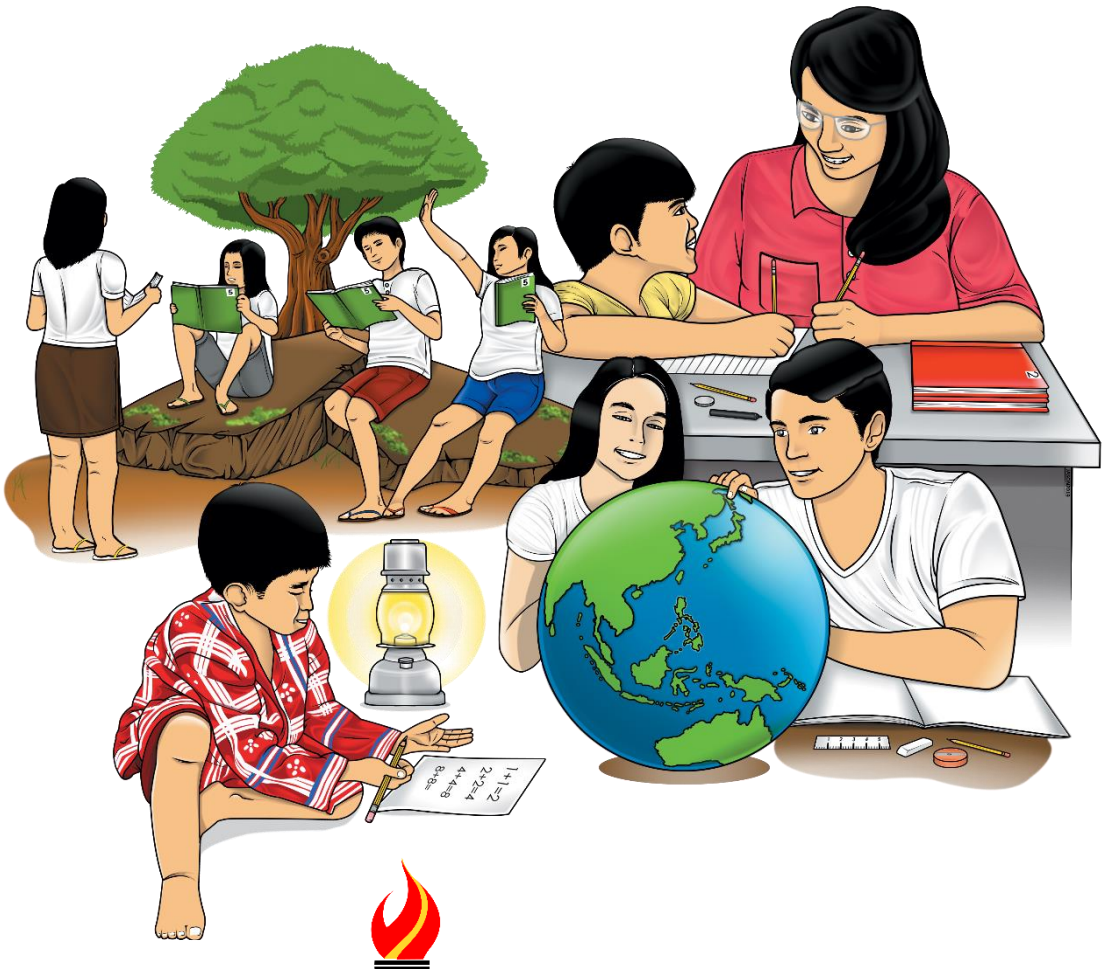
9

DepED
DEPARTMENT OF EDUCATION

Mathematics

Quarter 3 – Module 3
Proving Theorems On The Different Kinds Of Parallelogram



ALTERNATIVE DELIVERY MODE
ADM

Mathematics – Grade 9
Alternative Delivery Mode
Quarter 3 – Module 3: Proving Theorems On The Different Kinds Of Parallelogram (Rectangle)
First Edition, 2020

Published by the Department of Education
Secretary: Leonor Magtolis Briones
Undersecretary: Diosdado M. San Antonio

Development Team of the Module

Writer: Maria Cecilia Villamiel Arpilleda

Editors: Corazon T. Misa, Ma. Theresa G. Mallari, Catherine C. De Guzman

Validators: Remylinda T. Soriano, Angelita Z. Modesto, George B. Borromeo

Illustrator: Maria Cecilia Villamiel Arpilleda

Layout Artists: Maria Cecilia Villamiel Arpilleda, Leonil Rechie P. Cahanap

Management Team: Malcolm S. Garma
Genia V. Santos
Dennis M. Mendoza
Maria Magdalena M. Lim
Aida H. Rondilla
Lucky S. Carpio

Printed in the Philippines by _____

Department of Education - National Capital Region

Office Address: Misamis St., Brgy. Bago Bantay, Quezon City
Telefax: (632) 8926-2213 /8929-4330 /8920-1490 and 8929-4348
E-mail Address: ncr@deped.gov.ph

9

Mathematics

Quarter 3 – Module 3
Proving Theorems On The Different Kinds Of Parallelogram

DepED
DEPARTMENT OF EDUCATION

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

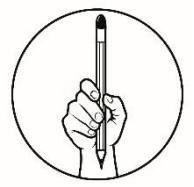
If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

What I Need to Know

This module contains:

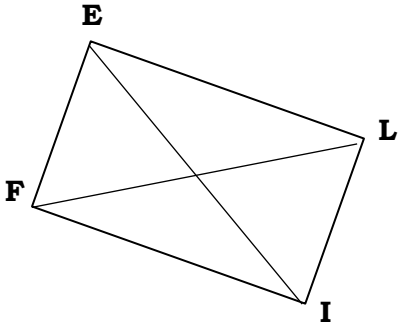
Lesson 1 - prove theorems on different kinds of parallelogram (Rectangle)



What I Know

Direction: Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. Which of the following information serves as sufficient condition for classifying quadrilateral LIFE as a rectangle?



   a. $\overline{LF}$ and $\overline{IE}$ bisect each other.
   b. $\overline{LF}$ and $\overline{IE}$ bisect each other and $\overline{LF} \perp \overline{IE}$
   c. $\overline{LF}$ and $\overline{IE}$ bisect each other and $\overline{LF} \cong \overline{IE}$
   d. $\overline{LF}$ and $\overline{IE}$ bisect each other, $\overline{LF} \perp \overline{IE}$, and $\overline{LF} \cong \overline{IE}$

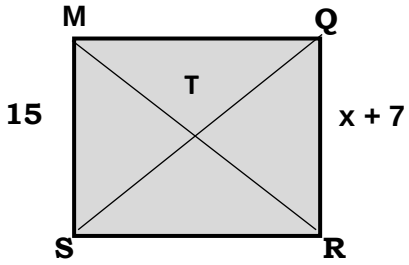
2. Which of the following quadrilaterals has diagonals that are sometimes **NOT** perpendicular?

   a. kite b. rectangle c. rhombus d. square

3. What condition makes parallelogram PATH a rectangle?
   a. $\overline{PA} \cong \overline{TH}$.
   b. $\angle$ T is a right angle.
   c. $\overline{PA} \parallel \overline{TH}$.
   d. $\overline{PT}$ and $\overline{AH}$ bisect each other.

4. If □ **MQRS** is a rectangle with $|QR|$ = x+7 cm. and $|MS|$ = 15 cm, find the value of x.

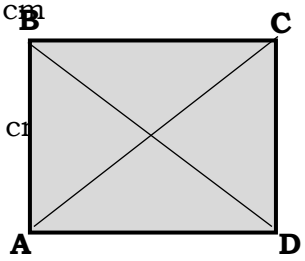
a. 7 b. 8 c. 9 d. 12



5. From the same figure on the right. If $|MR|$ = 20 cm what is the measure of $\overline{TQ}$?

a. 10 cm b. 20 cm c. 30 cm d. 40

For nos. 6-8, find the value of the indicated expression using rectangle □ABCD.



6. What is the value of $|AC| + |AD|$, if $|BD|$ = 28 cm and $|AD|$ =21 cm

a. 18 cm b. 49 cm c. 59 cm d. 70 cm

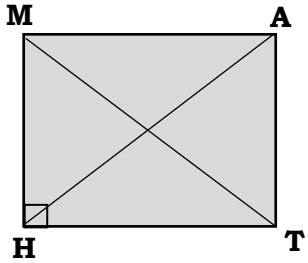
7. What is the value of $|BD| - |AC|$, if $|BC|$ = 40 cm and $|CD|$ = 30 cm

a. 0 b. 10 cm c. 20 cm d. 30 cm

8. What is $|BD|$ if $|BC|$ = 40 cm and $|AC|$ = 50 cm?

a. 10 cm b. 25 cm c. 50 cm d. 70 cm

9. Given rectangle MATH. What is the measure of $\overline{MT}$, if $|MH|$ = 5 cm and $|AH|$ = 9 cm?



a. 3 cm b. 4 cm c. 5 cm d. 9 cm

10. On the same figure, if $|MT|$ = 5(x - 4) and $|AH|$ = 2(x-1), what is the value of x?

a. 3 b. 4 c. 5 d. 6

Lesson 1

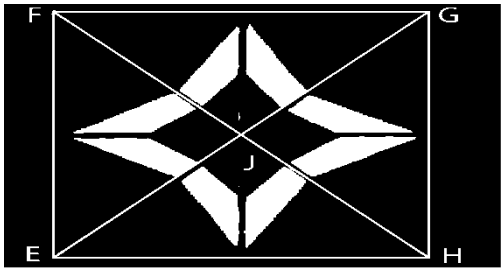
PROVING THEOREMS ON THE DIFFERENT KINDS OF PARALLELOGRAM (Rectangle)

In the previous module, you learned to use the properties of parallelogram to find the measures of angles, sides and other quantities that are conceptually related to the real world. In the next lessons, you will prove theorems on the different kinds of parallelograms such as rectangle, rhombus and square. In this module, you will learn to supply the missing statements and proofs that will validate the theorem on rectangle.

What's In

RECTANGULAR STAINED GLASS

Mariah is an artist who connects stained glass pieces with lead strips. In this rectangle , the strips are cut so that $|FH|$ = 34 in, $|GH|$ = 16 in and $|EH|$ = 30 in long. She wanted to get the total measure of the lead strips that she used. Can we help her in finding the total length of the strips?



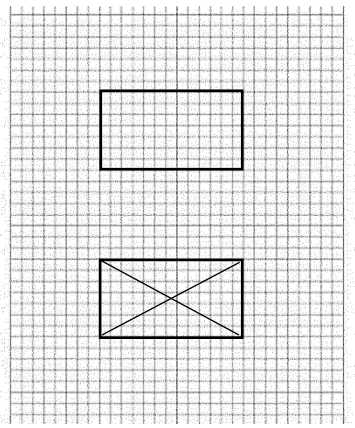
Let's Investigate!

1. What are the given measures in this rectangular stained glass? What particular part of the rectangle does each given measure stand for?
2. What properties of parallelogram are you going to use in order to find the missing measures of the sides of the rectangle?
3. How do you get the total length of the strips used?

What's New

Let us explore on how to prove a theorem on rectangle through this activity.

ACTIVITY: RECTANGLE PROBE
Materials: graphing paper
ruler



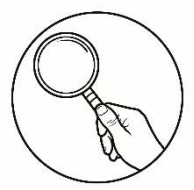
STEP 1: Draw a rectangle using the lines of the graphing paper as guide.

STEP 2: Draw both diagonals with your ruler and compare the lengths of the two diagonals.

STEP 3: Draw another rectangle and repeat step 2 to find out your conclusion inductively.

STEP 4: Complete the following conclusion:

The diagonals of a rectangle are _____.

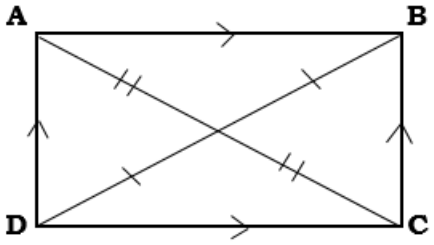


What is It

To elaborate more on this theorem, let us prove this using a two-column proof:

Given: Rectangle ABCD

Prove: $\overline{AC} \cong \overline{BD}$



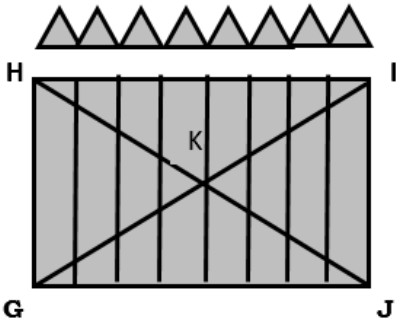
STATEMENTS	REASONS
1. □ABCD is a rectangle	1. Given
2. $\overline{AD} \cong \overline{BC}$	2.Property of Rectangle
3. $\overline{AB} \cong \overline{AB}$	3.Reflexive Property
4.$\angle D$AB and $\angle C$BA are right angles	4.Definition of Rectangle
5. $\angle D$AB ≅ $\angle C$BA	5. All right angles are congruent.
6. Δ DAB ≅ ΔCBA	6.SAS Postulate
7. $\overline{AC} \cong \overline{BD}$	7.CPCTC

Now that we have verified that the diagonals of a rectangle are congruent, let us try to apply this theorem to the following examples:

Example 1: The rectangular gate has diagonal braces. Given $|GI|$=47 in and $|HJ|$ = 3x+2 in, find the value of x and$|HJ|$

a

3



Solution:

Since diagonals of a rectangle

congruent, then $|HJ| = |GI|$

Thus, $|HJ|$ = 47 and

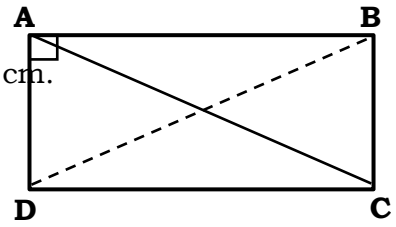
Solving for x: 3x = 47 – 2

3x = 45

x = 15.

Example 2: Given rectangle ABCD,
with $|AC|$ = 2x+28 cm and $|BD|$ = 4(5x-2) cm.

a. What is the value of x?
b. What are the measures of $\overline{AC}$ and $\overline{BD}$?



Solution: **a.** $|AC| = |BD|$

2x +28 = 4(5x-2)

2x +28 = 20x -8

28+8 = 20x-2x

36 = 18x

x = 2

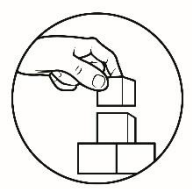
b. $|AC|$ = 2x + 28

$|AC|$ = 2(2) +28

$|AC|$ = 4 +28

$|AC|$ = **32 cm**

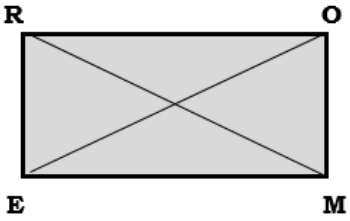
$|\boldsymbol{BD}| = |\boldsymbol{AC}|$, $|\boldsymbol{BD}|$ = **32 cm**



What's More

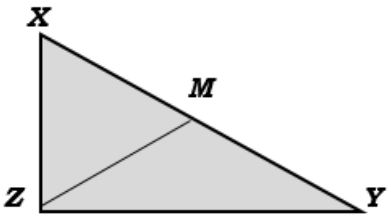
Activity 1: Supply the missing statements or reasons to prove that the diagonals of a rectangle are congruent.

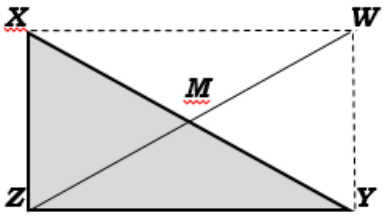
Given: Rectangle ROME
With diagonals $\overline{OE}$ and $\overline{RM}$
Prove: $\overline{OE} \cong \overline{RM}$



STATEMENTS	REASONS
1.	1. Opposite sides of a rectangle are parallel and congruent.
2. $\overline{EM} \cong \overline{ME}$	2.
3.	3. Definition of rectangle
4. ∠REM ≅ ∠OME	4.
5. ΔREM ≅ ΔOME	5.
6.	6. CPCTC

Activity 2. Given: M is the midpoint of hypotenuse $\overline{XY}$ and $|XY|$ = 10 m.
How long is $\overline{MZ}$?
Hint: Draw the rectangle XWYZ as shown.





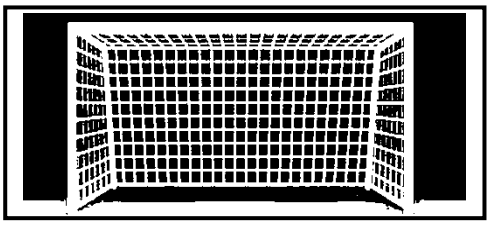
How did you find the activities? Did you find it challenging? How did you cope with it?

What I Have Learned

Theorem: A parallelogram is a rectangle if and only if its diagonals are congruent.

What I Can Do

SOCCER GOAL



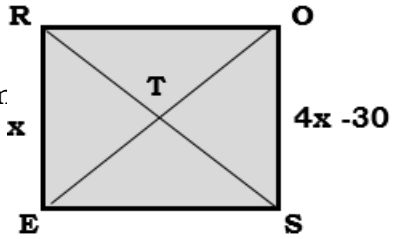
The opening of a soccer goal is shaped like a rectangle. The horizontal distance between the goal posts is 3 times the height of the soccer goal from the top of the goal post to the ground. If its perimeter is 64 feet, what is the distance between the goal posts? What is the total length of the diagonals that can be drawn in the given rectangular goal?

Assessment

Directions: Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. If □ **ROSE** is a rectangle with $|OS|$ = 4x-30 and $|RE|$ = x, find the value of x.

   a. 7 b. 8 c. 9 d. 10



2. From the same figure on the right, if $|EO|$ = 20 cm what is the measure of $\overline{RS}$?

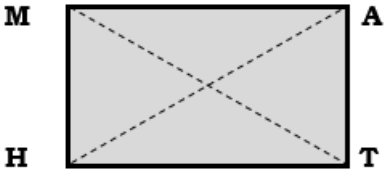
   a. 10 cm b. 20 cm c. 30 cm d. 40 cm

3. If $|RT| + |TS|$ = 56 cm, what is the measure of $\overline{ET}$?

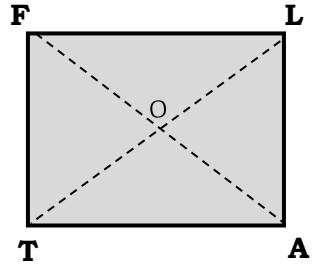
   a. 28 cm b. 36 cm c. 56 cm d. 112 cm

4. What condition makes parallelogram MATH a rectangle?

   a. $\angle M$TH is a right angle.
   b. $\overline{MA} \cong \overline{TH}$.
   c. $\overline{MA} \parallel \overline{TH}$.
   d. $\overline{MT} \cong \overline{AH}$.



5. Given rectangle FLAT. What is the measure of $\overline{OL}$ if $|FA|$ = 30 cm?



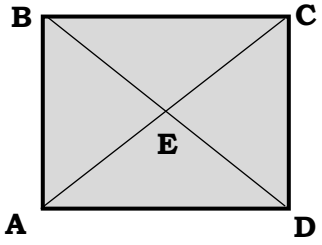
a. 3 cm b. 9 cm c. 12 cm d. 15 cm

6. On the same figure, if $|FA|$ = 3(x - 10) and $|TL|$ = 2(x -5), what is the value of x?

a. 10 b. 20 c. 30 d. 40

For numbers 7-10. Find the value of the indicated expression using rectangle ABCD.

7. What is the value of $|AC|$ - $|BD|$, if $|AB|$ = 21cm and $|AD|$ = 28 cm?



a. 0 b. 35 c. 50 d. 70

8. What is $|BD|$, if $|EC|$ = 25 cm?

a. 0 cm b. 35 cm c. 50 cm d. 70 cm

9. What is $|AC|$, if $|AE|$ = 25 cm?

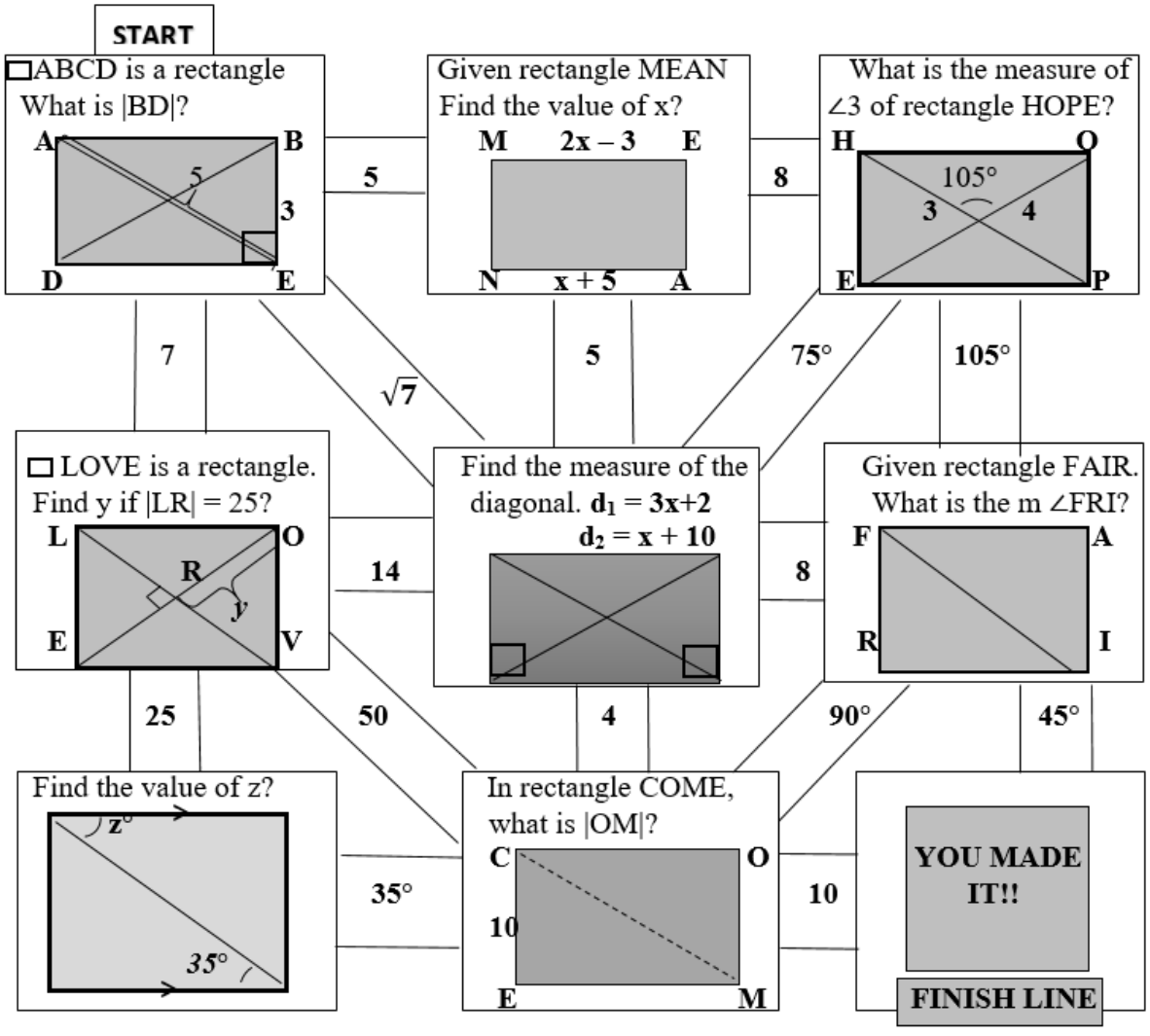
a. 10 cm b. 25 cm c. 50 cm d. 70 cm

10. If $|AC|$ = 5x and $|BD|$ = 9x – 80 cm, what is $|AE|$?

a. 60 cm b. 50 cm c. 40 cm d. 30 cm

Additional Activities

AMAZING: Answer each of the given problems on rectangle then put a cross on the path of your correct answer from the starting point up to the finish line (exit point). **Good Luck!**



How was the activity? How did you manage to get to the finish line?

Problem-Based Learning Worksheet

LET'S ANALYZE!

1. Suppose $|MO|$ = 5x -2 and $|PN|$ = 12x-23. What is the value of x?

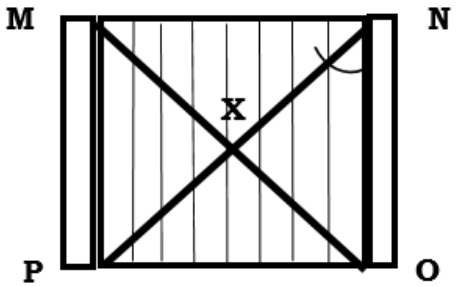
2. If $|MX|$ = 100 cm, what is the total measure of the two diagonals?

3. If the length of the two diagonal cables equals to 6.54 meters, calculate the length of each cable.

4. Given $|NO|$ = 16 meters and $|PN|$ = 20 meters, find $|MO|$?

TURNBUCKLE

Parallelograms are structures that can be "squared up" by a turnbuckle. For the gate in the picture, you tighten or loosen the turnbuckle on the diagonal cable so that the cable stays congruent to the other diagonal.



E-Search

https://www.shutterstock.com

https://mathbitsnotebook.com

https://1.cdn.edl.io *Prove theorems about parallelogram*

https://vectorstock.com

https://www.shutterstock.com

https://www.gettyimages.com

https://en.m.wikipedia.org>wiki

https://images.app.goo.gl/hMqLN79GgrFmusdQA

What I Know

1. C
2. B
3. B
4. B
5. A
6. B
7. A
8. C
9. D
10. D

What's In

$\overline{FG} \cong \overline{HI}$; $\overline{FI} \cong \overline{GH}$; $\overline{FH} \cong \overline{GI}$

$\overline{FG} + \overline{HI} + \overline{FI} + \overline{GH} + \overline{FH} + \overline{GI}$

34 + 34 +16 +16 +30 +30

= 160 inches

(Total length of lead strip used in the stained glass)

What's More

Activity 1

STATEMENTS	REASONS
1. $RE \parallel OM$; $RE \cong OM$	1. Opposite sides of a rectangle are parallel and congruent.
2. $EM \cong ME$	2. Reflexive Property
3. ∠ REM and ∠ OME are right ∠'s	3. Definition of rectangle
4. Δ REM ≅ Δ OME	4. SAS Postulates
5. $OE \cong RM$	5. CPCTC

Activity 2

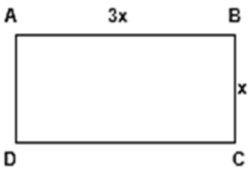
Rectangle XWYZ has two diagonals $\overline{XY}$ and $\overline{ZW}$.
Since $\overline{XY}$ is one of the two diagonals, therefore $\overline{XY} = \overline{ZW}$
If $\overline{XY}$ = 10, then $\overline{ZW}$ = 10
Given that point M is the midpoint of $\overline{XY}$, it is also the midpoint of $\overline{ZW}$ since their diagonals bisect each other.
Thus, $\overline{MZ} = ½ (\overline{ZW}) = ½ (10) \rightarrow \overline{MZ} = 5$

WHAT'S NEW (Rectangle Probe)
Diagonals of rectangle are congruent.

What I Can Do

P=2(x)+2(3x)

P = $\overline{AB} + \overline{BC} + \overline{CD} + \overline{DA}$
64 feet = 3x + x + 3x + x
64 feet = 8x
x = 64/8
x = 8 feet
3x = 24 feet
Length of the goal post = 24 feet



Using Pythagorean Theorem:
AC = $\sqrt{24^2 + 8^2}$
AC = $\sqrt{640}$ = 25.3 ft
AC + BD = 50.6 ft

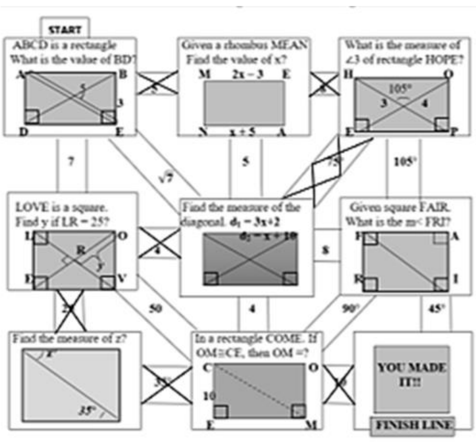
Assessment

1. D
2. B
3. A
4. D
5. D
6. B
7. A
8. C
9. C
10. B

Problem-Based Learning Worksheet

1. x = 3
2. 400 cm
3. 3.27 meters
4. 20 meters

Additional Activity



Answer Key

References

Ferdinand C. Pascual , Gilda C. Galangue, Nenita N. de Leon (2007) Worktext in Mathematics III Geometry. pp. 159-161, Innovative Educational Materials, Inc. Sta. Ana, Manila

Gladys C. Nivera (2013) Grade 9 Mathematics -Patterns and Practicalities. pp. 374-376 , SalesianaBOOKS by Don Bosco Press, Inc. Makati City

For inquiries or feedback, please write or call:

Department of Education - Bureau of Learning Resources (DepEd-BLR)

Ground Floor, Bonifacio Bldg., DepEd Complex
Meralco Avenue, Pasig City, Philippines 1600

Telefax: (632) 8634-1072; 8634-1054; 8631-4985

Email Address: blr.lrqad@deped.gov.ph * blr.lrpd@deped.gov.ph