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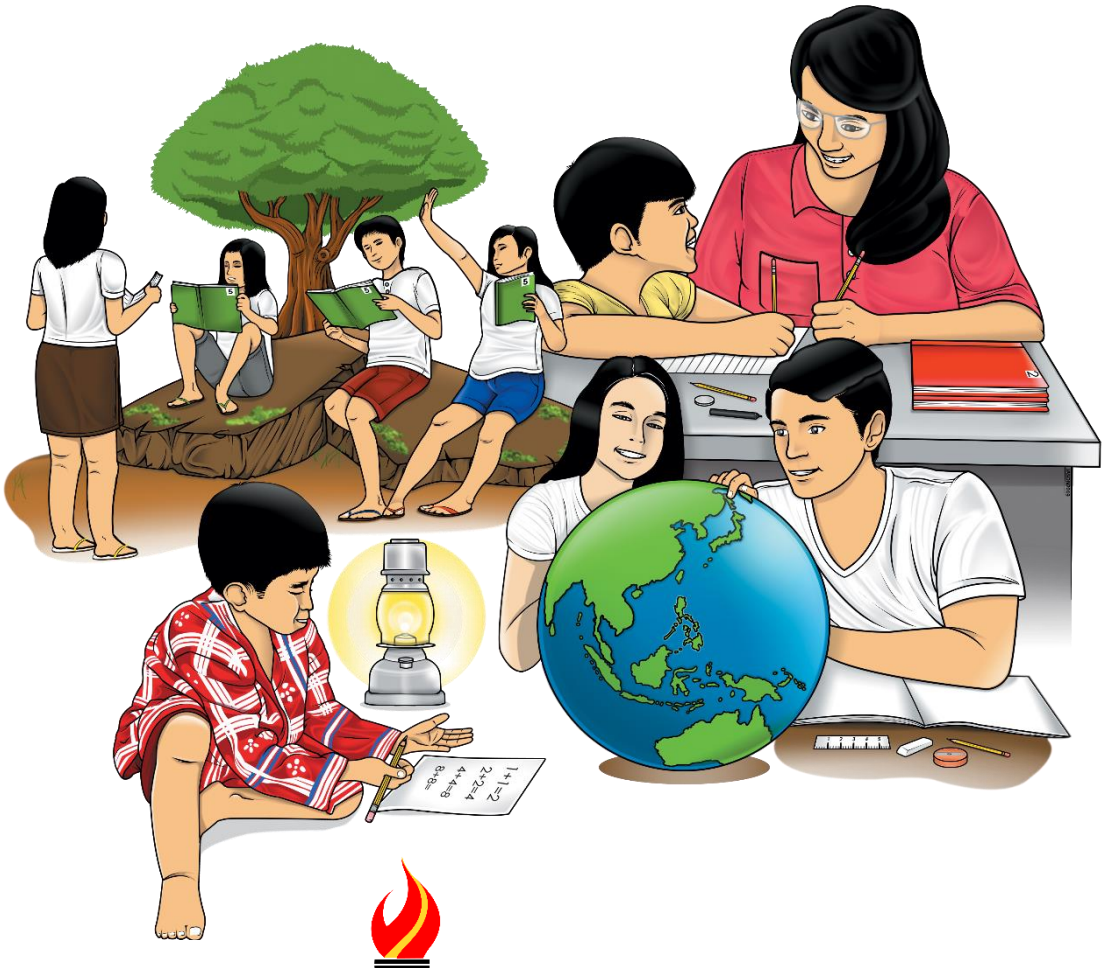
Mathematics

Quarter 3 – Module 4
Proving Theorems On The Different Kinds Of Parallelogram



ALTERNATIVE DELIVERY MODE
ADM

Mathematics – Grade 9
Alternative Delivery Mode
Quarter 3 – Module 4: Proving Theorems On The Different Kinds Of Parallelogram (Rhombus)
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9

Mathematics

Quarter 3 – Module 4
Proving Theorems On The Different Kinds Of Parallelogram

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Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

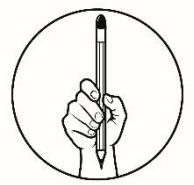
If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

What I Need to Know

This module contains:

Lesson 1 - prove theorems on different kinds of parallelogram (Rhombus)



What I Know

Direction: Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

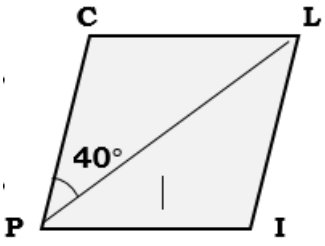
1. Diagonals of a rhombus are _____.

   a. Parallel c. congruent

   b. perpendicular d. complementary

2. Given rhombus **CLIP**, what is the measure of ∠LIP?

   a. 40° b. 60° c. 100° d. 140°



3. From the same figure in no. 2, find the measure of ∠ILP?

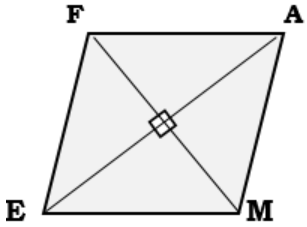
   a. 40° b. 60° c. 100° d. 140°

4. The diagonal of a rhombus _____ its opposite angles.

   a. bisects c. contains

   b. connects d. dissects

For items 5 – 7, consider the rhombus FAME on the right.



5. If m$\angle FAM = 70^0$, what is the measure of ∠AMF?

   a. 35° b. 55° c. 60° d. 110°

6. If m$\angle MFA = 65^0$, what is the measure of ∠MFE?

   a. 35° b. 65° c. 70° d. 130°

7. If m∠AFE = 120°, what is the measure of ∠AME?

   a. 60° b. 90° c. 120° d. 180°

Use rhombus TRAP on the right for items 8-10.

8. What is the measure of ∠2?

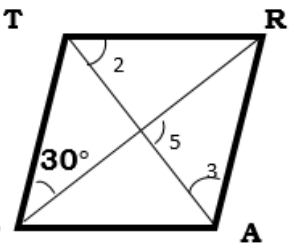
   a. 45° b. 60° c. 90° d. 12(

9. What is the measure of ∠5?

   a. 45° b. 60° c. 90° d. 12(

10. What is the measure of ∠3?

    a. 45° b. 60° c. 90° d. 12(



Lesson 1

PROVING THEOREMS ON THE DIFFERENT KINDS OF PARALLELOGRAM (Rhombus)

In the previous module, you proved a theorem on rectangle to claim that its diagonals are congruent. You used this theorem to find the lengths of the diagonals and sides of the rectangle. In this module, you are going to learn theorems on another kind of parallelogram, the rhombus. The two-way proof will still be used to prove a new set of theorems involving rhombus.

What's In

THE ARKANSAS FLAG

Jacynth is a Grade 9 exchange student who needs to stay with her foster parent in Arkansas. She's been doing well in school. One time her Math teacher gave an assignment on how to get the exact area of the white region at the center of an Arkansas state flag which happens to be in shape of a rhombus. On one flag that she has measured, the diagonals are 24 inches and 36 inches long. Can we help her find its area?

Let's Investigate!

1. What are the lengths of the diagonals of the rhombus based from the given data above?

2. What can be said about the four small triangles formed when the diagonals of the rhombus

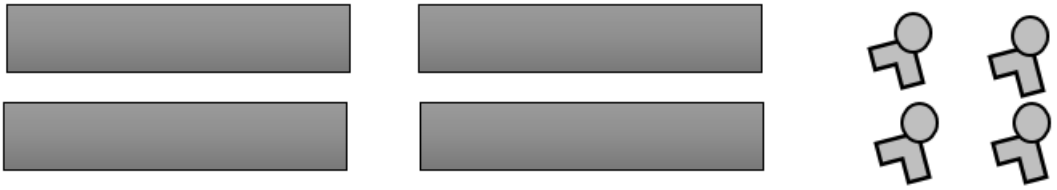
   are drawn?

3. What is the exact area of the white rhombus-shaped region at the center of the flag?

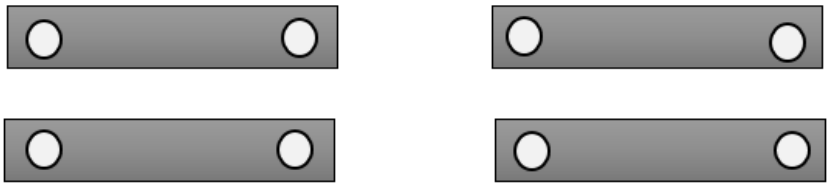
What's New

Let us explore on how to prove some theorems on rhombus through this activity.

ACTIVITY: RHOMBUS PROBE
Materials: four strips of cardboard of equal length,
four round fasteners, puncher

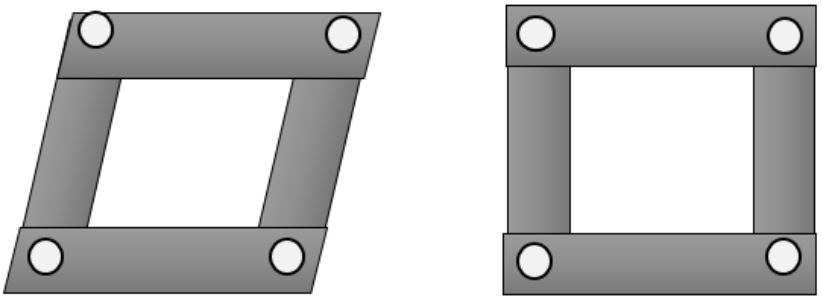


STEP 1: Punch holes on each of the four cardboards at both ends.



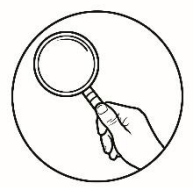
STEP 2: Connect these four cardboards 2 at a time to form an equilateral parallelogram.

What kind of parallelogram is formed?



STEP 3: Draw the diagonals of the rhombus. Measure the angles formed by the diagonals and the opposite angles of the rhombus. Use a ruler and a protractor to complete the following conclusion.

a. The diagonals of a rhombus are _____ to each other.
b. The diagonal of a rhombus _____ its opposite angles.



What is It

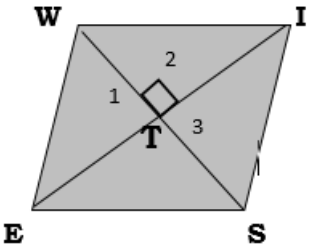
Based on the previous activity, we were able to conclude that: 1. The diagonals of a rhombus are perpendicular to each other; and 2. The diagonal of a rhombus bisects its opposite angles. Let us verify these statements using the following proofs.

Theorem 1: The diagonals of a rhombus are perpendicular to each other.

Proof:

Given : Rhombus WISE with diagonals $\overline{WS}$ and $\overline{EI}$

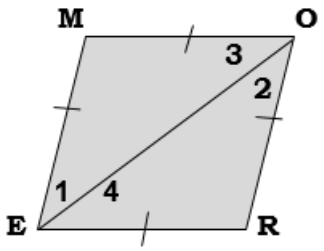
Prove: $\overline{WS} \perp \overline{EI}$



STATEMENTS	REASONS
1. Rhombus WISE w/ diagonals $\overline{WS}$ and $\overline{EI}$	1. Given
2. $\overline{WI} \cong \overline{IS} \cong \overline{SE} \cong \overline{EW}$	2. Definition of a rhombus
3. $\overline{WS}$ and $\overline{EI}$ bisect each other at T.	3. Diagonals of a parallelogram bisect each other.
4. $\overline{WT} \cong \overline{ST}$	4. Definition of bisector
5. $\overline{TI} \cong \overline{TI}$	5. Reflexive Property
6. $\Delta WTI \cong \Delta STI$	6. SSS Postulate
7. $\angle 3 \cong \angle 2$	7. CPCTC
8. $\angle 3$ and $\angle 2$ form a linear pair	8. Definition of angles forming a linear pair
9. $\angle 2$ and $\angle 3$ are supplementary	9. Linear Pair Postulate
10. $\angle 3$ and $\angle 2$ are right angles	10. If two angles are both congruent and supplementary, then they are right angles
11. $\overline{WS} \perp \overline{EI}$	11. Definition of perpendicular lines

Theorem 2: Each diagonal of a rhombus bisects its opposite angles.

Given: Rhombus MORE with diagonal $\overline{EO}$
Prove : $\angle 1 \cong \angle 4$, $\angle 3 \cong \angle 2$



Proof:

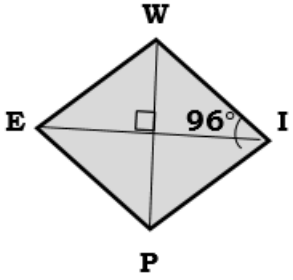
STATEMENTS	REASONS
1. MORE is a rhombus with diagonal $\overline{EO}$	1. Given
2. $\overline{MO} \cong \overline{RE} \cong \overline{ME} \cong \overline{RO}$	2. All sides of a rhombus are ≅. Definition of a rhombus
3. $\angle M \cong \angle R$	3. Opposite angles of parallelogram are ≅.
4. $\Delta MEO \cong \Delta ROE$	4. SAS Postulate
5. $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$	6. CPCTC
6. $\Delta MEO \cong \Delta ROE$ are isosceles Δs	5. Definition of isosceles Δ
7. $\angle 1 \cong \angle 3$ **;** $\angle 2 \cong \angle 4$	7. Base angles of isosceles Δ are ≅.
8. $\angle 1 \cong \angle 4$; $\angle 3 \cong \angle 2$	8. Transitive Property

Now, let us apply these theorems.

Example 1:

$\square$ WIPE is a rhombus. Find the measures of the following angles if m∠WIP = 96°

a. m∠WIE
b. m∠IWE
c. m∠ EWP
d. m∠ WPE



Solution:

a. Since diagonal $\overline{IE}$ is the bisector of ∠WIP and m∠WIP = 96°, m∠WIE = ½ (96°), then **m∠WIE = 48°.**

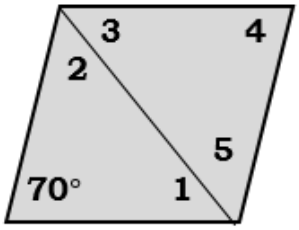
b. Since consecutive angles of a parallelogram are supplementary and m∠WIP = 96°, m∠ IWE = 180° - 96°, then **m∠IWE = 84°**

c. Since diagonal $\overline{WP}$ is the bisector of ∠IWE which measures 84° then m∠EWP = ½(84°), then **m∠EWP = 42°.**

d. Since ∠EWP ≅ ∠WPE, then **m∠ WPE = 42°**.

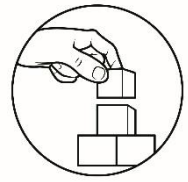
Example 2:

Find the measure of each numbered angle in the rhombus.



Solution:

- m∠4 = 70° (Opposite angles are congruent.)
- m∠1 + m∠5 + 70° = 180° (Consecutive angles are supplementary.)
- m∠1 + m∠5 = 180° – 70° = 110° (Subtraction Property)
- m∠1 = m∠5 = 55° (110° ÷ 2, Diagonal of rhombus bisects opposite ∠*s*)
- m∠1 + m∠5 = m∠2 + m∠3 (Opposite angles are congruent.)
- m∠2 = m∠3 = 55° (110° ÷ 2, Diagonals of rhombus bisect opposite ∠*s*)

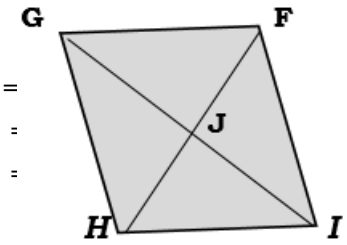


What's More

Activity 1: Write TRUE if the statement is *always true*, otherwise, write FALSE.

Given: □ GFIH is a rhombus and diagonals $\overline{GI}$ and $\overline{FH}$ intersect at J.

___1. $\overline{FH} \cong \overline{GI}$.
___2. $\overline{FJ} \perp \overline{IJ}$.
___3. ∠FHG ≅ ∠FHI.
___4. ∠FHG ≅ ∠IFH.
___5. m∠FGH + m∠ GHI = 180°.
____ 6. $\overline{IJ} \cong \overline{GJ}$
____ 7. m∠FIH =
____ 8. m∠FJG =
____ 9. m∠GFJ =
____10. $\overline{FI} \cong \overline{IH}$

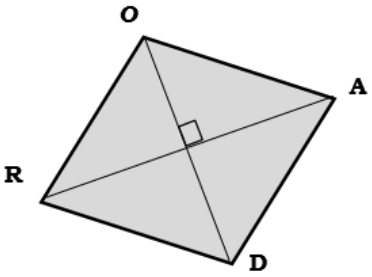


Activity 2 :

Given: Rhombus ROAD

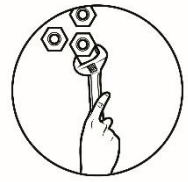
If m∠ROA = 108°, find:

a. m∠ROD
b. m∠ORA
c. m∠OAD



What I Have Learned

THEOREM: Diagonals of a rhombus are perpendicular. Each diagonal bisects each of the opposite angles.



What I Can Do

CREATE A JOURNAL **THE TESSELLATED ROOF**

(Answer the following questions)

Look at the picture at the right.

Do you see rhombus figures in the picture?

What makes a rhombus very useful as a shape?

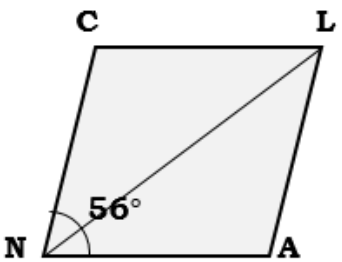
What makes it flexible?

Assessment

Directions: Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. The diagonal of a rhombus _____ its opposite angle
   a. conjugates c. bisects
   b. connects d. combines

2. Given the rhombus CLAN, what is the measure of ∠NAL?
   a. 28° b. 56° c. 84° d. 124°



3. From the same figure in no. 2, find the measure of ∠CNL?

a. 28° b. 56° c. 84° d. 124°

Use the figure on the right to get the measure of each numbered angle in the given rhombus SOUL.

4. What is the measure of ∠3?

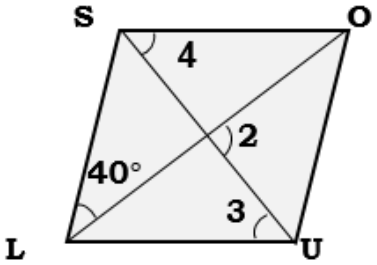
a. 45° b. 50° c. 90° d. 120

5. What is the measure of ∠2?

a. 45° b. 60° c. 90° d. 12(

6. What is the measure of ∠4?

a. 45° b. 50° c. 90° d. 12(



7. Diagonals of a rhombus are _____.

a. parallel c. congruent

b. perpendicular d. complementary

8. In rhombus **DRIP**, if $m\angle DIP = 65°$, what is the measure of∠RIP ?

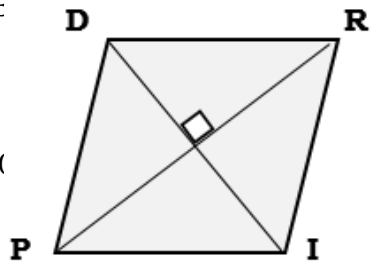
a. 35° b. 55° c. 65° d. 13

9. From the same figure, if $m\angle DRP = 35°$, what is the measure of ∠IRP?

a. 35° b. 65° c. 70° d. 13(

10. If m∠PDR = 130° , what is the measure of ∠PIR?

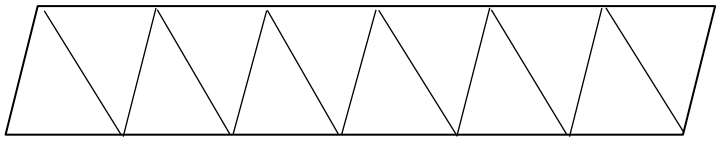
a. 65° b. 90° c. 130° d. 180°



Additional Activities

ACTIVITY: CONSTRUCTING A TESSELATION

A Tessellation is created when a shape is repeated over and over again covering a plane surface without any gaps or overlaps. A Regular Tessellation is made up of congruent regular polygons.



What to do: Construct a tessellation of rhombuses on a piece of bond paper. Make your own design, with different colors creating a mosaic.

Problem-Based Learning Worksheet

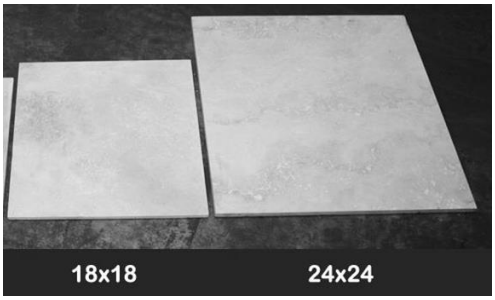
LET'S ANALYZE!

RHOMBUS TILES

Joshua wanted to put tiles on their floor and have rhombus design as finishing touches to their new house. He is undecided which tile to choose among the selections. There are 18-cm x 18-cm, 24-cm x 24-cm, and 30-cm x 30-cm tiles.

Can we help him decide which option is best for his flooring? The floor area is 100 square meters.

1. If he is to choose the smallest 18-cm x 18-cm size, how many tiles would he need?
2. If he is to buy the 24-cm x 24-cm size, how many tiles would he use to cover the entire floor?
3. How about the 30-cm x 30-cm size, how many tiles would fit the entire floor?
4. If you were Joshua, what size of tile do you prefer?
5. If you are the seller, which tile would you recommend? Why?



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Answer Key

What I Know

1. B
2. C
3. A
4. A
5. B
6. B
7. C
8. B
9. C
10. B

What's In

1. 36 inches and 24 inches
2. 4 congruent right triangles.
3. Area = 432 sq. in

WHAT'S NEW (RHOMBUS PROVE)

1. The diagonals of a rhombus are perpendicular to each other.
2. The diagonals of a rhombus bisect each opposite angles.

What's More

Activity 1

1. FALSE	6. TRUE
2. TRUE	7. FALSE
3. TRUE	8. TRUE
4. TRUE	9. FALSE
5. TRUE	10. TRUE

Activity 2:

1. Diagonal OD bisects $\angle ROA$.

$$m\angle ROD = \frac{1}{2}m\angle ROA$$

$$\angle ROD = 54^\circ$$

2. $\angle ROD$ and $\angle ORA$ are complementary.

$$\angle ORA = 90 - 54 = 36^\circ$$

3. $\angle OAD$ is opposite of $\angle ORD$

$$\angle OAD = 2(36) = 72^\circ$$

Assessment

1. C
2. D
3. A
4. B
5. C
6. B
7. B
8. D
9. A
10. C

Problem-Based Learning Worksheet

1. 3087 tiles
2. 1737 tiles
3. 1112 tiles
4. Answers may vary
5. Answers may vary

References

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Gladys C. Nivera (2013) Grade 9 Mathematics -Patterns and Practicalities. pp. 374-376, SalesianaBOOKS by Don Bosco Press, Inc. Makati City

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