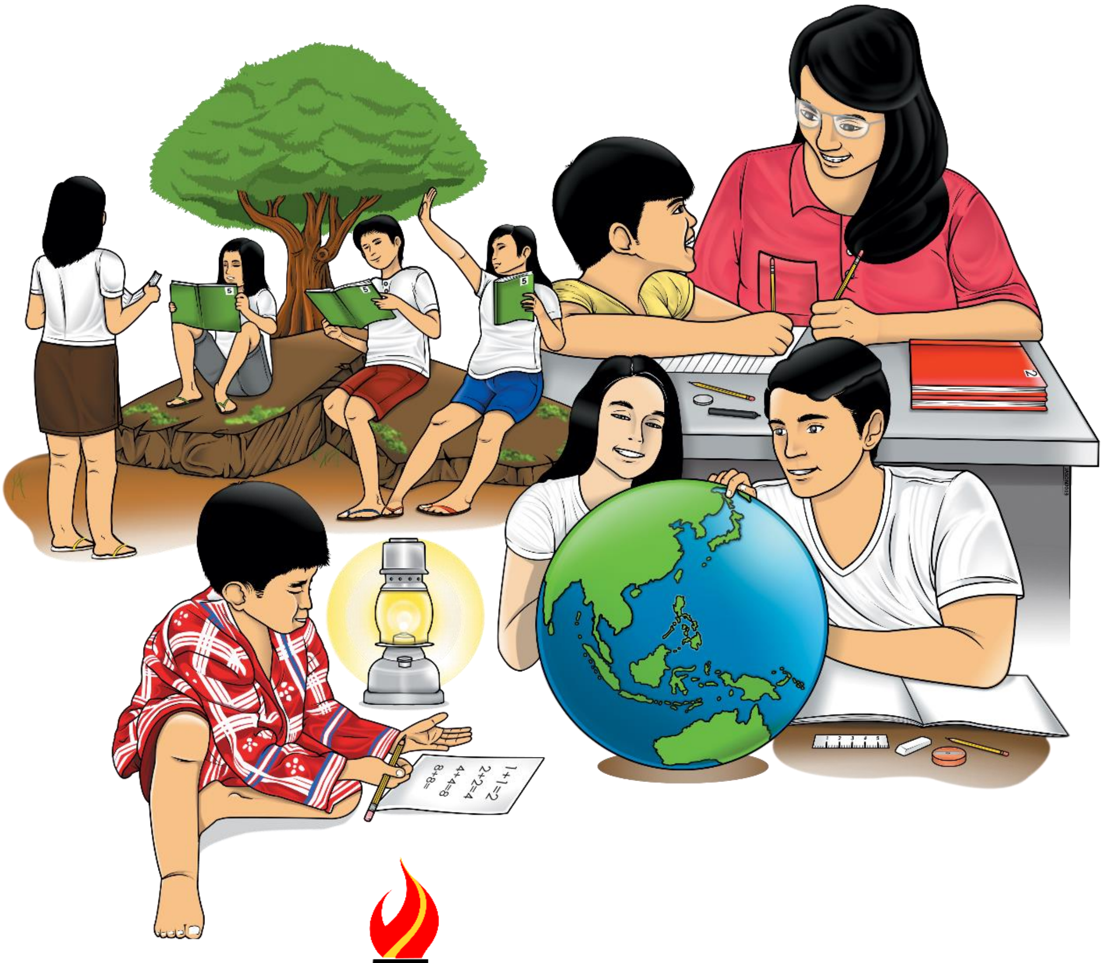
9

Mathematics

Quarter 3 – Module 5:
Proving Theorems On the Different Kinds of Parallelogram (Rectangle, Rhombus, Square)



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Alternative Delivery Mode
Quarter 3 – Module 5: Proving Theorems On the Different Kinds of Parallelogram (Rectangle, Rhombus, Square)
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Mathematics

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Proving Theorems On the Different Kinds of Parallelogram (Rectangle, Rhombus, Square)

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Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

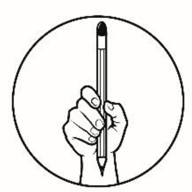
Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

What I Need to Know

The learners will be able to prove theorems on different kinds of parallelogram

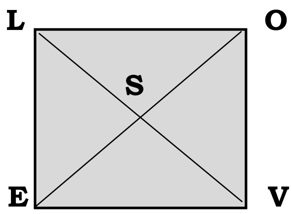


What I Know

Directions: Read and answer each of the following questions below. Write the letter of the best answer to each question. Answer directly here in the module.

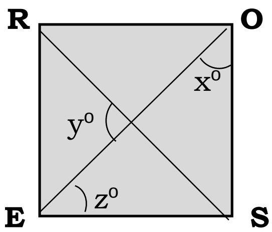
1. Which of the following statements describes that a quadrilateral is a square?
   a. One which is equiangular.
   b. One whose diagonals are congruent.
   c. One whose diagonals are perpendicular to each other.
   d. One that has four right angles and four congruent sides.

Use square **LOVE** on the right for items 2-6.



2. Find the length of diagonal LV if |OS| = 12 cm.
   a. 6 cm b. 12 cm c. 24 cm d. 36 cm
3. What is the m∡OVL given that m∡EVL = 45^0?
   a. 45° b. 55° c. 60° d. 90°
4. What is the measure of ∡OLE?
   a. 100° b. 45° c. 80° d. 90°
5. What is the measure of ∡LSO?
   a. 90° b. 80° c. 75° d. 45°
6. What is the length of diagonal OE if diagonal LV = 30 cm
   a. 15 cm b. 20 cm c. 30 cm d. 60 cm

Refer to square ROSE for items 7-9. Find the measures of the lettered angles.



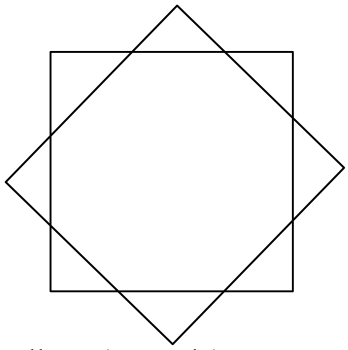
7. What is the measure of angle x?
   a. 30° b. 45° c. 60° d. 90°
8. What is the measure of angle y?
   a. 30° b. 45° c. 80° d. 90°
9. What is the measure of angle z?
   a. 30° b. 45° c. 80° d. 90°
10. If one diagonal of a square has a length of 5(n + 3) and the other diagonal has a length of 4(3n - 5), then what is the value of n?
    a. 3 b. 5 c. 7 d. 10

Lesson 1	Proving Theorems On the Different Kinds of Parallelogram (Rectangle, Rhombus, Square)

From the previous modules, you learned that *special* parallelograms like rectangle and rhombus have other properties than those general properties of parallelograms. For instance, the diagonals of a rectangle are always congruent and sometimes perpendicular, whereas in a rhombus, diagonals are always perpendicular and sometimes congruent. You also learned that each diagonal of a rhombus bisects the opposite angles. This time we are going to explore other properties related to square. The activities in this module were prepared for you to understand the lesson. The two-way proof will be used to prove theorems involving square.

What's In

PETRONAS TWIN TOWERS OF MALAYSIA



The square shaped-figure is used in arts as well as in architecture. One example of the use of squares as design is the Petronas Tower of Malaysia, one of the famous structures in Asia. The design of the tower is composed of an 8-point star formed by two overlapping squares, which is a common design of a *Muslim Architecture*.

Questions to think about!

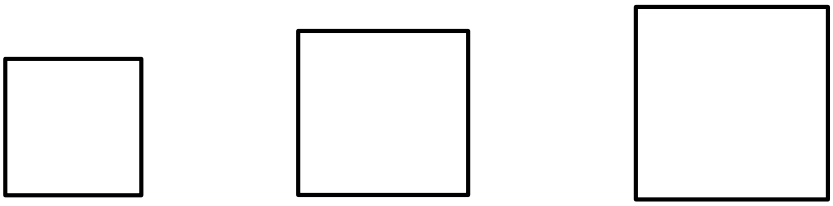
1. Where else in real life do you usually find squares?
2. Cite an example where square is a better option than the other parallelograms in terms of Architecture.
3. If you are going to design your artwork and need to choose one from rectangle, rhombus, and square, as medium, which among those parallelograms are you going to choose? Why?

What's New

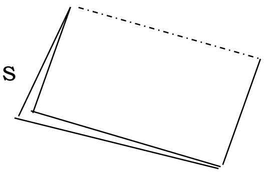
Let us explore on how to prove the theorems on square through this activity.

ACTIVITY: A CLOSER LOOK AT THE SQUARE

Make three different sizes of square. Investigate the relationship of the sides and the angles.



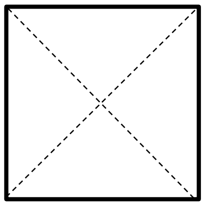
QUESTIONS TO ANSWER:



1. Fold the biggest square so that a pair of opposite sides coincide.
   Are these opposite sides congruent?

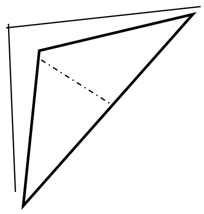
   Unfold and fold it again so that this time, the other pair of opposite sides coincide.
   Are the other opposite sides also congruent?

   Do the same thing for the other squares.
   Is your observation the same for the smaller squares?



2. Draw the diagonals and measure.
   What can you say about the lengths of the diagonals?

3. Fold the square along a diagonal so that two opposite vertices coincide.
   Are the angles congruent?



   Unfold and fold it again along the other diagonal so that the other pair of opposite vertices coincide.
   Are the other pair of opposite angles also congruent?

   What is the measure of each of the angles?

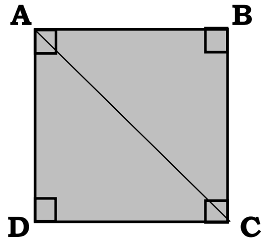
4. Fold the square half along the diagonal.
   How many triangles are formed?

   What kind of triangle is formed?
   Are the triangles congruent?

5. Complete the statements:
   In a square,
   a. the diagonals are ________ and ________ to each other.
   b. if it is folded through a diagonal, two congruent _________ triangles are formed.

What is It

In a square, if it is folded through a diagonal, two ≅ isosceles right triangles are formed.



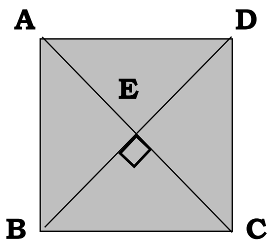
Given: ABCD is a square

Prove: ΔADC ≅ ΔCBA, and

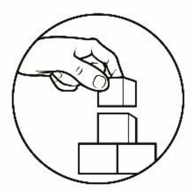
ΔADC and ΔCBA are isosceles right triangles.

STATEMENTS	REASONS
1. ABCD is a square	1. Given
2. $\overline{AD} \cong \overline{CB} \cong \overline{DC} \cong \overline{BA}$	2. All sides of a square are ≅.
3. $\overline{AC} \cong \overline{AC}$	4. Reflexive Property
5. ΔADC ≅ ΔCBA	4. SSS Postulate
5. ΔADC and ΔCBA are isosceles triangles	5 Definition of isosceles triangle.
6 $\angle D$ and $\angle B$ are right angles.	6. All angles of a square are ≅ and right angles.
7. ΔADC and ΔCBA are isosceles right triangles	7. Definition of isosceles right triangle.

Example: Given square ABCD and its diagonals intersect at E, the following statements are TRUE.

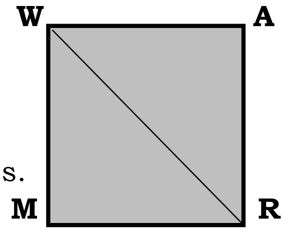


1. $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{DA}$
2. $m\angle$BCD = $m\angle$CDA = $m\angle$DAB = $m\angle$ABC = 90°
3. m∡BCE = 45°
4. m∡BEA = 90°
5. $\overline{AC} \cong \overline{BD}$
6. $\overline{BE} \cong \overline{ED} \cong \overline{AE} \cong \overline{EC}$
7. ΔBEA ≅ Δ DEA ≅ Δ DEC≅ ΔCEB
8. ΔACD ≅ ΔBDA ≅ ΔCAB ≅ ΔBDC
9. ΔABC ≅ ΔCDA; ΔBAD ≅ ΔDCB
10. $\overline{BD} \perp \overline{AC}$



What's More

Activity 1: Supply the missing statements or reasons to prove that in a square, if it is folded through a diagonal, two congruent isosceles right triangles are formed.



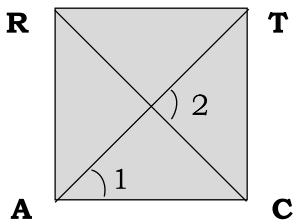
Given: ☐WARM is a square with diagonal WR

Prove: ΔWMR ≅ ΔRAW, and
ΔWMR and ΔRAW are isosceles right triangles.

STATEMENTS	REASONS
1.	1. Given
2. $\overline{WA} \cong \overline{RM} \cong \overline{WM} \cong \overline{RA}$	2.
3	3. Reflexive Property
4. Δ WMR ≅ ΔRAW	4.
5	5. Definition of isosceles triangle
6.	6. All angles of a square are ≅ and right.
7.	7. Definition of isosceles right triangle

Activity 2:

☐CART is a square. Find:



1. $m\angle 1$ = _____
2. $m\angle 2$ = _____
3. Why is ∠2 a right angle?

4. If |RC| = 3m+5 cm and |TA| = 2(m+15) cm, then:

   a. what is the value of m?
   b. what is the actual length of $\overline{RC}$?
   c. what is the actual length of $\overline{TA}$?
   d. why is $\overline{RC} \cong \overline{TA}$?

What I Have Learned

In a square,

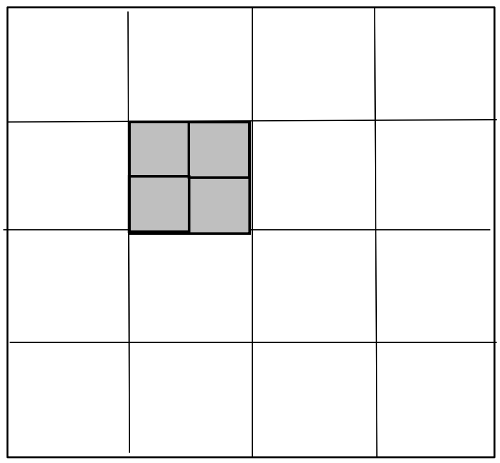
a. the diagonals are congruent and perpendicular to each other.

b. if it is folded through a diagonal, two congruent isosceles right triangles are formed.

What I Can Do

Tiling Challenge

You are tasked to study the number and the size of tiles needed for the floor of the receiving room. The room is a square with an area of 64 square meters. The whole area must be divided into sixteen congruent squares, where each square must be divided into 4 congruent smaller squares.



1. What is the size of the smallest square in the pattern?
2. How many 1-meter tiles are needed to cover the floor?

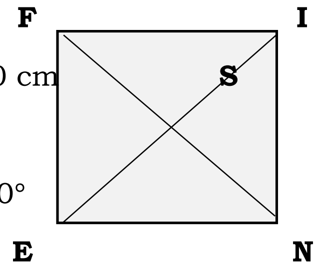
Assessment

Directions: Read and answer each of the following questions accurately. Write the letter of the best answer.

1. In a square, if it is folded through a diagonal, two congruent isosceles ______ triangles are formed.

   a. acute c. oblique
   b. right d. obtuse

For numbers 2-6, refer to square **FINE** on the right.



2. Find the length of diagonal FN if |ES| = 20 cm.

   a. 20 cm b. 40 cm c. 60 cm d. 80 cm

3. What is the measure of $\measuredangle$INE if m$\measuredangle$INF= 45°?

   a. 45° b. 55° c. 60° d. 90°

4. What is the measure of $\measuredangle$FSI?

   a. 45° b. 60° c. 90° d. 145°

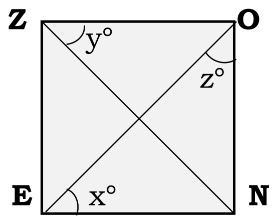
5. What is the measure of $\measuredangle$ISN?

   a. 90° b. 100° c. 120° d. 135°

6. What is the measure of diagonal IE if diagonal FN = 60 cm?

   a. 15 cm b. 20 cm c. 30 cm d. 60 cm

Refer to square ZONE for items 7-9. Find the measures of the lettered angles.



7. What the measure of $\measuredangle$y?

   a. 30° b. 45° c. 60° d. 90°

8. What is the measure of $\measuredangle$x?

   a. 30° b. 45° c. 80° d. 90°

9. What is the measure of $\measuredangle$z?

   a. 30° b. 45° c. 80° d. 90°

10. If one diagonal of a square measures 2(4k–2) cm and the other diagonal measures 3(k+7) cm, then what is the value of k?

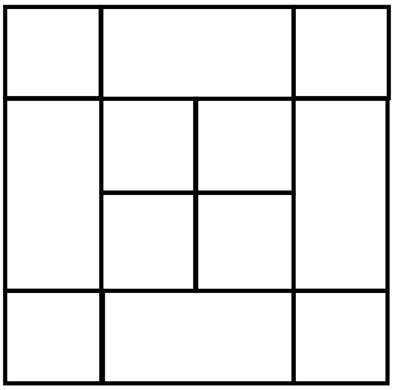
    a. 10 b. 7 c. 5 d. 3

Additional Activities

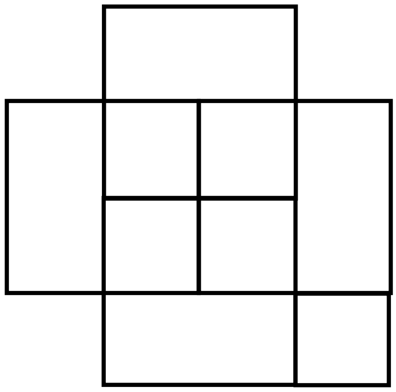
Activity 1: Squares can be seen and used everywhere. One of those is through the eye test.

How many squares do you see?

A.



B.



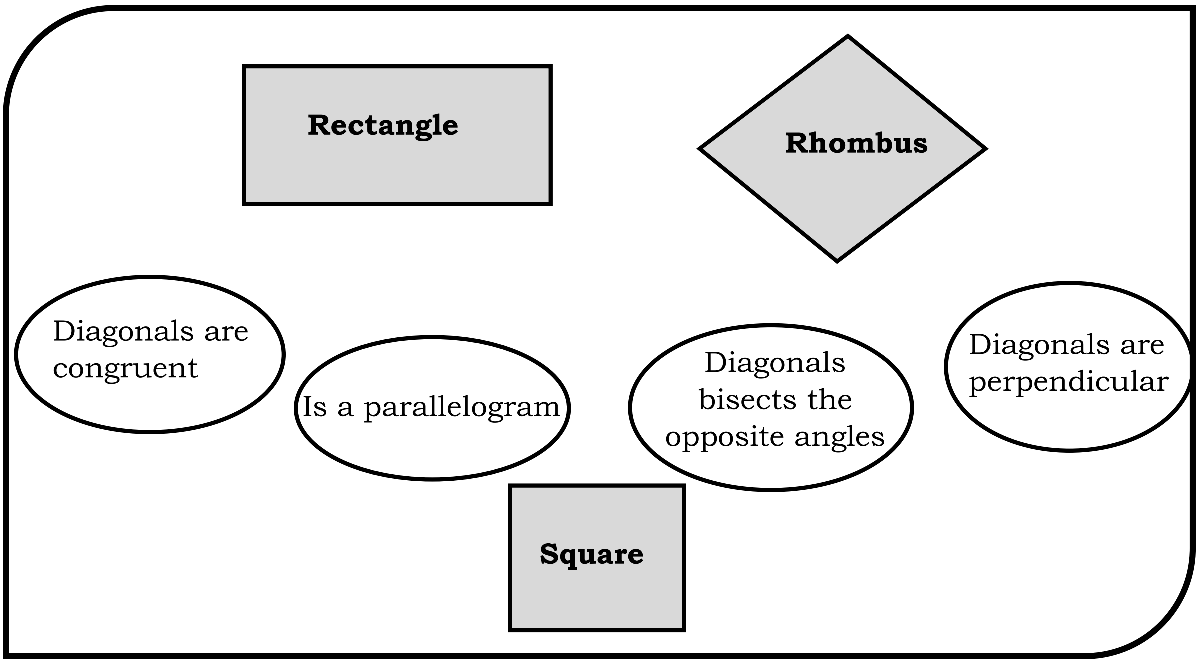
Hint: Count the number of squares from the biggest down to the smallest size.

Activity 2: Put an X in the box if the parallelogram ALWAYS exhibits the property.

Property	Rectangle	Rhombus	Square
1. Diagonals are congruent.			
2. Diagonals bisect each other.			
3. Diagonals are perpendicular.			
4. Diagonal bisects opposite angles.			
5. Diagonals are perpendicular bisectors to each other.			
6. Diagonal cuts the parallelogram into two congruent triangles.			
7. Diagonal cuts the parallelogram into two congruent right triangles.			
8. Diagonal cuts the parallelogram into two congruent isosceles right triangles.			

Based from your answers in Activity 2, what can you say about the diagonals of a square? Is it related to both rectangle and rhombus? In what way?

ACTIVITY 3: Connect the parallelograms with lines linking to its properties.



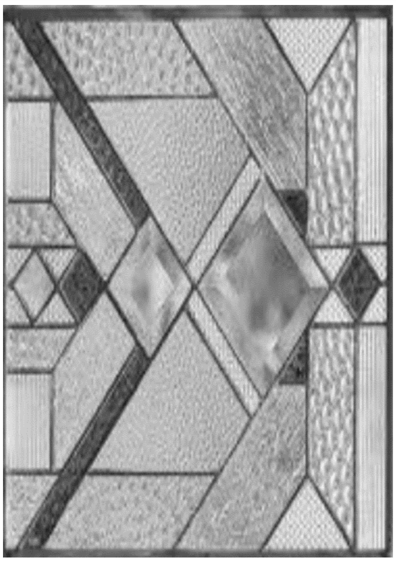
PROBLEM-BASED LEARNING WORKSHEET

LET'S DESIGN AND CREATE!

Grade 9-Magalang were given the task to do their own design of a stained-glass window provided they would use the parallelograms rectangle, rhombus, and square as pattern. Let us help them to make one with the following conditions:

a. The stained window must contain at least 10 parallelograms which are composed of rectangle, rhombus and square.
b. Parallelogram of the same kind must be of the same color (Example: rectangle is *red*, rhombus is *green* and square is *yellow*).

Note: You can choose the color that you want for each kind. Put your output on an A4 size bond paper.



Answer Key

What I Kow

1. D
2, C
3. A
4. D
5. A
6. C
7. B
8. D
9. B
10. B

What's New

1. In a square,

a. the diagonals are congruent and perpendicular to each other.

b. each diagonal forms two congruent isosceles right triangles.

What's More

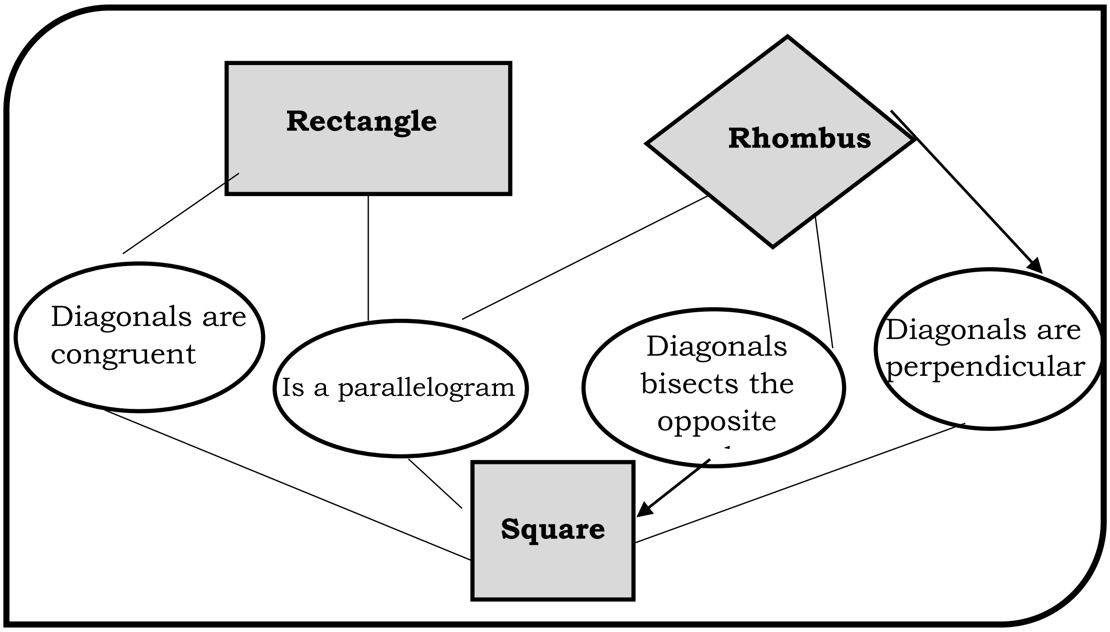
ACTIVITY 1:

STATEMENTS	REASONS
1. WARM is a square.	1. Given
2. WA ≅ MR ≅ WM≅ AR	2. Definition of square
3. WR ≅ RW	3. Reflexive Property
4. Δ WMR ≅ ΔRAW	4. SSS Postulate
5. WMR and RAW are isosceles triangles.	5. Definition of isosceles triangle
6. $\angle M$ and $\angle A$ are right angles.	6. A square is a quadrilateral with four right angles and four ≅ sides.
7. Δ WMR and ΔRAW are isosceles right triangles.	7. Definition of isosceles right triangle

ACTIVITY 2:

1. $\angle 1 = 45^o$
2. $\angle 2 = 90^o$
3. Because the diagonals RC and TA are perpendicular to each other.
4. a. 25
   b. 80
   c. 80
   d. Because the diagonals of a square are congruent

Activity 3 Connect with lines



Additional activities

Activity 1;

A. 18 squares B. 11 squares

Activity 2:

Property	Rectangle	Rhombus	Square
1. Diagonals are congruent.	x		x
2. Diagonals bisect each other.	x	x	x
3. Diagonals are perpendicular.		x	x
4. Diagonal bisects opposite angles.		x	x
5. Diagonals are perpendicular bisectors to each other.		x	x
6. Diagonal cuts the parallelogram into two congruent triangles.	x	x	x
7. Diagonal cuts the parallelogram into two congruent right triangles.	x		x
8. Diagonal cuts the parallelogram into two congruent isosceles right triangles.			x

Assessment

1. B
2. B
3. D
4. C
5. A
6. D
7. B
8. B
9. B
10. C

What I Can Do

1. 1 x 1 or 1 sq. meter
2. 64 tiles

References

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