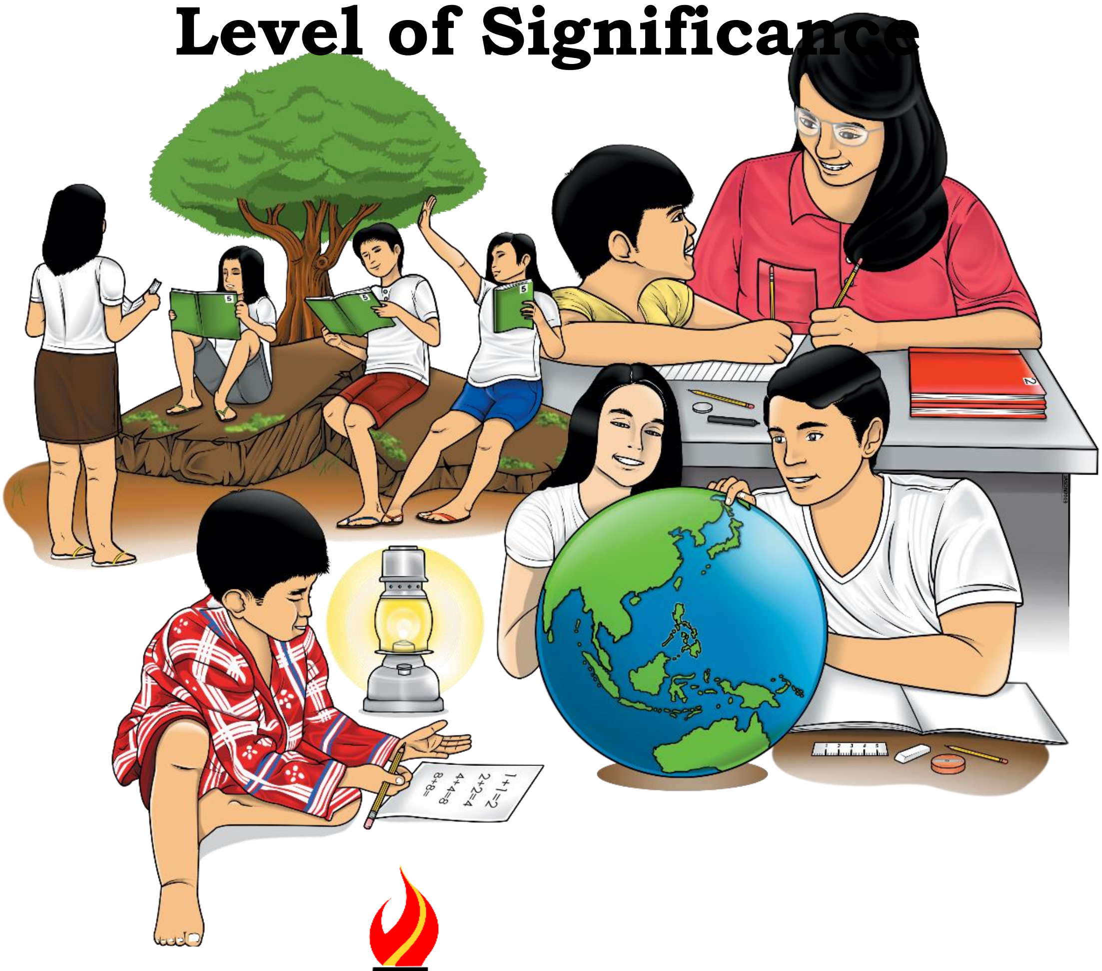
Senior High School

# Statistics and Probability

## Quarter 4 – Module 5: Identifying the Appropriate Rejection Region for a Given Level of Significance



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Senior High School

# Statistics and Probability

## Quarter 4 – Module 5: Identifying the Appropriate Rejection Region for a Given Level of Significance

DEPARTMENT OF EDUCATION

# Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

## *What I Need to Know*

In the previous module, you have learned to identify the appropriate test statistic when the population variance is known or unknown. You were able to define different statistical concepts related to z-test and t-test as the tools for computing value in hypothesis testing problem. The steps in choosing correct statistical test were also discussed. Moreover, the test used for Central Limit Theorem was explained.

Since you already know how to choose the test statistic applicable in hypothesis testing, you are now ready to identify the appropriate rejection region when population variance is known or unknown. In determining rejection region, you will also be defining other statistical concepts such as critical value.

After going through this module, you are expected to:

1. define the critical values, level of significance, hypothesis test, and rejection region;
2. identify the critical value when population variance is known or unknown; and
3. determine the appropriate rejection region for a given level of significance when population is known/unknown and Central Limit Test is to be used.

## *What I Know*

Choose the best answer to the given questions or statements. Write the letter of your choice on a separate sheet of paper.

Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. In a right-tailed test with $\alpha = 0.01$, the critical value of $z$ is:
   a. 1.28  b. 1.65  c. 1.96  d. 2.33

2. The value that separates a rejection region from an acceptance region is called a ___________.
   a. Parameter  c. critical value
   b. Hypothesis  d. significance level

3. For a two-tailed test with variance unknown, n= 19, and $\alpha = 0.05$, what is the critical value.
   a. $\pm 2.092$  b. $\pm 2.101$  c. $\pm 2.145$  d. $\pm 2.878$

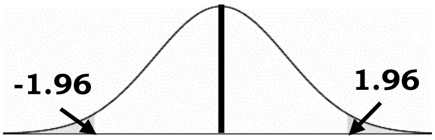
4. For a two-tailed test with a sample size of 40, the null hypothesis will be rejected at 5% level of significance if the test statistic is.
   a. $z \leq -1.28\ or\ z \geq 1.28$ c. $\leq -1.96\ or\ z \geq 1.96$
   b. $z \leq -1.645\ or\ z \geq 1.645$  d. $z \leq -2.33\ or\ z \geq 3.33$

5. If the alpha level is increased from 0.01 to 0.05, then the boundaries for the critical region move farther away from the center of the distribution.
   a. True  c. both A and B
   b. False  d. cannot be determined

6. In the two-tailed test, the rejection region lies on ___________ of the normal distribution:
   a. center  b. left tail  c. right tail  d. both tails

7. Given the illustration at the right, which of the following is NOT TRUE?
   a. This is a left-tailed test.
   b. This is a right-tailed test.
   c. This has a critical value of 1.645
   d. This has a level of significance of 0.5.

8. Given the normal curve at the right, what is the rejection region?
   a. $\leq 1.645$ *or* $z \geq 1.645$
   b. $z \geq -1.645$ *or* $z \geq 1.645$
   c. $z \geq -1.96$ *or* $z \leq 1.96$
   d. $z \leq -1.96$ *or* $z \geq 1.96$



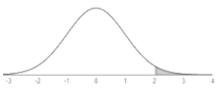
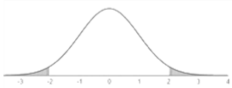
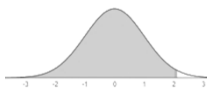
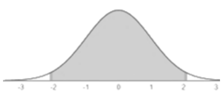

9. What is the critical value if the population variance is unknown, $n = 13$, $\alpha = 0.05$, and it is a one-tailed test?
   a. $t$ =1.782  b. $t$ =2.179  c. $t$ =2.681  d. $t$ =3.055

10. Given a two-tailed test, population variance is known, and $\alpha = 0.10$, what is critical region
   a. $z \geq 1.28$
   b. $z \leq -1.96$
   c. $\leq -2.33$ or $z \geq 2.33$
   d. $z \leq -1.645$ or $z \geq 1.645$

11. Which of the following is the sketch of the normal curve if $z \geq 1.645$?

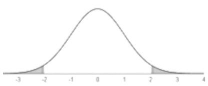
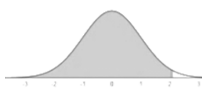
a. 
b. 
c. 
d. 

12. Which of the following graphs of rejection region show $t \geq 2.074$?

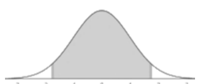
a. 
b. 
c. 
d. 

13. In the given problem below, identify the rejection region. It is claimed that the mean distance of a certain type of vehicle is 35 miles per gallon of gasoline with population standard deviation $\sigma = 5$ miles. What can be concluded about the claim using $\alpha = 0.1$ if a random sample of 49 such vehicles has sample mean, $\bar{x} = 36$ miles?
   a. $z \leq -1.28$
   b. $z \geq 2.33$
   c. $\leq -1.645$ *or* $z \geq 1.645$
   d. $z \leq -2.575$ *or* $z \geq 2.575$

14. Based on the problem in no. 13, which is the correct graph?

a. 
b. 
c. 
d. 

15. In a modeling agency, a researcher wishes to see if the average height of female models is less than 67 inches, as the coach claims. A random sample of 20 models has an average height of 65.8 inches. The standard deviation of the sample is 1.7 inches. At $\alpha = 0.05$, which of the following shows the appropriate rejection of the given problem?

a. 
b. 
c. 
d. 

# Lesson 1 Identifying the Appropriate Rejection Region for a Given Level of Significance

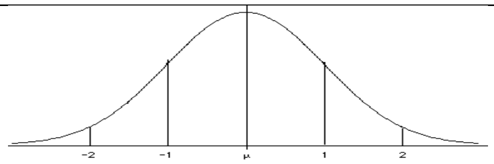
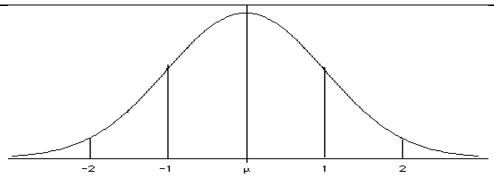
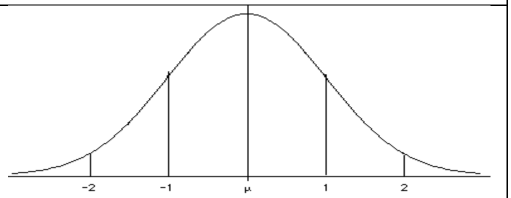
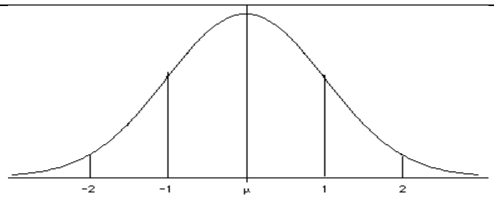
In hypothesis testing, a researcher collects sample data. From the given data, the researcher formulates the null and alternative hypotheses. Then, s/he chooses appropriate test statistic and computes it. If the statistics fall within the specific range of values, the researcher rejects the null hypothesis. The range of values that leads the researcher to reject the null hypothesis is called region of rejection. What is rejection region and how is it important in the process of hypothesis testing?

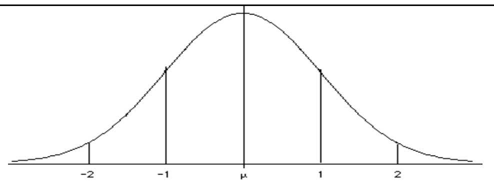
Before we discuss the topic, let us recall some concepts that will lead you to the concept of rejection region.

## *What's In*

### Activity 1: You Bring Color to My Life!

Given a standard normal curve, shade the required area with color GREEN and for the remaining area, use color RED.

| | |
|---|---|
| 1. between $z = -1.56$ and $z = +1.56$ |   |
| 2. to the left of $z = 2.05$ |   |
| 3. to the right of $z = -1.3$ |   |
| 4. between $z = -1.58$ and $z = 1.58$ |   |

| | |
|---|---|
| 5. to the left of $z = 1.96$ |   |

***Notes to the Teacher***

Check the level of readiness of your students. If the students did not pass the first activity, provide other activities that will help them recall how to determine the areas of normal curve.

# *What's New*

## Activity 2: Let Me Read and Understand!

Carefully read the problem and answer the questions that follow.

**Problem 1.** A banana company claims that the mean weight of its banana is 150 grams with a standard deviation of 18 grams. Data generated from a sample of 49 bananas randomly selected indicated a mean weight of 153.5 grams per banana. Is there a sufficient evidence to reject the company's claim? Use $\alpha = 0.05$.

1. What are the hypotheses?
2. Is it two-tailed or one-tailed test?
3. What is the level of significance?
4. Is the population standard deviation known?
5. What appropriate test statistic (z-test or t-test) can you use?
6. Based on the level of significance, hypothesis test, and test statistic, what is the critical value?
7. Draw the rejection region.

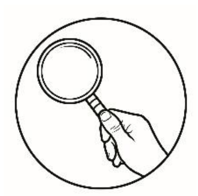
**Problem 2.** The manufacturer of an airport baggage scanning machine claims it can handle an average of 530 bags per hour. At $\alpha = 0.05$ in a left-tailed test, would a sample of 16 randomly chosen hours with a mean of 510 and a standard deviation of 50 indicate that the manufacturer's claim is an overstatement?

1. What are the hypotheses?
2. Is it two-tailed test or one-tailed test?
3. What is the level of significance?
4. Is the population standard deviation known or unknown?

5. What appropriate test statistic (z-test or t-test) can you use?
6. Based on the level of significance, hypothesis test, and test statistic, what is the critical value?
7. Draw the rejection region.

***Guide Questions:***

1. How did you find the activity?
2. What are the similarities and differences of the two problems?
3. Have you encountered previously learned statistical concepts? If yes, will you discuss those concepts?
4. Were you able to answer all the follow-up questions? If not, why?
5. What are the concepts that seemed to be familiar and unfamiliar to you?
6. How do these concepts relate to the rejection region?



# *What is It*

To be able to answer the questions in the next activities, please take time to read and understand this section that discusses the next steps in hypothesis testing.

## Critical Value, Significance Level, and Rejection Region

In hypothesis testing, a **critical value** is a point on the test distribution that is compared to the test statistic to determine whether to reject the null hypothesis. Critical values for a test of hypothesis depend upon the test statistic, which is specific to the type of the test and **significance level** ($\alpha$) which defines the sensitivity of the test. A value of $\alpha = 0.05$ implies that the null hypothesis is rejected 5% of the time when it is in fact true. In practice, the common values of $a$ are 0.1, 0.05, and 0.01.

## Critical Value of z-Distribution

A **critical value of z (Z-score)** is used when the sampling distribution is normal or close to normal. Z-scores are used when the **population standard deviation is known** or when you have larger sample sizes. While the z-score can also be used to calculate probability for unknown standard deviations and small samples, many statisticians prefer using the t-distribution to calculate these probabilities.

*Table of Critical Values (Z-Score)*

| **Test Type** | **Level of Significance** | | | |
|---|---|---|---|---|
| | $\alpha = 0.01$ | $\alpha = 0.025$ | $\alpha = 0.05$ | $\alpha = 0.10$ |
| **left-tailed test** | $-2.33$ | $-1.96$ | $-1.645$ | $-1.28$ |
| **right-tailed test** | 2.33 | 1.96 | 1.645 | 1.28 |
| **two-tailed test** | $\pm 2.575$ | $\pm 2.33$ | $\pm 1.96$ | $\pm 1.645$ |

a. **left-tailed test:** If the alternative hypothesis $H_a$ contains the less-than inequality symbol (<), the hypothesis test is a left-tailed test.
b. **right-tailed test:** If the alternative hypothesis $H_a$ contains the greater-than inequality symbol (>), the hypothesis test is a right-tailed test.
c. **two-tailed test:** If the alternative hypothesis $H_a$ contains the not-equal-to symbol (≠), the hypothesis test is a two-tailed test. In a two-tailed test, each tail has an area of $\frac{1}{2}\alpha$.

**Examples:**

Find the critical z values. In each case, assume that the normal distribution applies.

1. left-tailed test with α= 0.01 $\boldsymbol{z = -2.33}$ (based on the table of critical value of z)
2. two-tailed test with α=0.05 $\boldsymbol{z = \pm 1.96}$
3. right-tailed test with α=0.025 $\boldsymbol{z = 1.96}$

## Critical Value of t-Distribution

The **t-distribution table** values are **critical values** of the **t-distribution**. The column header is the **t-distribution probabilities** (alpha). The row names are the **degrees of freedom** (df).

To find critical values for t-distribution:
1. Identify the level of significance.
2. Identify the degrees of freedom, d.f. = n -1.
3. Find the critical value using t-distribution in the row with n-1 degrees of freedom. If the hypothesis test is:
   a. **left-tailed,** use "α one tail" column with a negative sign.
   b. **right-tailed,** use "α one tail" column with a positive sign.
   c. **two-tailed,** use "α two tails" column with a negative and a positive sign.

*Critical Value Table for t-Distribution*

| $\alpha$ **for one-tailed test** | ***0.05*** | ***0.025*** | ***0.01*** | ***0.005*** |
|---|---|---|---|---|
| $\alpha$ **for two-tailed test** | ***0.10*** | ***0.05*** | ***0.02*** | ***0.01*** |
| **df = (n – 1)** | | | | |
| **1** | 6.311 | 12.706 | 31.821 | 63.657 |
| **2** | 2.920 | 4.303 | 6.065 | 9.925 |
| **3** | 2.353 | 3.182 | 4.541 | 5.841 |
| **4** | 2.132 | 2.776 | 3.747 | 4.604 |
| **5** | 2.025 | 2.571 | 3.365 | 4.032 |
| **6** | 1.943 | 2.447 | 3.143 | 3.707 |
| **7** | 1.895 | 2.365 | 2.998 | 3.499 |
| **8** | 1.860 | 2.306 | 2.896 | 3.355 |
| **9** | 1.833 | 2.262 | 2.821 | 3.250 |
| **10** | 1.812 | 2.228 | 2.764 | 3.169 |
| **11** | 1.796 | 2.201 | 2.718 | 3.106 |
| **12** | 1.782 | 2.179 | 2.681 | 3.055 |
| **13** | 1.771 | 2.160 | 2.650 | 3.012 |
| **14** | 1.761 | 2.145 | 2.624 | 2.977 |
| **15** | 1.753 | 2.134 | 2.602 | 2.947 |

| | | | | |
|---|---|---|---|---|
| **16** | 1.746 | 2.120 | 2.583 | 2.921 |
| **17** | 1.740 | 2.110 | 2.567 | 2.898 |
| **18** | 1.734 | 2.101 | 2.552 | 2.878 |
| **19** | 1.729 | 2.093 | 2.539 | 2.861 |
| **20** | 1.725 | 2.086 | 2.528 | 2.845 |
| **21** | 1.721 | 2.080 | 2.512 | 2.831 |
| **22** | 1.717 | 2.074 | 2.508 | 2.819 |
| **23** | 1.714 | 2.069 | 2.500 | 2.807 |
| **24** | 1.711 | 2.064 | 2.492 | 2.797 |
| **25** | 1.708 | 2.060 | 2.485 | 2.787 |
| **26** | 1.706 | 2.056 | 2.479 | 2.779 |
| **27** | 1.703 | 2.052 | 2.473 | 2.771 |
| **28** | 1.701 | 2.048 | 2.467 | 2.763 |
| **29** | 1.699 | 2.045 | 2.462 | 2.756 |
| **30** | 1.697 | 2.042 | 2.457 | 2.750 |

**Examples:**

a) Find the critical t-value for a left-tailed test with α= 0.05 and n =21.
   **Answer:** $\boldsymbol{t = -1.725}$
b) Find the critical t-value for a right-tailed test with α=0.01 and n = 17.
   **Answer:** $\boldsymbol{t = 2.583}$
c) Find the critical t-values for a two-tailed test with α=0.05 and n =26.
   **Answer:** $\boldsymbol{t = \pm 2.060}$

## Critical Regions/Rejection Regions

**Critical region**, also known as the **rejection region,** describes the entire area of values that indicates you reject the null hypothesis. In other words, the critical region is the area encompassed by the values not included in the acceptance region. It is the area of the "tails" of the distribution.

The "tails" of a test are the values outside of the critical values. In other words, the tails are the ends of the distribution and they begin at the greatest or least value in the alternative hypothesis (the critical values).

## Rejection Region If Population Variance Is Known

To determine the critical region for a normal distribution, we use the table for the standard normal distribution. If the level of significance is $\alpha = 0.10$, then for a one-tailed test, the critical region is below $z = -1.28$ or above $z = 1.28$. For a two-tailed test, use $\frac{\alpha}{2} = 0.05$ and the critical region is below $z = -1.645$ and above $z = 1.645$. If the absolute value of the calculated statistics has a value equal to or greater than the critical value, then the null hypotheses $H_o$ should be rejected and the alternate hypothesis $H_a$ is assumed to be supported.

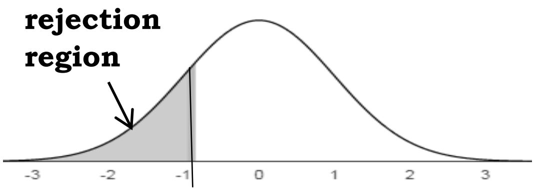
## Rejection Region If Population Variance Is Unknown

To determine the critical region for a t-distribution, we use the table of the t-distribution. (Assume that we use a t-distribution with 20 degrees of freedom.) If the level of significance is $\alpha = .10$, then for a one-tailed test, $t = -1.325$ or $t = 1.325$. For a two-tailed test, use $\frac{\alpha}{2} = 0.05$ and then $t = -1.725$ and $t = 1.725$. If the absolute value of the calculated statistics has a value equal to or greater than the critical value, then the null hypotheses $H_o$ will be rejected and the alternate hypotheses $H_a$ is assumed to be correct.

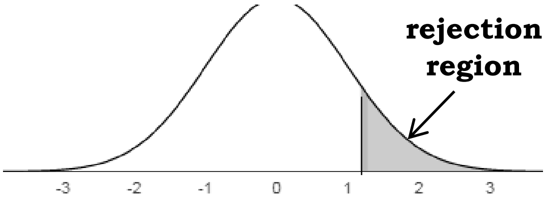
## Hypothesis Test and Their Tails

There are three types of test from a "tails" standpoint:

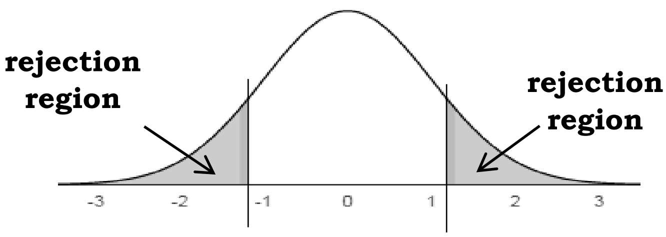
- A left-tailed test only has a tail on the left side of the graph.




- A right-tailed test only has a tail on the right side of the graph




- A two-tailed test has tails on both ends of the graph. This is a test where the null hypothesis is a claim of a specific value.



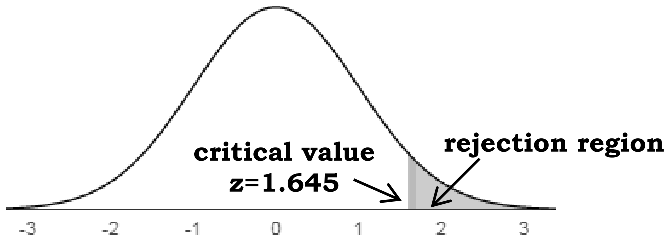

**Illustrative Examples:**

Determine the critical values and the appropriate rejection region. Sketch the sampling distribution.

1. **Right-tailed test where $\sigma$ is known, $\alpha = 0.05$, and $n = 34$**

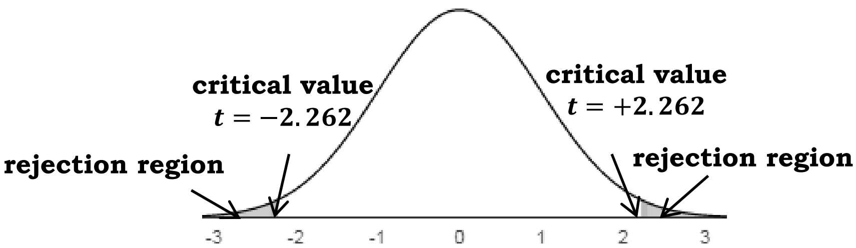
*In this example, the population standard deviation is known. Therefore, the test statistic would be z-test. To obtain the critical value for the level of significance of 0.05 and one-tailed test, z-value from the table is 1.645. The hypothesis test is right-tailed,* *so the inequality symbol would be ≥. Hence, the rejection region for a one-tailed test is* $\boldsymbol{z \geq 1.645}$.

*To sketch the graph, locate first the critical value of 1.645 which is between the 1 and 2 in the normal curve. Then, shade the region greater than the critical value because it is a right-tailed test.*



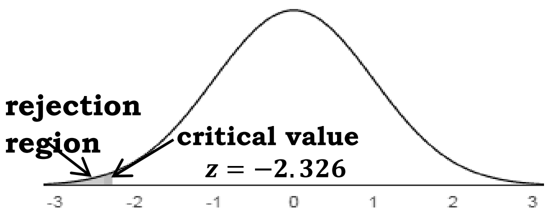

**2. Two-tailed test where $\sigma$ is unknown, $\alpha = 0.05$, and $n = 10$**

*Since this is a two-tailed test, ½ of 0.05= 0.025 of the values would be in the left and the other 0.025 would be in the right tail. Looking up t-score (n=10-1=9) associated with 0.025 on the reference table, we find 2.262. Therefore, +2.262 is the critical value of the right tail and -2.262 is the critical value of the left tail. The rejection region* $\boldsymbol{t \leq -2.262 \text{ or } t \geq 2.262}$.




**3. Left-tailed test where $\sigma$ is known, $\alpha = 0.01$, and $n = 40$**

*A one-tailed test with 0.01 would have 99% of the area under the curve outside of the critical region. Since the variance is known, we use z-score as the reference to find the critical value. This is a left-tailed test, so the critical value we need is negative. The solution is z= -2.326. The rejection region is* $\boldsymbol{z \leq -2.326}$.



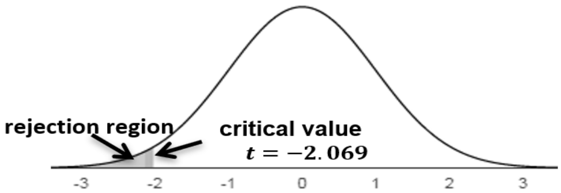

In the first three examples, you were able to find rejection region given the hypothesis test, population variance known or unknown, number of sample, and level of significance. The following example will discuss on how to determine the appropriate rejection region in a real-life problem.

**4. A survey reports a customer in the drive thru lane of one fast food chain spends eight minutes to wait for his/her order. A sample of 24 customers at the drive thru lane showed a mean of 7.5 minutes with a standard deviation**

**of 3.2 minutes. Is the waiting time at the drive thru lane less than that of the survey made? Use 0.05 significance level.**

| Hypotheses | Hypothesis Test | Population Standard Known/Unknown | Level of Significance | Number of Sample | z-value or t-value |
|---|---|---|---|---|---|
| $\boldsymbol{H_0: \mu = 8,}$ $\boldsymbol{H_a: \mu < 8,}$ | **left-tailed test** | **σ is unknown.** | $\boldsymbol{\alpha = 0.05}$ | $\boldsymbol{n = 24}$ | **t-value** |

*A one-tailed test with 0.05 level of significance has 95% of the area under the curve outside of the critical region. Since the variance is unknown, we use t-score with df = 24-1=23 as the reference to determine the critical value. This is a left-tailed test, so the critical value we need is negative. The critical value is 2. 069 and the rejection region is* $\boldsymbol{t \leq -2.069}$.




5. A banana company claims that the mean weight of its banana is 150 grams with a standard deviation of 18 grams. Data generated from a sample of 49 bananas randomly selected indicated a mean weight of 153.5 grams per banana. Is there sufficient evidence to reject the company's claim? Use $\alpha = 0.05$.

| Hypotheses | Hypothesis Test | Population Standard Known/Unknown | Level of Significance | Number of Sample | z-value or t-value |
|---|---|---|---|---|---|
| $\boldsymbol{H_0: \mu = 150}$ $\boldsymbol{H_a: \mu \neq 150}$ | **two-tailed test** | **σ is known.** | $\boldsymbol{\alpha = 0.05}$ | $\boldsymbol{n = 49}$ | **z-value** |

The rejection region is $\boldsymbol{z \leq -1.96}$ **or** $\boldsymbol{z \geq 1.96.}$


After you find the appropriate rejection region, you will then compute the standard (z or t) value based on the given data in the hypothesis problem. If the computed value is in the rejection region, then reject the null hypothesis and if not, do not reject the null hypothesis. More discussions about this decision making will be on the next module.

## *What's More*

### Activity 1: What is My Value?

Find the critical value of the following.

1. right-tailed test $\alpha = 0.05$ n=25
2. two-tailed test $\alpha = 0.01$ n=20
3. two-tailed test $\alpha = 0.10$ n=29
4. left-tailed test $\alpha = 0.05$ n=50
5. two-tailed test $\alpha = 0.01$ n=67
6. one-tailed test, $\sigma$ known $\alpha = 0.05$, n=34
7. two-tailed test, $\sigma$ unknown $\alpha = 0.01$ n=23
8. right-tailed test, $\sigma$ unknown $\alpha = 0.01$ n=15
9. one-tailed test, $\sigma$ known $\alpha = 0.025$ n=37
10. left-tailed test, $\sigma$ known $\alpha = 0.05$ n=36

### Activity 2. Reject It!

Find the rejection region for each hypothesis test based on the information given.

1. $H_o$: $\mu$=121 $H_a$:$\mu$ >121 $\alpha$=0.01 n=39 $\sigma$=known
2. $H_o$: $\mu$=98.6 $H_a$:$\mu \neq$98.6 $\alpha$=0.05 n= 25 $\sigma$=unknown
3. $H_o$: $\mu$=27 $H_a$: $\mu$ <27 $\alpha$=0.05 n=12 $\sigma$=known
4. $H_o$: $\mu$=65 $H_a$: $\mu\neq$65 $\alpha$=0.05 n=9 $\sigma$=unknown
5. $H_o$: $\mu$=2.9 $H_a$: $\mu$>2.9 $\alpha$=0.01 n=50 $\sigma$=known

### Activity 3. Let's Do Sketch!

Sketch the graph given the critical value and rejection region.

1. $z \geq 2.33$
2. $z \leq -1.645$ or $z \geq 1.645$
3. $t \leq -2.145$
4. $t \leq -1.771$ or $t \geq 1.771$
5. $z \leq -1.28$

### Activity 4. Think Critically!

Identify the critical value of each given problem. Find the rejection region and sketch the curve on a separate sheet of paper.

1. $H_O: \mu = 90$
   $H_a: \mu \neq 90$
   The sample mean is 69 and sample size is 35. The population follows a normal distribution with standard deviation 5. Use $\alpha = 0.05$.

2. A survey reports the mean age at death in the Philippines is 70.95 years old. An agency examines 100 randomly selected deaths and obtains a mean of 73 years with a standard deviation of 8.1 years. At 1% level of significance, test

whether the agency's data support the alternative hypothesis that the population mean is greater than 70.95.

3. The mean time a customer waits in line before checking in a grocery chain is less than 10 minutes. To verify the performance of the store, the obtaining mean time of 25 costumers is 9.5 minutes with a standard deviation of 1.6 minute. Use these data to test the null hypothesis that the mean time is 10 minutes, at 0.01 level of significance.

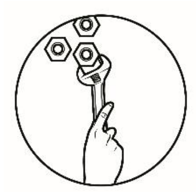
4. A fast food restaurant cashier claimed that the average amount spent by the customers for dinner is ₱125.00. Over a month period, a sample of 50 customers was selected and it was found that the average amount spent for dinner was ₱130.00. Using 0.05 level of significance, can it be concluded that the average amount spent by customers is more than ₱125.00? Assume that the population standard deviation is ₱7.00.

5. According to the radio announcer, the average price of kilogram of pork *liempo* is more than ₱210.00. However, a sample of 15 prices randomly collected from different markets showed an average of ₱215.00 and standard deviation of ₱9.00. Using 0.05 level of significance, is there sufficient evidence to conclude that the average price of pork *liempo* is more than ₱210.00?

## What I Have Learned

Complete the following statements.

1. _____________ is a point on the test distribution that is compared to the test statistic to determine whether to reject or accept the null hypothesis.
2. A _____________ may be defined as the sensitivity of the test.
3. The most used levels of significance are _______, _______, and _______.
4. Z-score is used when the population standard deviation is _____________ while t-score is used when the population standard deviation is_____________.
5. _____________, also known as the critical region, describes the entire area of values that indicates you reject the null hypothesis.
6. The values outside the critical values are the _____________.
7. To determine the critical region if population variance is known, use table for _____________ distribution while if the variance is unknown, use table for _____________ distribution.
8. If the hypothesis test is a right-tailed test, then the z-values or t-values on the rejection region are _____________ the critical value.
9. When the given hypothesis test is a two-tailed test, then the rejection regions are on ___________________ tails of the distribution.
10. To sketch the graph of the rejection region, locate first the _____________.



## *What I Can Do*

Create a meme about concepts in hypothesis testing such as hypothesis, test statistic, or rejection region.

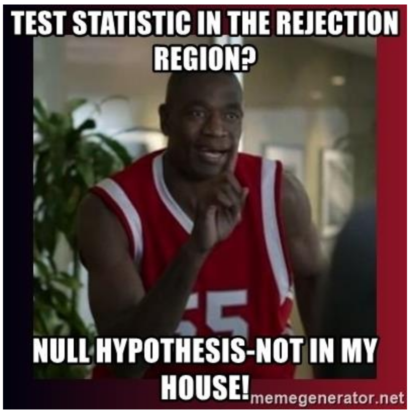
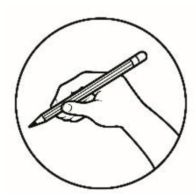



Photo taken from https://memegenerator.net/instance/48001959/dikembe-mutombo-test-statistic-in-the-rejection-region-null-hypothesis-not-in-my-house

| Criteria for Creating a Meme | Equivalent Points |
|---|---|
| Design | 15 points |
| Appropriateness | 15 points |
| Uniqueness | 10 points |
| Effectiveness | 10 points |
| **Total** | **50 points** |



## *Assessment*

Choose the best answer to the given questions or statements. Write the letter of your choice on a separate sheet of paper.

1. In a left-tailed test with $\alpha = 0.01$, the critical value of $z$ is:
   a. -2.576 B. -2.330 C. -1.960 D. -1.645
2. Which of the following defines the area encompassed by the values not included in the non-rejection region or also the area of the tails of the distribution?
   a. critical value C. level of significance
   b. rejection region D. population variance

3. For a two-tailed test with variance unknown, n= 16, and $\alpha = 0.05$, what is the critical value?
   a. $\pm 2.092$ B. $\pm 2.134$ C. $\pm 2.145$ D. $\pm 2.145$

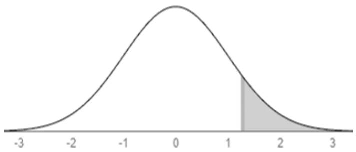
4. For a one-tailed test with a sample of 15, the null hypothesis will not be rejected at 5% level of significance if the test statistics is:
   a. $t \leq -1.761$ B. $t \leq -1.753$ C. $t \leq -1.703$ D. $t \leq -1.697$

5. If the level of significance decreased from 0.1 to 0.05, then the boundaries for the critical region move farther away from the center of the distribution.
   a. true B. false C. both A and B D. cannot be determined

6. In a right-tailed test, the rejection lies in the ________ tails of distribution.
   a. Up B. left C. right D. down

7. Based on the graph, which of the following is TRUE?
   a. This is a two-tailed test.
   b. This is a right-tailed test.
   c. Level of significance is 0.025.
   d. The rejection region is $t \leq$ -1.725.

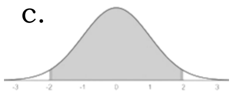

8. What is the rejection region of the given normal curve at the right?
   a. $z \geq 1.28$
   b. $z \geq 1.645$
   c. $z \geq 1.96$
   d. $z \leq 2.33$



9. Given a left-tailed test, population standard deviation is unknown, $n = 27$, $\alpha = 0.01$, what is the critical value?
   a. $t = -2.528$ B. $t = -2.479$ C. $t = -1.706$ D. $t = 2.479$

10. What is the critical value if the population variance is known, $a = 0.025$, and it is a two-tailed test?
   a. $z = \pm 1.28$ B. $z = \pm 1.645$ C. $z = \pm 1.96$ D. $z = \pm 2.33$

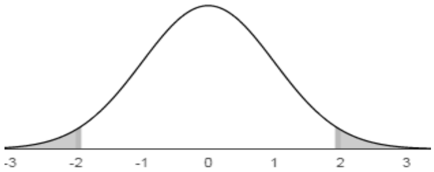
11. Which of the following is the correct illustration of rejection region $t \leq -1.943$?

a. 
b. 
c. 
d. 

12. Which of the following is the sketch of the normal curve if $z < -1.645$ *or* $z > 1.645$?

a. 
b. 
c. 
d. 

13. Given the graph of the normal curve at the right, what are the directional test of hypothesis and critical z value if $\alpha = 0.01$?
   a. two-tailed test, $\pm 2.33$
   b. two-tailed test, $\pm 2.575$
   c. left-tailed test, $-1.645$
   d. right-tailed test, $1.645$



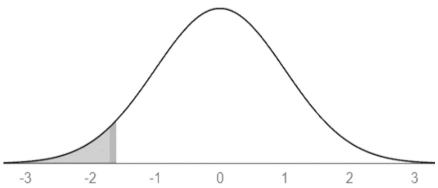

14. In the given problem below, what is the rejection region?

   The factory owner claimed that their bottled fruit juice has the capacity of less than an average of 280 ml. To test the claim, a group of consumers gets a sample of 80 bottles of the fruit juice, calculates the capacity, and then finds the mean capacity to be 265ml. The standard deviation is 8ml. Use $\alpha = 0.05$ level of significance to test the claim.

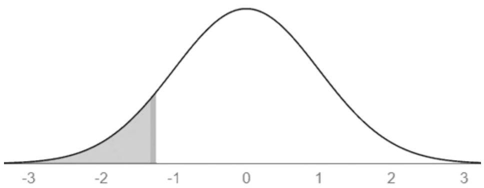
   a. $z \leq -1.645$
   b. $z \leq -1.28$
   c. $z \geq 1.645$
   d. $z \geq 2.33$

15. Based on the given problem in no. 14, which is the appropriate rejection region?

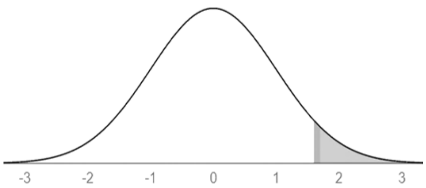
a.



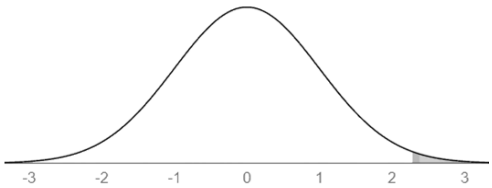

c.




b.




d.




# *Additional Activities*

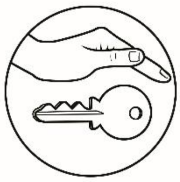
## Activity 5. Do It Now!

Read and analyze the given problem. Supply the data being asked for on the items that follow.

1. Effects of drug and alcohol on the nervous system have been the subject of significant research. A neurologist wants to test the effect of a drug by injecting 100 rats with a unit dose of the drug, subjecting each rat to stimulus, and recording its response time. It has been found out that the mean is $\bar{x} = 1.05$ with standard deviation of $s = 0.5$. The mean response time of a rat not to respond is 1.2 seconds. She wishes to test whether the mean response time

for drug-injected rats differs from 1.2 seconds. Assume that the population is normal using α = 0.05.

a. null and alternative hypotheses: ______________________________
b. test of hypothesis: ______________________________
c. level of significance: ______________________________
d. population standard deviation: ______________________________
e. sample standard deviation: ______________________________
f. number or sample: ______________________________
g. test statistic: ______________________________
h. critical value: ______________________________
i. rejection region: ______________________________
j. graph:



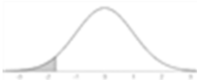
# Answer Key

## What's New

Problem 1

1. $\mu = 150$
   $\mu \neq 150$
2. two-tailed test
3. $\alpha = 0.05$
4. Yes
5. z-test
6. $z = \pm 1.96$
7. 

Problem 2

1. $\mu = 530$
   $\mu < 530$
2. one-tailed test
3. $\alpha = 0.05$
4. unknown
5. t-test
6. $t = -1.753$
7. 

## What's More

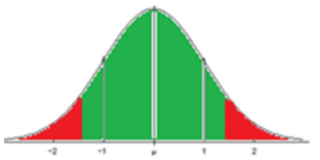
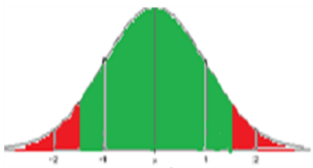
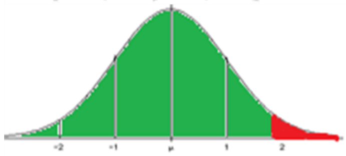
Activity 1

1. $t = 1.711$
2. $t = 2.861$
3. $t = 1.701$
4. $z = -1.645$
5. $z = \pm 2.575$
6. $z = \pm 1.645$
7. $t = \pm 2.819$
8. $t = 2.625$
9. $z = \pm 1.96$
10. $z = -1.645$

Activity 2

1. $z \geq 2.33$
2. $t \leq -2.064$ or
   $t \geq 2.064$
3. $z \leq -1.645$
4. $t \leq -2.306$ or
   $t \geq 2.306$
5. $z \geq 2.33$

## What's In

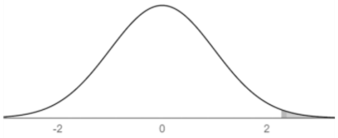
1. 
2. 
3. 
4. 
5. 

## What I Know

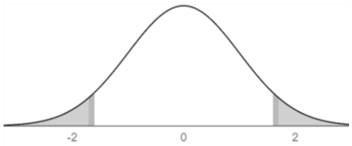
1. d
2. c
3. b
4. c
5. b
6. d
7. a
8. d
9. a
10. d
11. d
12. a
13. c
14. b
15. a

**Activity 3**

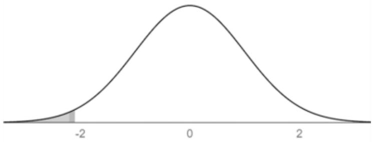
1. $z \geq 2.33$



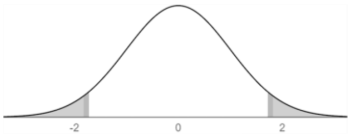
2. $z \leq -1.645$ or $z \geq 1.645$



3. $t \leq -2.145$



4. $t \leq -1.771$ or $t \geq 1.771$



5. $z \leq -1.28$

**Activity 4**

1. $z \leq -1.96$ or $z \geq 1.96$

2. $z \geq 2.33$

3. $t \leq -2.492$

4. $z \geq 1.645$

5. $t \geq 1.761$

**Assessment**

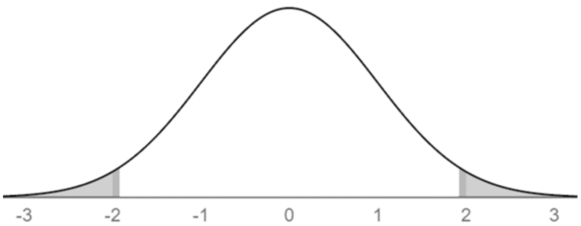
1. B
2. B
3. B
4. A
5. A
6. C
7. D
8. A
9. B
10. D
11. A
12. A
13. B
14. A
15. A

## What I Have Learned

1. Critical Value
2. level of significance
3. 0.1,0.05, and 0.01
4. known, unknown
5. rejection region
6. tails of the test
7. z, t
8. greater than or equal
9. left and right
10. critical value

**Additional Activities**

a. *null and alternative hypotheses:* $H_o: \mu = 1.2$ $H_a: \mu \neq 1.2$
b. *test of hypothesis:* *two-tailed test*
c. *level of significance:* $\alpha = 0.05.$
d. *population standard deviation:* *none*
e. *sample standard deviation:* $s = 0.5$
f. *number or sample:* $n = 100$
g. *test statistic:* *z-test*
h. *critical value:* $z = \pm 1.96$
i. *rejection region:* $z \leq -1.96$ *or:* $z \geq 1.96$
j. *graph:*



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