



Mathematics

Quarter 4 – Module 1: **Determining the Relationship of Volume** Between a Rectangular Prism and a Pyramid; a Cylinder, and a Cone; and a Cylinder and a Sphere



Mathematics – Grade 6 Alternative Delivery Mode Quarter 4 – Module 1: Determining the relationship of volume between a rectangular prism and a pyramid; a cylinder, and a cone; a cylinder and sphere. First Edition, 2020

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Mathematics

Quarter 4 – Module 1: Determining the Relationship of Volume Between a Rectangular Prism and a Pyramid; a Cylinder, and a Cone; and a Cylinder and a Sphere



Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-bystep as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.



What I Need to Know

This module was designed and written with you in mind. It is here to help you master the skills in determining the relationship of volume between a rectangular prism and a pyramid; a cylinder, and a cone; a cylinder and sphere. The scope of this module allows you to use it in many different learning situations. The language used recognizes your diverse vocabulary level. The lessons are arranged to follow the standard sequence of your course. But the order in which you read them can be changed to match with the textbook you are now using.

After going through this module, you are expected to determine the relationship of volume between a rectangular prism and a pyramid; a cylinder, and a cone; a cylinder and sphere (M6ME-IVa-95).



What I Know

Read each item carefully. Answer the following questions by choosing the letter of your answer and writing it on your answer sheet.

- 1. Which of the following statements correctly describe the volume of prisms and pyramids with the same dimensions?
 - A. The volume of a pyramid is one-third of the volume of a prism.
 - B. The volume of a pyramid is one-fourth of the volume of a prism.
 - C. The volume of a pyramid is one-third greater than the volume of a prism.
 - D. The volumes of a pyramid and prism are the same because their bases are the same.
- 2. A sphere is inscribed in a cylinder, such that the point on a curve line touches the inner surface of the cylinder. What is the ratio of the volume of the sphere to the volume of the cylinder?
 - A. $\frac{4}{3}$: 1 B. $1:\frac{4}{3}$ C. $\frac{2}{3}:1$ D. $1:\frac{2}{3}$
- 3. A cylinder and a cone have the same radius and height. How will you compare the volume of the cone to the volume of the cylinder?
 - A. The volume of the cone is two-third the volume of the cylinder.
 - B. The volume of the cone is four-third the volume of the cylinder.
 - C. The volume of the cone is one-third the volume of the cylinder.
 - D. The relationship of the volumes of the cylinder and cone cannot be determined.
- 4. Consider the radius and height of a cone, a cylinder and a sphere to be the same. Which correctly arranges the figures from smallest to largest?
 - A. Sphere, Cylinder, Cone C. Cylinder, Cone, Sphere
- D. Cone, Cylinder, Sphere
- B. Cone, Sphere, Cylinder
- 5. A cone and a cylinder have congruent bases and altitudes. What is the ratio of the volume of the cone to the cylinder?
 - A. $\frac{1}{3}$ B. $\frac{2}{3}$ C. $\frac{4}{3}$ D. $\frac{5}{3}$

Lesson

Determining the Relationship of Volume Between a Rectangular Prism and a Pyramid; a Cylinder, and a Cone; a Cylinder and Sphere

In the previous lessons, you have learned how to find the surface area of cubes, prisms, pyramids, cylinders, cones, and spheres. This time, we will focus on how to determine the relationship of volume between a rectangular prism and a pyramid; a cylinder, and a cone; a cylinder and a sphere.



What's In

Fill in the blanks. Choose your answer inside the box and write it in your answer sheet.

cylinder	sphere		pyramid	height
	$V = \frac{4}{3}\pi r^3$	$V = \frac{1}{3}\pi r^2 h$	V= $\pi r^2 h$	

- 1. The volume of ______ is one-third of the volume of a rectangular prism with the same base and height.
- 2. In finding the volume of soccer a ball, the formula to use is
- 3. A cone exactly fills one-third of a ______ whose base and height are equal.
- 4. Having the same radius and height, the volume of a ______ is two-thirds of the volume of a cylinder.
- 5. The volume of a cylinder is the product of the circular base area and the _____.



Given the following illustrations, what can you say about the relationship between the volume of a rectangular prism and a pyramid? A cylinder and a cone? A cylinder and a sphere?





The volume of a solid figure is the amount of space inside it. Volume is measured in cubic units, which means it tells the number cubes in a given size it takes to fill the solid figure.

Relationships of the volumes between solid figures can be determined as illustrated in the **What is in** section of this module. Let us explore how we can establish the relationship between the following pairs of figures.

A. Relationship of the Volume between Rectangular Prism and Pyramid

1. Exploring Volume of a rectangular prism.

Study the figure at the right

The volume of a rectangular prism is equal to the product of its base area and its height.

Volume = (area of the base) x height Volume = (length x width) x height Volume = l w h



Let us determine the volume of the cylinder at the right with the given dimensions. To solve for the volume of a given rectangular prism, follow the formula and perform the operations V = l w hV = 5 cm x 3 cm x 2 cm $V = 30 \text{ cm}^3$ Let's try to solve another example.



Find the volume of the rectangular prism shown below with the illustrated dimensions:

1 = 9 cm w= 4 cm and h = 6 cm.

Solution:

 $V = l \cdot w \cdot h$ V = 9 cm x 4 cm x 6 c m V = 216 cm³



2. Exploring the Volume of a Pyramid

Consider the following pyramid and the rectangular prism having equal altitudes (height) and bases with equal areas.



If the pyramid is filled with water or sand and its content is poured into a rectangular prism, only a third of the prism will be filled. Thus, the volume of a pyramid is $\frac{1}{3}$ the volume of a prism.

 $Volume = \frac{1}{3}(base \ area \ x \ height)$ or $V = \frac{lwh}{3}$

To find the volume of the given pyramid whose length measures 9 cm, width of 4 cm and height of 6 cm, study the solution below.

Solution:

$$Volume = \frac{1}{3}(base area x height)$$

$$Volume = \frac{1}{3}(length x width x height)$$

$$Volume = \frac{1}{3}(9 \text{ cm } x 4 \text{ cm } x 6 \text{ cm})$$

$$Volume = \frac{1}{3}(216\text{ cm}^{3})$$

$$Volume = \frac{216 \text{ cm}^{3}}{3}$$

$$Volume = 72 \text{ cm}^{3}$$

B. Relationship between the volume of the Cylinder and Cone

1. Exploring the Volume of a Cylinder

Cylinders have circular bases. Finding the volume of a cylinder is just like finding the volume of a prism.

Volume = (area of the base) x height Volume = $(\pi \cdot r^2)$ h Volume = $\pi \cdot r^2 \cdot h$



h = 15 cm

r = 2 cm

Example:

Find the volume of a cylinder with radius of 2 cm and a height of

15 cm. (Use
$$\pi$$
 = 3.14)

Solution:

 $V = \pi \cdot r^2 \cdot h$

 $V = 3.14 \text{ x} (2 \text{ cm})^2 \text{ x} 15 \text{ cm}$

 $V = 3.14 \text{ x} 4 \text{ cm}^2 \text{ x} 15 \text{ cm}$

 $V = 188.4 \text{ cm}^3$

2. Exploring the Volume of a Cone

If a cone is filled with water or sand, and then its content is poured into a cylinder with equal base area and height, only a third of the cylinder will be filled.



This shows that the volume of a cone is $\frac{1}{3}$ of the volume of a cylinder with the same base area and height.

$$V = \frac{1}{3}\pi r^2 h$$
 or $V = \frac{Bh}{3}$



 $V = 188.4 \text{ cm}^3$

Let's find the volume of the given cone whose height measures 15 cm and radius measures 2 cm.

Solution:

$$V = \frac{Bh}{3}$$

$$V = \frac{(\pi r^2)h}{3}$$

$$V = \frac{3.14 x (2 cm)^2 x 15 cm}{3}$$

$$V = \frac{3.14 x 4 cm^2 x 15 cm}{3}$$

$$V = \frac{188.4 cm^3}{3}$$

$$V = 62.8 cm^3$$

Notice that the volume of the cone, 62.8 cm^3 is one-third of the volume of the cylinder with the same base and height, 188.4 cm^3 . This confirms the relation that a cone's volume is 1/3 the volume of a cylinder.

C. Relationship Between the Volume of the Cylinder and Sphere

1. Exploring the Volume of a Sphere

Fill a cylinder with water. Push a sphere with the same radius into the cylinder. Notice that about $\frac{2}{3}$ of the water will be displaced. So the volume of the sphere is $\frac{2}{3}$ of the volume of the cylinder.



The volume of the cylinder would be the area of its base times its height, which is $(\pi r^2)(2r)or 2\pi r^3$. The sphere does not fill the whole cylinder. In fact, the volume of the sphere is $\frac{2}{3}$ of the volume of the cylinder: $\frac{2}{3}(2\pi r^3)$.

Therefore, the volume of the sphere is $V = \frac{4}{3}\pi r^3$

Let's find the volume of the sphere whose radius measures 3 cm. Note that a cylinder with the same radius and height double the radius has a volume of 169.56 cm^3 .

v = 115.04 cm³

Notice that the volume of the sphere, 113.04 cm^3 is two-thirds of the volume of the cylinder, 169.56 cm³. This confirms that the volume of a sphere is 2/3 the volume of a cylinder with the same radius.

A. Complete the needed data in the table below by writing the missing formula or height to show the relationship between volumes of different solids figures. Write your answer on your answer sheet.

Quantities	Prisms	Pyramid	Cylinder	Cone	Sphere
Base Area	\bigcirc	$1 \cdot w$	$\pi \cdot r^2$	$\pi \cdot r^2$	$\pi\cdot\mathrm{r}^2$
Height	h	h	h	\bigcirc	h= 2r
Volume = B x h	$l \cdot w \cdot h$	\bigcirc	\bigcirc	$\frac{\frac{1}{3}(\pi \cdot r^2}{\cdot h)}$	\bigcirc

B. Find the volume of each solid figure found in the table below. Write your answer on your answer sheet. (Use π =3.14). Write your answer on a sheet of paper.

Solid	Prism	Pyramid	Cylinder	Cone	Sphere
length	4 cm	4 cm			
width	5 cm	5 cm			
radius			2 cm	2 cm	3 cm
Height	3 cm	3 cm	3 cm	3 cm	
Volume	1	2	3	4	5

What I Have Learned

The volume of a solid figure refers to the amount of space inside it. Volume is measured in cubic units, which means that it tells the number of cubes in a given size it takes to fill the solid figure.

We can determine the relationships of volume between a rectangular prism and a pyramid; a cylinder and a cone, and a cylinder and a sphere by:

The volume of a rectangular prism is equal to the product of the base area and its height. Having the same dimensions, the volume of a rectangular prism is three times the volume of the pyramid. Thus, the volume of a pyramid is one-third of the volume of a rectangular prism with the same dimensions.

The volume of a prism can be expressed as:

$$V = B h \text{ or } V = l \cdot w \cdot h$$

while the volume of a pyramid can be expressed as:

$$V = \frac{1}{3}Bh$$
 or $V = \frac{lwh}{3}$.

The volume of a cylinder is just like finding the volume of a prism. The volume of a cylinder is the product of the height *h* and the area of the base (B). That is, V = B h or $V = \pi r^2 h$.

The volume of a cone is one third of the volume of a cylinder with the same dimensions. That is, $V = \frac{1}{3}Bh$ or $V = \frac{1}{3}\pi r^2h$.

And the volume of the sphere is $\frac{2}{3}$ of the volume of a cylinder with the same radius and height double of the radius: $\frac{2}{3}(2\pi r^3)$. Therefore, the volume of the sphere is $V = \frac{4}{3}\pi r^3$

What I Can Do

Read and answer each item below. Choose the letter of your answer and write it on your answer sheet.

- 1. Claire makes models to show the relationship between the volume of a rectangular prism and the volume of a rectangular pyramid. The rectangular prism model has a base area of 8 square inches and a height of 12 inches. She also makes a rectangular pyramid with a base and height congruent to the corresponding parts of the prism. Which conclusion is INCORRECT about the relationship between the volumes of the models?
 - A. The volume of the rectangular prism is triple the volume of the rectangular pyramid.
 - B. The volume of the rectangular pyramid is one-third the volume of the rectangular prism.
 - C. The volume of the rectangular prism is three times the volume of the rectangular pyramid.
 - D. The volume of the rectangular pyramid is three times the volume of the rectangular prism.
- 2. Dino has a cone-shaped container that he fills with water. He pours the water into the cylindrical shaped container. Both containers have the same height and bases. Which of the following statements is CORRECT after he pours the water from the cone to the cylinder?
 - A. The cylindrical shaped container is completely filled with water.
 - B. Only three-fourths of the volume of cylindrical container is filled with water.
 - C. One-third of the volume of the cylindrical container is not filled with water.
 - D. Two-thirds of the volume of the cylindrical container is not filled with water.
- 3. A spherical tank for natural gas has a volume of about 1500 m³. How will you compare its volume to a cylindrical type of tank having the same dimensions?
 - A. The volume of the spherical tank is two-thirds of the volume of the cylindrical tank.
 - B. The volume of the spherical tank is greater than the volume of the cylindrical tank.
 - C. The volumes of the spherical and the cylindrical tanks are the same.
 - D. The volume of the spherical tank is one-third of the volume of the cylindrical tank.
- 4. The area of the base of a rectangular prism is 15 m² and the height measures 5 m. What is the volume of the pyramid with the same dimensions as the rectangular prism?
 - A. The volume of pyramid is 25 m^3 .
 - B. The volume of pyramid is 35 m^3 .
 - C. The volume of pyramid is 45 m^3 .
 - D. The volume of pyramid is 55 m^3 .

Assessment

Read and answer each item. Choose the letter of your answer and write it on your answer sheet.

- 1. Oscar fills a cone-shaped container with water. He pours the water into the cylindrical-shaped container. The height and the bases of the containers have the same measurements. Which of the following statements is INCORRECT after he pours the water from the cone to the cylinder?
 - A. The cylindrical-shaped container is one-third filled with water.
 - B. The cylindrical shaped container is completely filled with water.
 - C. The volume of the cone-shaped container filled with water is one-third of the volume of the cylindrical-shaped container.
 - D. The volume of the cylindrical-shaped container filled with water is three times the volume of the cone-shaped container.
- 2. A rectangular prism and a pyramid have congruent bases and equal altitudes. What is the ratio between the volume of the pyramid to the volume of the rectangular prism?

A. $\frac{1}{3}$	C. $\frac{4}{3}$
B. $\frac{2}{3}$	D. $\frac{5}{3}$

3. The cylinder below has a base and vertical height congruent to that of the cone. Which best describes the volume of the cone?

- 4. Which of the following describes the relationship of the volume of cylinder and sphere with the same dimensions?
 - A. The volume of sphere is one-half the volume of cylinder.
 - B. The volume of sphere is one-third of the volume of cylinder.
 - C. The volume of sphere is two-thirds of the volume of cylinder.
 - D. The volume of sphere is the same with the volume of cylinder.
- 5. Elena has a cylindrical shaped container that is 9 inches high with a diameter of 10 inches. Reggie has a sphere-shaped container that has a radius of 4 inches. About how much more volume does¹ Elena's container have than Reggie's?
 - A. 438.55 inches³
 - B. 325.23 inches³
 - C. 342.54 inches3
 - D. 121.56 inches3

Additional Activities

Answer the following questions in your own words. Write your answer in your answer sheet.

1. Define volume.

2. How is the volume of cylinder related to the volume of the cone?

3. Compare the volume of a cylinder and a sphere. How are they related?

4. What is the relationship between the volume of a prism and a pyramid?

5. What can you say about the volume of prism and cylinder? How are they related?

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