ∠E and ∠F?
2. What is the included angle in $\overline{AC}$ and $\overline{BC}$? in $\overline{DF}$ and $\overline{EF}$?
3. What is the sum of the interior angles of ΔABC? ΔDEF?
4. If ∠B ≅ ∠E, and ∠C ≅ ∠F, what additional information is required to tell that the triangles are congruent using SAS Congruence?
5. If $\overline{AC} \cong \overline{DF}$ and $\overline{BC} \cong \overline{EF}$, what additional information is required to tell that the triangles are congruent using SSS Congruence?

Questions:

1. Did you find any difficulty in the conduct of the activity?
2. What did you do to overcome this difficulty?

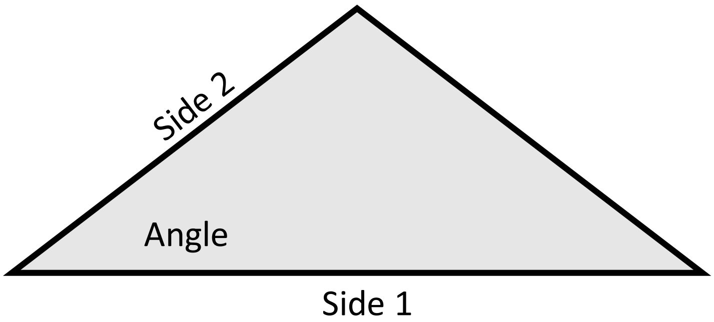
What's New

The activity ***Investigate Me*** is a recall of your understanding on the different angles and sides of triangles. In the next activity you will be using some measuring tools, pen and paper to further understand the concept behind the previous activity.

Measuring Angles and Sides

Materials needed: protractor, ruler, paper and pencil/pen

Directions: Using measuring tools, construct an angle based on the given measurements of the angle and the length of the sides. Lastly connect the end point of each side to form a triangle and find its measurement. Write your answer on a separate sheet of paper. An illustration is provided to guide you in how to do the measurement.



1. Given that Side 1 is 3 inches and Side 2 is 2 inches. Find the length between the end points of the side when the angle measures:

Angle:	30°	45°	90°	120°	150°
Length between the end points					

2. Given that both sides of the angle measures 2 inches. Find the length between the end points of the side when the angle measures:

Angle:	30°	45°	90°	120°	150°
Length between the end points					

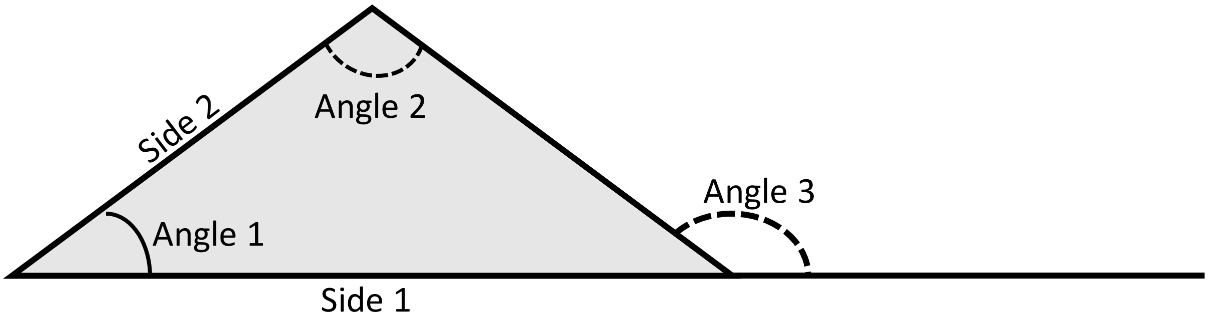
Questions:

1. What can you say about the length of the side connecting the endpoints? What pattern were you able to establish?
2. What can you say about the relationship of the measurement of the angle and the length of the opposite side?
3. Is there a relationship between the length of a side of a triangle and the measure of the angle opposite it? If your answer is YES, indicate its relationship.

Extending the triangle

Materials needed: protractor, ruler, paper and pencil/pen

Directions: Using measuring tools extend side 1 of the triangles formed in the activity **Measuring Angles and Sides** as illustrated below. Supply the measurements of the angle 2 and angle 3 in each table. Write your answer on a separate sheet



1. Given that Side 1 is 2 inches and Side 2 is 3 inches. Find measurement of angles 2 and 3 when the measurement of angle 1 is:

Angle 1:	30°	45°	90°	120°	180°
Angle 2:					
Angle 3:					

2. Given that both sides of the angle measures 2 inches. Find measurement of angles 2 and 3 when the measurement of angle 1 is:

Angle 1:	30°	45°	90°	120°	180°
Angle 2:					
Angle 3:					

Questions:

1. Is there a relationship between the measurement of angle 1 and angle 2? How about between angle 1 and angle 3? Is the relationship also true between angle 2 and angle 3?
2. What generalization can you draw from the measurement of angle 3 to the measurement of angle 1 and measurement of angle 2?

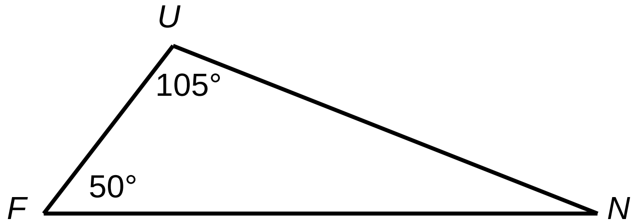
What is It

There are various theorems on triangle inequalities. These theorems can be illustrated in one triangle and in two triangles. The most commonly used theorems in one triangle are Angle-Side Relationship Theorem, Triangle Inequality Theorem, and Exterior Angle Inequality Theorem. On the other hand, the Hinge Theorem (SAS Inequality Theorem) and the Converse of Hinge Theorem (SSS Inequality Theorem) illustrate the inequalities in two triangles. To fully understand these theorems, read and understand the figures and illustrations below.

Inequalities in One Triangle

In your activity in ***Measuring Angles and Sides*** you found out that there is a relationship between the measure of the angle and the length of the side of a triangle. This theorem is referred to as Angle-Side Relationship Theorem which states that: In a triangle, the side opposite the larger angle is the longer side and vice versa. To know more about this theorem, examine the following examples:

Example 1: Compare the length of the sides of the following triangle.



Solution:

Step 1: Find the measure of the third angle. The sum of all the angles in any triangle
is 180°.

$\angle F + \angle U + \angle N$	$=$	$180°$	Sum of interior angles of a triangle
$50° + 105° + \angle N$	$=$	$180°$	By substitution
$\angle N + 155°$	$=$	$180°$	Adding 50° and 105°
$\angle N + 155° + (-155°)$	$=$	$180° + (-155°)$	Addition property of equality
$\angle N + 0$	$=$	$180° - 155°$	Identity property of addition
$\angle N$	$=$	$25°$	By simplifying

Step 2: Look at the relative sizes of the angles and compare.

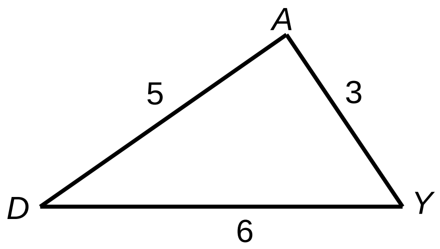
$$\angle N < \angle F < \angle U$$

Step 3: Following the angle-side relationship we can order the sides accordingly. Remember it is the side opposite the angle.

$\overline{FU} < \overline{UN} < \overline{FN}$

Thus, $\overline{FN}$ is the longest side since it is the opposite side of the largest angle, $\angle U$, while $\overline{FU}$ is the shortest side whose opposite angle, $\angle N$ measures $25°$.

Example 2: Compare the measure of the angles of the following triangle.



Solution:

Step 1: Since the length of the sides were given, we can easily compare the lengths from shortest to longest.

$$\overline{AY} < \overline{DA} < \overline{DY}$$

Step 2: Following the angle-side relationship we can order the angles opposite to these sides accordingly.

$$\angle D < \angle Y < \angle A$$

Therefore, $\angle A$ is the largest angle whose is opposite to side DY whose side is 6 units. It follows with the other angles. **Example 1** and **Example 2** illustrate the angle-side relationship theorem.

Angle-Side Relationship Theorem

If two angles of a triangle are not congruent, then the larger side is opposite the larger angle.

If two sides of a triangle are not congruent, then the larger angle is opposite the larger side.

How can we form triangles?

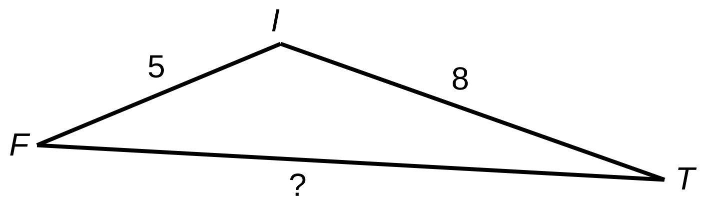
Not any three lengths can form the sides of a triangle. Let a, b, and c be the lengths of the sides of a triangle. A triangle can only be formed under the following conditions:

$$a + b > c$$

$$a + c > b$$

$$b + c > a$$

Example 3: In Δ FIT, find the range of the possible lengths of $\overline{FT}$.



(Note: The figure is not drawn to scale.)

Solution: Let a = 5 and b = 8

Step 1: Using the triangle inequality theorem for the above triangle gives us three statements:

$a + b > c \quad \Rightarrow \quad 5 + 8 > c \quad \Rightarrow \quad c < 13$

$a + c > b \quad \Rightarrow \quad 5 + c > 8 \quad \Rightarrow \quad c > 3$

$b + c > a \quad \Rightarrow \quad 8 + c > 5 \quad \Rightarrow \quad c > -3$ *(disregard because lengths must be positive)*

Step 2: Combining the two valid statements:

$3 < c < 13$ (The length of $\overline{FT}$ is greater than 3 and less than 13)

Triangle Inequality Theorem

The sum of the lengths of any two sides of a triangle is greater than the length of the remaining side.

Let a, b, and c be the lengths of a triangle. These lengths may only form a triangle if the three conditions are satisfied:

$$a + b > c;$$
$$a + c > b;$$
$$c + b > a.$$

Let b an unknown side of a triangle. To find the range of possible measure of side b, the inequality below may be used:

$$(a - c) < b < (a + c)$$

Example 4: Which length/s can form a triangle?

A.) 3, 4, 6 B.) 5, 6, 11 C.) 2, 3, 9

Solution: Check the lengths given if it will form a triangle using Triangle Inequality Theorem.

Given lengths	$a = 3, b = 4, c = 6$		$a = 5, b = 6, c = 11$		$a = 2, b = 3, c = 9$	
$a + b > c$	$3 + 4 > 6$	True	$5 + 6 > 11$	False	$2 + 3 > 9$	False
$a + c > b$	$3 + 6 > 4$	True	$5 + 11 > 6$	True	$2 + 9 > 3$	True
$b + c > a$	$4 + 6 > 3$	True	$6 + 11 > 5$	True	$3 + 9 > 2$	True
Decision	Triangle		Not a Triangle		Not a Triangle	

Based on the Triangle Inequality Theorem, only option A satisfies the condition that will form a triangle.

Alternative Solution: Using a broomstick or a barbecue stick, cut them to each desired length and try to form them into triangles.

As we form them, these are the figures formed:

A.) 3, 4, 6

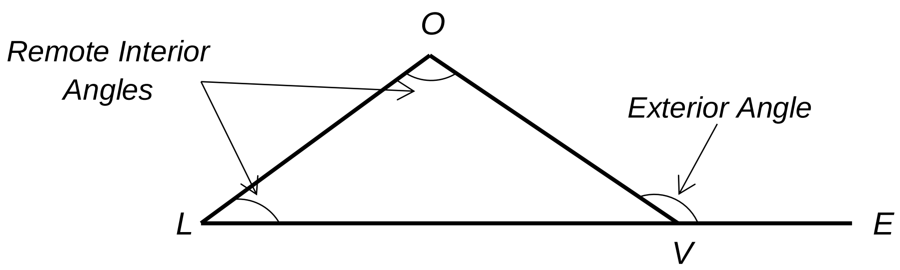
B.) 5, 6, 11

C.) 2, 3, 9

Based on the figures, only option A formed a triangle. Option B formed a straight line instead of a triangle and option C is short of sides that it cannot form a triangle. If we compare the sum of the two sides of options A, B, and C, only option A satisfied the conditions of Triangle Inequality Theorem.

Exterior Angle Inequality Theorem

The measure of an exterior angle of a triangle is greater than the measure of either remote interior angle.



Example 5: In the figure above, if m∠L= 45° and m∠O= 105°, then by Exterior Angle

Inequality Theorem:

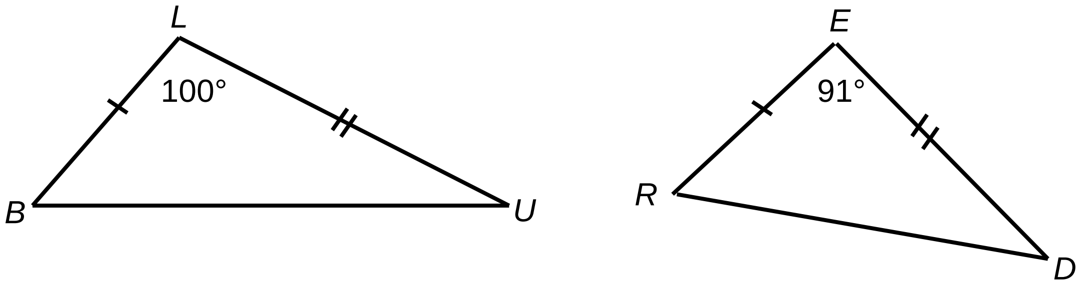
$m\angle OVE > m\angle L$ or $m\angle OVE > m\angle O$

$m\angle OVE > 45°$ or $m\angle OVE > 105°$

Inequalities in Two Triangle

The previous theorems deal with inequalities in one triangle, this time, we will be dealing with inequalities in two triangles.

Example 6: Compare the lengths of the third side of Δ BLU and Δ RED.



(Note: The figure is not drawn to scale.)

Given: $\overline{BL} \cong \overline{RE}$

$\overline{LU} \cong \overline{ED}$

$\overline{BU}$? $\overline{RD}$

Solution: $\angle L > \angle E$

$100° > 91°$

$\overline{BU} > \overline{RD}$

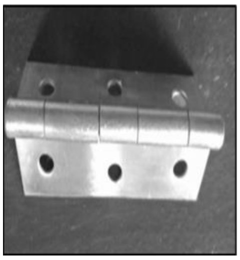
Based on the angle-side theorem, the opposite side of the greater angle is longer side.

From this example, the corresponding two sides of two triangles are congruent but the measure of their included angles differ, it follows that the opposite side of the greater angle is longer. Thus, the opposite side of ∠L which is $\overline{BU}$ is greater than the opposite side of ∠E which is $\overline{RD}$. This theorem is what we called the **Hinge Theorem** or the **SAS Inequality Theorem.**

Hinge Theorem or SAS Inequality

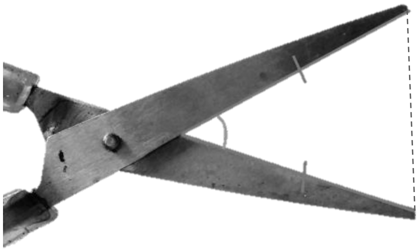
If two sides of one triangle are congruent to two sides of another triangle, but the included angle of the first triangle is greater than the included angle of the second, then the third side of the first triangle is longer than the third side of the second.

Are you familiar with hinge devices?

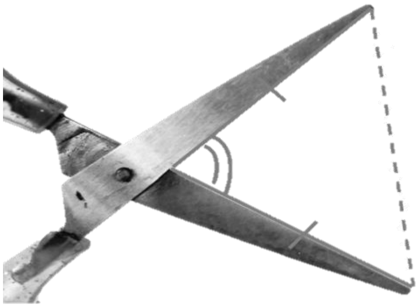


Hinge devices are used to fasten or join two things together and allow adjustment, rotation, twisting or pivoting. The picture in the left is called hinge which is attached to our doors at home. It is responsible for the opening and closing of our doors. Scissors, compass and folding ladder are just some of the examples of hinge

Example 7: Which of the following pictures show a wide opening of the scissors?



A



B

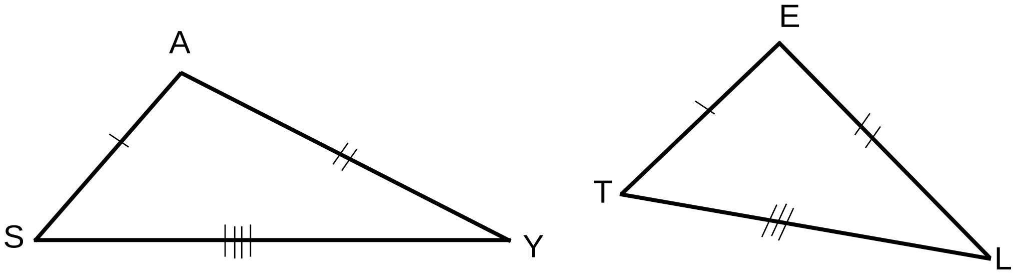
Answer: Obviously, the answer is Picture B since its angle is greater than that of Picture A. Although, a pair of scissors has the same length, its opening (dashed line) depends on the angle of adjustment of the user.

Conversely, we can also tell which included angle is greater based on the length of the third side given that corresponding two sides of two triangles are congruent. This theorem is the converse of Hinge Theorem or the SSS Inequality Theorem.

Converse of Hinge Theorem or SSS Inequality Theorem

If two sides of one triangle are congruent to two sides of another triangle, but the third side of the first triangle is longer than the third side of the second, then the included angle of the first triangle is larger than the included angle of the second.

Example 8: Given that ΔSAY and ΔTEL have two congruent sides as shown in the figure, which angle is greater, $\angle A$ or $\angle E$?

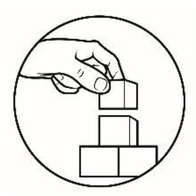


Answer:

In the figure, we can observe that $\overline{SA} \cong \overline{TE}$ and $\overline{AY} \cong \overline{EL}$. It is also given that $\overline{SY}$ is longer than $\overline{TL}$. With these, the Converse of Hinge Theorem or SSS Inequality Theorem tells us that $\angle A$ is greater than $\angle E$.

In summary, the following are theorems in triangle inequalities.

INEQUALITIES	THEOREM	IF-THEN STATEMENT
Inequalities in One Triangle	Angle-Side Relationship	If two angles of a triangle are not congruent, then the larger side is opposite the larger angle.
		If two sides of a triangle are not congruent, then the larger angle is opposite the larger side.
	Triangle Inequality Theorem	The sum of the lengths of any two sides of a triangle is greater than the length of the third side.
	Exterior Angle Inequality Theorem	The measure of an exterior angle of a triangle is greater than the measure of its either remote interior angle.
Inequalities in Two Triangles	Hinge Theorem or SAS Inequality Theorem	If two sides of one triangle are congruent to two sides of another triangle, but the included angle of the first triangle is greater than the included angle of the second, then the third side of the first triangle is longer than the third side of the second.
	Converse of Hinge Theorem or SSS Inequality Theorem	If two sides of one triangle are congruent to two sides of another triangle, but the third side of the first triangle is longer than the third side of the second, then the included angle of the first triangle is larger than the included angle of the second.

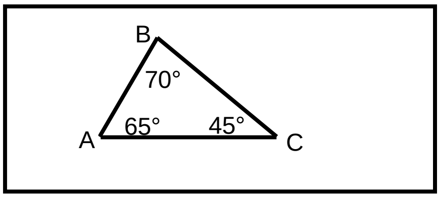


What's More

Activity 1: Identify Me

Directions: Write your answer on a separate sheet of paper. Answer what is asked in the given problem.

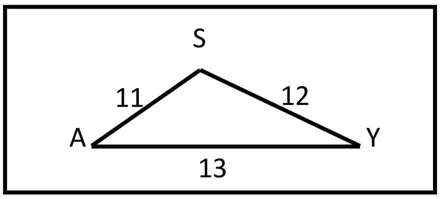
1.



a. What is the longest side in a ΔABC? How about the shortest side?

b. What theorem did you apply in determining the longest and shortest side?

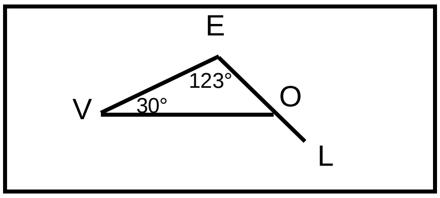
2.



a. What is the smallest angle of ΔSAY? How about the largest angle?

b. What theorem did you apply in determining the smallest and largest angle?

3.

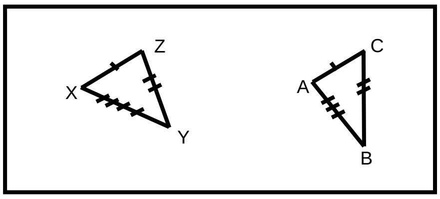


a. What is the measurement of $\angle LOV$?

b. Using the relation symbols $>, <,$ or $=$, complete the statements below: $m\angle LOV$ ___ $m\angle E$ and $m\angle LOV$____ $m\angle V$.

c. What theorem did you use to justify the statements above?

4.

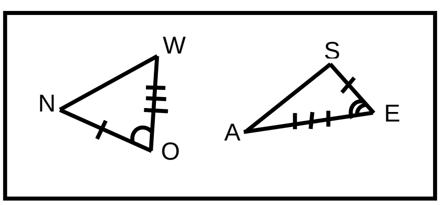


a. Using the relation symbols $>, <,$ or $=$, complete the statement below:

$m\angle Z$___ $m\angle C$

b. What theorem did you use to justify the statement above?

5.



a. Using the relation symbols $>, <,$ or $=$, complete the statement below:

$\overline{NW}$ ___ $\overline{AS}$

b. What theorem did you use to justify the statement above?

Activity 2: Am I a Triangle?

Directions: Which of the following could be the lengths of the sides of a triangle? Put a triangle (△) if it forms a triangle and (X) if it does not form a triangle.

_______1. 1, 2, 3

_______2. 17, 16, 9

_______3. 9, 11, 18

________4. 4, 8, 11

________5. 5, 13, 6

Activity 3: Make It True

Directions: Modified True or False. Write True if the statement is correct, but if it's False, change the underlined words to make it right.

__________1. The measure of an exterior angle of a triangle is <u>less</u> than the measure of either remote interior angle.

__________2. If one side of a triangle is longer than a second, then the angle opposite the first side is <u>larger</u> than the angle opposite the second side.

__________3. The sum of the lengths of any two sides of a triangle is <u>greater</u> than the length of the third side.

__________4. If one angle of a triangle is larger than a second angle, then the side <u>adjacent</u> the first angle is longer than the side <u>adjacent</u> the second angle.

__________5. If two sides of one triangle are congruent to two sides of another triangle, but the included angle of the first triangle is greater than the included angle of the second, then the third side of the first triangle is <u>longer</u> than the third side of the second.

What I Have Learned

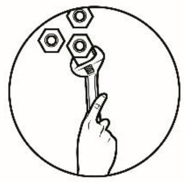
Fill in the blanks!

Directions: Complete the paragraph below by filling in the blanks with correct word/s or figure/s which you can choose from the box below. Each word or figure may be used repeatedly. Write your answer on a separate sheet.

longer than	included angle	greater than	Hinge Theorem
Triangle Inequality Theorem	Angle-Side Relationship	Exterior Angle Inequality	SSS Inequality Theorem
remote interior angle	opposite	inequalities	remote exterior angle

There are two types of theorems of inequalities in triangles. One is the Inequalities in One Triangle. One theorem under the inequalities in one triangle is the ________________ in which if two angles of a triangle are not congruent, then the larger side is __________ to the larger angle. The second one is the ____________________ which states that the sum of the lengths of any two sides of a triangle is ____________ the length of the third side. The third one is the _____________________ Theorem which states that the measure of an exterior angle of a triangle is greater than the measure of its ____________________.

On the other hand, there are also theorems of the inequalities in Two Triangles. One of its theorem is the _______________ or SAS Inequality Theorem in which if the two sides of one triangle are congruent to two sides of another triangle, but the included angle of the first triangle is greater than the included angle in the second, then the third side of the first triangle is _____________ the third side of the second. The other theorem is the Converse if Hinge Theorem or ______________________ which states that if two sides of one triangle are congruent to two sides of another triangle, but the third side of the first triangle is longer than the third side of the second, then the _____________ of the first triangle is larger than the included angle of the second.

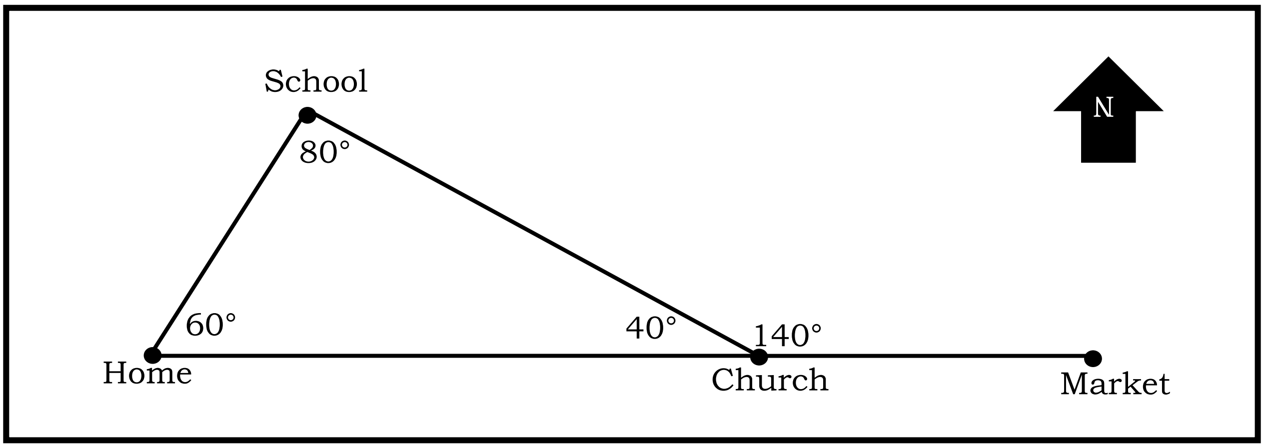


What I Can Do

Activity 1

Directions: Read the problem below and answer the questions that follow.

In the map below, the locations form a triangle. Dina, a student, is at home while Alvin, her classmate is at school. In this situation, answer the questions that follow.



Questions:

1. Who is farther from Church given their locations, Dina or Alvin? What triangle inequality theorem is applied in this situation?
2. Dina requested Alvin to pick her up in going to Church. Will Alvin be able to travel the shortest path to go to Church given Dina's request? What triangle inequality theorem is applied in this situation?
3. Given the same time started and same rate of walking, who among the two can arrive at the Market first?
4. If a new road will be constructed connecting the School and the Market, is the constructed road longer than the road between Home and Church? What triangle inequality theorem is applied in this situation?

Activity 2

Directions: Hinges are used to fasten two things together and allow adjustment, rotation, twisting, or pivoting. Your task is to identify and list down hinge devices you know and determine its function and its relation to Hinge Theorem. You can list hinge devices as many as you can but not less than 3. Write your answer on a separate sheet of paper. For your guidance, you may use the template on the next page.

Hinge Devices	Function	Relationship to Hinge Theorem

For rating your output, please refer to the rubric provided.

	4	3	2	1
Content	The discussion of the function of hinge devices listed is correct, clear, and realistic and the relationship to Hinge Theorem is properly illustrated.	The discussion of the function of hinge devices listed is correct, clear, and realistic and the relationship to Hinge Theorem is not properly illustrated.	The discussion of the function of hinge devices listed is correct, clear, and realistic but did not indicate the relationship to Hinge Theorem.	The discussion of the function of hinge devices listed is not clear and did not indicate the relationship to Hinge Theorem.
Quantity	Listed more than 3 hinge devices with complete function and discussion to its relation to Hinge Theorem.	Listed only 3 hinge devices with complete function and discussion to its relation to Hinge Theorem.	Listed only 2 hinge devices with complete function and discussion to its relation to Hinge Theorem.	Listed only 1 hinge devices with complete function and discussion to its relation to Hinge Theorem.

Assessment

Directions: Read the questions carefully. Write the letter that corresponds to your answer on a separate sheet of paper.

1. Which of the following statements is true based on Angle-Side Relationship Theorem?
    A. The largest side of a triangle is opposite with the smallest angle.
    B. The largest side of a triangle is adjacent with the smallest angle.
    C. The smallest side of a triangle is opposite with the smallest angle.
    D. The smallest side of a triangle is adjacent with the smallest angle.

2. The sum of the lengths of any two sides of a triangle is always _________.
    A. half of the length of the third side
    B. equal to the length of the third side
    C. less than the length of the third side
    D. greater than the length of the third side

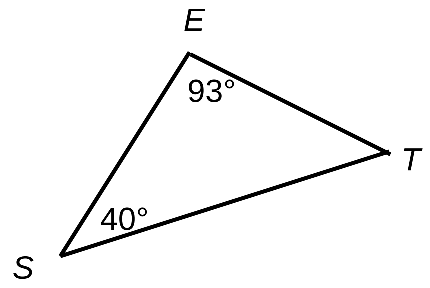
3. The measure of an exterior angle of a triangle is always _________.
    A. equal to its adjacent interior angle
    B. less than its adjacent interior angle
    C. less than either remote interior angle
    D. greater than either remote interior angle

4. If two sides of one triangle are congruent to two sides of another triangle, but the included angle of the first triangle is greater than the included angle of the second, then the third side of the first triangle is ________________.
    A. equal to the third side of the second triangle.
    B. less than the third side of the second triangle.
    C. longer than the third side of the second triangle.
    D. greater than or equal to the third side of the second triangle.

5. Which of the following inequalities theorem is applicable in two triangles?
    A. Angle-Side Relationship
    B. SSS Inequality Theorem
    C. Triangle Inequality Theorem
    D. Exterior Angle Inequality Theorem

In items 6 to 8, refer to the figure at the right.

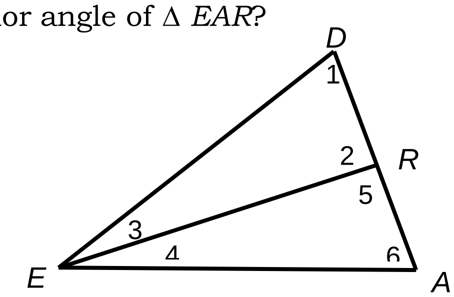
6. Which side of Δ *SET* is the longest?
   A. $\overline{SE}$
   B. $\overline{ET}$
   C. $\overline{ST}$
   D. Cannot be determined



7. Which side is the shortest?
   A. $\overline{SE}$
   B. $\overline{ET}$
   C. $\overline{ST}$
   D. Cannot be determined

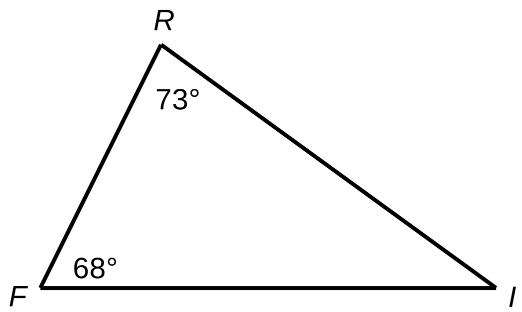
8. What theorem is applied?
   A. Angle-Side Relationship
   B. SSS Inequality Theorem
   C. Triangle Inequality Theorem
   D. Exterior Angle Inequality Theorem

9. Which of the following angles is an exterior angle of Δ *EAR*?
   A. $\angle 1$
   B. $\angle 2$
   C. $\angle 3$
   D. $\angle 4$



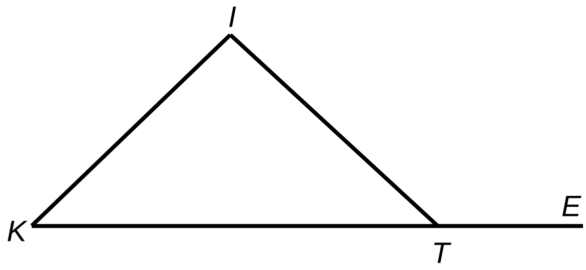
10. Which of the following could NOT be used as the length of the sides of a triangle?
   A. 4, 4, 4 C. 8, 14, 10
   B. 11, 12, 13 D. 6, 5, 12

11. List the sides of ΔTRI in order from longest to shortest measure.
   A. $\overline{FI}$, $\overline{FR}$, $\overline{RI}$
   B. $\overline{FI}$, $\overline{RI}$, $\overline{FR}$
   C. $\overline{FR}$, $\overline{RI}$, $\overline{FI}$
   D. $\overline{FR}$, $\overline{FI}$, $\overline{RI}$



12. In the figure below, which of the following statements is true?

A. $\angle KIT = \angle ETI$
B. $\angle KIT > \angle ITE$
C. $\angle ETI > \angle KIT$
D. $\angle IKT > \angle ITE$



13. Hinges are used to fasten two things together and allow adjustment, rotation, twisting, or pivoting. Which of the following is NOT a hinged device?

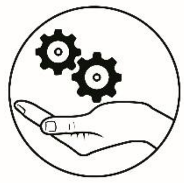
A. scissor
B. folding ladder
C. screw
D. compass

14. Jason found out that two sides of two triangles are congruent. As he measures the included angle between these sides, he found out that the measure of the side opposite to the included angle of the first triangle is longer than the measure of the side to the included angle of the second triangle. What theorem did Jason use?

A. Triangle Inequality Theorem
B. Exterior Angle Inequality Theorem
C. Hinge Theorem or SAS Inequality Theorem
D. Converse of Hinge Theorem or SSS Inequality Theorem

15. Each of Angel, Anne, Luis and Sam was given a 15-inch piece of stick. They were instructed to create a triangle. Each cut the stick in their own chosen lengths as follows: Angel—*5 in, 5 in, 5 in*; Anne—*3 in, 5 in, 7 in*; Luis—*6 in, 4 in, 5 in*; and Sam—*3 in, 4 in, 8 in*. Who among them was not able to form a triangle?

A. Angel
B. Anne
C. Luis
D. Sam

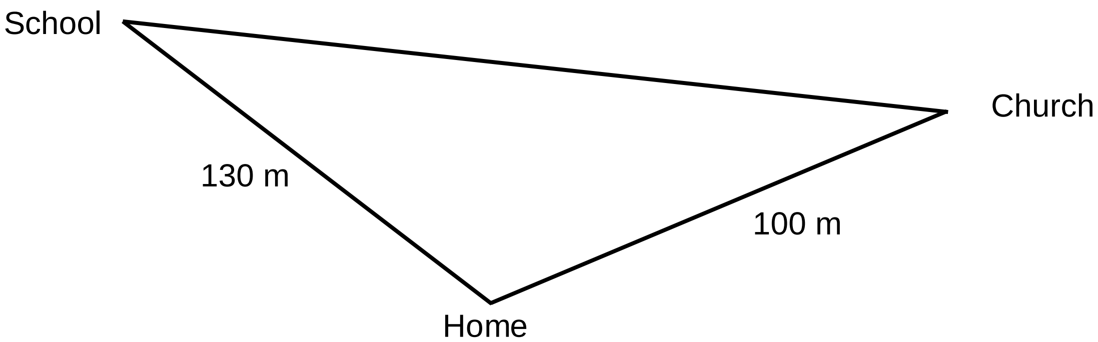


Additional Activities

This section of this module will help you deepen your understanding about the importance of inequalities in triangle.

Activity 1

Directions: The distance Ronie walks from home to school is 130 meters and 100 meters when he goes to church from home. Romeo estimates that the distance Ronie walks when he goes directly to Church, coming from school, is 200 meters. Avelino's estimation is 240 meters. Which estimation is feasible? Justify your answer. Write your answer on a separate sheet of paper.



To assess your answers, please refer to the rubric provided.

4	3	2	1
Use appropriate theorem with complete solution leading to correct answer.	Use appropriate theorem with solution but with incorrect answer.	Use inappropriate theorem with correct answer.	Use inappropriate theorem with incorrect answer.

Answer Key

What I Know

1. B
2. A
3. C
4. A
5. B
6. A
7. D
8. C
9. B
10. C
11. D
12. C
13. C
14. C
15. B

What's In

1. $\overline{BC}, \overline{EF}$
2. ∠C, ∠F
3. 180°, 180°
4. $\overline{BC} \cong \overline{EF}$
5. $\overline{AB} \cong \overline{DE}$

What's New

Measure Me:
$\angle I \approx 65°$, $\angle F \approx 90°$, $\angle T \approx 25°$
$\angle X \approx 50°$, $\angle A 100°$, $\angle M \approx 30°$
See Triangle Inequality Theorem 1 and 2

Compare Me:

1. 14>10
2. 18>6
3. 16>8
4. 13>10
5. 18>5
6. 15>8

See Triangle Inequality Theorem 3

Complete Me:
ΔFLY $\angle 1 \approx 67°$, $\angle 2 \approx 132°$, $\angle 3 \approx 161°$, $\angle 4 \approx 48°$, $\angle 5 \approx 113°$, $\angle 6 \approx 19°$
ΔFRE $\angle 1 \cong 70°$, $\angle 2 \cong 143°$, $\angle 3 \approx 147°$, $\angle 4 \approx 37°$, $\angle 5 \approx 110°$, $\angle 6 \approx 33°$
See Exterior Angle Inequality Theorem

Form Me:
The length of the third side opposite 45°≈ $5\ inches$ long, while the length of the third side opposite 75° ≈ $8.5\ inches\ long$.

1. Congruent
2. Included angle
3. Triangle
4. See Hinge Theorem

For 5 and 6, see Converse of Hinge Theorem
Note: Students' answer may closely equal to the answer

What's More

Activity 1

1. a. $\overline{AC}$ –longest side
   $\overline{AB}$ – shortest side
   b. Angle-Side Relationship Theorem
2. a. ∠Y –smallest
   ∠S – largest
   b. Angle-Side Relationship Theorem
3. a. 153°
   b. > (both statement)
   c. Exterior Angle Inequality Theorem
4. a. >
   b. Converse of Hinge Theorem
5. a. <
   b. Hinge Theorem

Activity 2

1. X
2. Δ
3. Δ
4. Δ
5. X

Activity 3

1. Greater
2. True
3. True
4. Opposite
5.

What I Have Learned

1. Angle-side Relationship
2. Opposite
3. Triangle Inequality Theorem
4. Greater than
5. Exterior Angle Inequality
6. Remote interior angle
7. Hinge Theorem
8. longer than
9. SSS Inequality Theorem

What I Can Do

Activity 1

1. Dina, Angle-Side Relationship
2. No, Triangle Inequality Theorem
3. Alvin
4. Exterior Angle Inequality Theorem

Activity 2
Answers may vary.

Assessment

1. C	9. B
2. D	10. D
3. D	11. B
4. B	12. C
5. B	13. B
6. C	14. C
7. B	15. D
8. A	

Additional Activities

Applying Triangle Inequality Theorem 3, Avelino's estimation is more feasible.

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