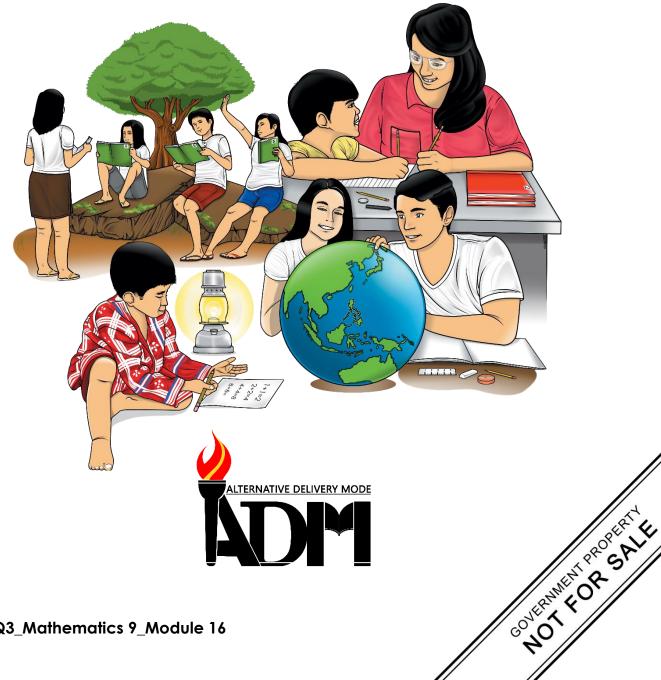




Mathematics

Quarter 3 – Module 16: **Solving Problems in Triangle Similarity and Right Triangles**



Mathematics – Grade 9 Alternative Delivery Mode Quarter 3 – Module 16: Solving Problems in Triangle Similarity and Right Triangles First Edition, 2020

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9

Mathematics

Quarter 3 – Module 16: Solving Problems in Triangle Similarity and Right Triangles



Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-bystep as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.



What I Need to Know

In this module, you will apply what you have learned about concepts and theorems in triangle similarity and right triangles. You will analyze and solve problems involving these concepts in real life situations. The scope of this module permits it to be used in many different learning situations. The language used recognizes the diverse vocabulary level of students. The lessons are arranged to follow the standard sequence of the course. But the order in which you read them can be changed to correspond with the textbook you are now using.

The module is comprised of only one lesson:

 \bullet solve problems that involve triangle similarity and right triangles. $\ensuremath{\textbf{M9GE-IIIj-1}}$

After going through this module, you are expected to:

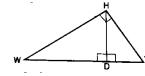
1. solve problems that involve triangle similarity and right triangles.



What I Know

Find out how much you already know about the module. Write the letter of the best answer to each question on a sheet of paper. Answer all items. After taking and checking this short test, take note of the items that you were not able to answer correctly and look for the right answers as you go through this module.

- 1.) ΔWHY is a right triangle with $\angle WHY$ as the right angle? $\overline{HD} \perp \overline{WY}$. Which of the following segments is a geometric mean?
 - I.HDIV. \overline{DW} II. \overline{DY} V. \overline{HW} III. \overline{HY} VI. \overline{WY} A.II, IV, VIB.I, III, V

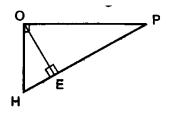


C. I only

D. All except VI

2.) In the figure, there are three similar triangles by Right Triangle Proportionality Theorem. Name the triangle that is missing in this statement: $\Delta HOP \sim __\Delta OEP.$

$OP \sim __ \Delta OEP.$	
A. <i>ΔΗΟΕ</i>	B. Δ <i>0EH</i>
С. <i>∆НОР</i>	D. Δ <i>ΗΕΟ</i>



3.) The lengths of the sides of a triangle are 6 cm, 10 cm, and 13 cm. what kind of a triangle is it?

A. Regular Triangle	C. Right Triangle
B. Acute Triangle	D. Obtuse Triangle

4.) What is the perimeter of a 30-60-90-degree triangle whose shorter leg is 5 cm long?

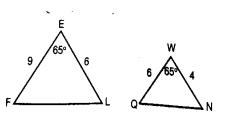
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A. 5\sqrt{3} cm B. 15 + 5\sqrt{3} cm C. 15 + \sqrt{3} cm D. 10 + 5\sqrt{3} cm
```

5.) The hypotenuse of an isosceles right triangle measures 7 cm. how long is each leg?

```
A. 7\sqrt{2} \ cm B. 3.5 cm C. \frac{7\sqrt{2}}{2} \ cm D. \frac{7\sqrt{3}}{3} \ cm
```

6.) What similarity concept justifies that $\Delta FEL \sim \Delta QWN$?

- A. Right Triangle Proportionality Theorem
- B. Triangle Proportionality Theorem
- C. SSS Similarity Theorem
- D. SAS Similarity Theorem



7.) At a certain time of the day, if a 1.5-m pole casts a shadow of 2.4 meters long, how tall is a tree that casts a shadow 16 meters long?

A. 10 m B. 24 m C. 25.6 m D. 38.4 m

8.) A rectangular field is 50 yards wide and 200 yards long. Paul walks diagonally across the field. How far does he walk?

A. 127.3 yards B. 115 yards C. 111.8 yards D.206.16 yards

9.) Which of the following triangles are always similar?

A. Isosceles Triangles	C. Equilateral Triangles
B. Right Triangles	D. Scalene Triangles

10.) A triangle has sides with lengths 6, 8, and 11 cm. a second triangle has sides with lengths 18, 16, and 22 cm. Are these triangles similar?

A. Yes B. No C. It depends. D. Cannot be determined

11.) Two poles lean against a wall such that they make the same angle with the ground. The 18-foot pole reaches 12 feet up the wall. How much further up the wall does the 22-foot pole reach, to the nearest tenth of a foot?

A. 2.7 feet B. 6.3 feet C. 4.3 feet D. 14.7 feet

12.) Which is **not** a triangle similarity theorem?

A. AA B. SAS C. SSS D. None

13.) Given the diagram at the right as labeled. Find the value of $x \swarrow$

A. 13 B. 13.5 C. 16.7 D. 17

20

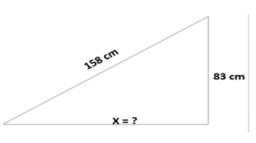
14.) How long is the diagonal of a flat screen TV with height and width of 27 in and 36 in, respectively?

A. 90 in B. 63 in

C. 45 in

D. 31 in

15.) Jay has a piano at the back of his truck. He plans to use a ramp to make it easier for him to move down the piano. The back of the truck is 83 centimeters from the ground and the ramp is 158 centimeters long. What is the horizontal distance x from the end of the ramp to the back of the truck?



A. 120.44 cm B. 134.44 cm C. 140.45 cm d. 154.45 cm

Lesson

1

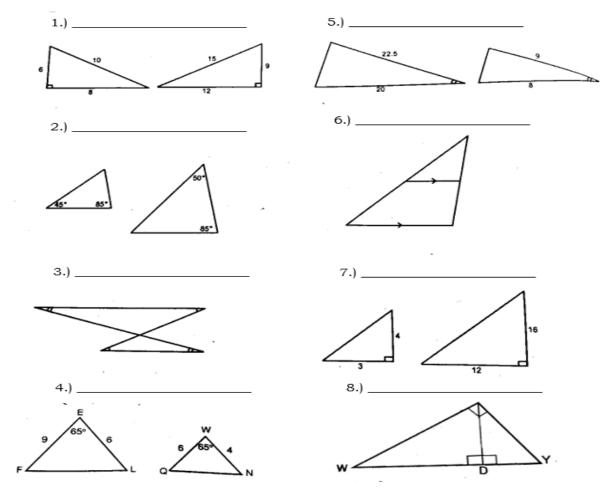
Right Triangle Similarity Theorems

In this module, you will apply what you have learned about concepts and theorems in triangle similarity and right triangles. You will analyze and solve problems involving these concepts in real life situations.



Solving problems involving triangle similarity and right triangles requires the knowledge you acquired in the past lessons. So, let's have a short review about triangle similarity theorems.

In each pair of triangles, use the given information to determine whether the triangles are similar. State the similarity theorem used.





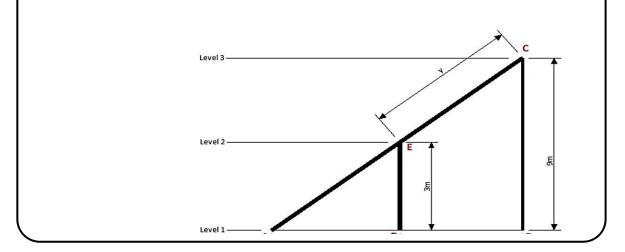
Read the selection below

"WALANG IWANAN": SMC'S COVID - 19 EFFORT

San Miguel Corporation (SMC) management assured the public that the company's supply chain is stable and that it has more than enough inventory to feed Filipinos nationwide for the next several months. They made the statement to underscore that there is no need to panic during this Covid-19 pandemic.

Amid the shortage in alcohol sanitizers, the management also said that SMC is looking to produce 70% ethyl alcohol at one of its facilities and distribute this for free to communities through local government units. The company is also donating to LGUs its own brand of disinfectant powder, which is safe for humans and can easily be used as extra protection against the virus.

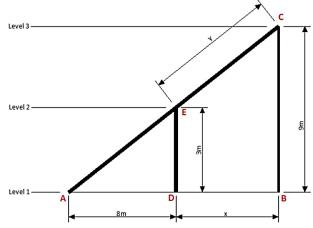
Suppose that a factory of SMC is using an inclined conveyor belt to transport its product from Level 1 to Level 2, which is 3m above Level 1 as shown in the figure below. The inclined conveyor is supported from one end at Level 1 to Level 2 by a post located 8 m away from Level 1 support point.



Notice that the given problem involves similar triangles. We can apply the concepts that you've learned about the theorems and postulates in similar triangles.

Here's the solution of the problem.

Solution: First, let us denote each intersection point by a letter as shown in red on the figure.



Following the same explanation provided in the examples above, we can conclude that the two triangles $\triangle ABC$ and $\triangle ADE$ are similar. Therefore,

$$\frac{|DE|}{|BC|} = \frac{|AD|}{|AB|}$$
$$\frac{3}{9} = \frac{8}{|AB|} \Rightarrow |AB| = \frac{(8)(9)}{3} = 24 m$$
$$x = |AB| - 8 = 24 - 8 = \mathbf{16 m}$$

Hence, the new post should be placed at a distance of 16 m from the existing post. Since the construction is forming right triangles, we can calculate the additional distance (y), which the product has to travel from Level 2 to Level 3 as follows:

y = |AC| - |AE|.

Solving for the distances |AE| and |AC|, :

$$|AE| = \sqrt{|AD|^2 + |DE|^2} = \sqrt{8^2 + 3^2} = 8.54 m$$

Similarly,

$$|AC| = \sqrt{|AB|^2 + |BC|^2} = \sqrt{24^2 + 9^2} = 25.63 m$$

y = $|AC| - |AE| = 25.63 - 8.54 = 17.09 m$

Thus,

which is the additional distance that the product has to travel to reach the new vel.

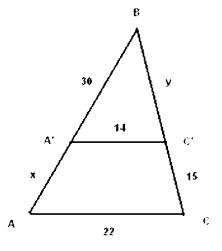


Let's have another sample problem.

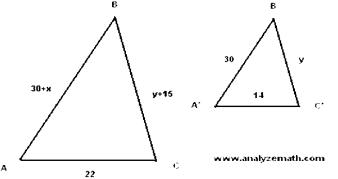
1. In triangle ABC, $\overline{A'C'}$ is parallel to \overline{AC} . Find the value of y or |BC'| and the value of x or |A'A|.

Solution:

 \overline{BA} is a transversal that intersects the two parallel lines $\overline{A'C'}$ and \overline{AC} , hence the corresponding $\angle BA'C'$ and $\angle BAC$ are congruent. \overline{BC} is also a transversal to the two parallel lines $\overline{A'C'}$ and \overline{AC} and therefore $\angle BC'A'$ and $\angle BCA$ are congruent.



These two triangles have two congruent angles and are therefore similar and the lengths of their sides are proportional. Let us separate the two triangles as shown below.



We now use the proportionality of the lengths of the sides to write equations to solve for the values of x and y.

An equation for x may be written as follows. (30 + x) = 22

$$\frac{(30+x)}{30} = \frac{22}{14}$$

Solve the equation for x. 420 + 14x = 660

x = 17.1 (rounded to one decimal place).

An equation for y may be written as follows.

$$\frac{22}{14} = \frac{(y+15)}{y}$$
Solve the equation for y.
$$22y = 14y + 210$$
$$8y = 210$$
$$\mathbf{v} = \mathbf{26.25}$$

2. Mathematics research team of Manila wishes to determine the altitude of a mountain found in Teresa, Rizal as illustrated as follows (see figure below). They use a light source at L, mounted on a structure of height 2 meters, to focus a beam of light through the top of a pole P' to the top of the mountain M'. The height of the pole is 20 meters. The distance between the altitude of the mountain and the pole is 1000 meters. The distance between the pole and the laser is 10 meters. We assume that the light source mount, the pole, and the altitude of the mountain are in the same plane. Find the altitude h of the mountain.

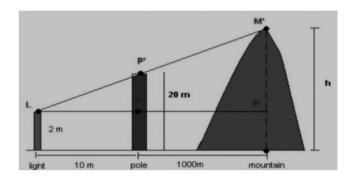


Image by Free Mathematics Tutorials https://www.analyzemath.com

Solution:

We first draw a horizontal line \overline{LM} . $\overline{PP'}$ and $\overline{MM'}$ are perpendicular to the ground and therefore parallel to each other. Since $\overline{PP'}$ and $\overline{MM'}$ are parallel, the triangles, Δ LPP' and Δ LMM' are similar. Hence, the proportionality of the sides is:

$$\frac{|LM|}{|LP|} = \frac{|M'M|}{|P'P|} \rightarrow \frac{1010}{10} = \frac{(h-2)}{18}$$

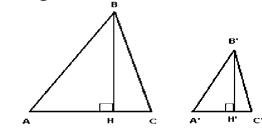
Solve for h:

10h - 20 = 18,18010h = 18,200

<u>h = 1820 meters</u>

The altitude of the mountain is 1820 meters.

The two triangles are similar and the ratio of the lengths of their sides is equal to k: $\frac{|AB|}{|A'B'|} = \frac{|BC|}{|B'C'|} = \frac{|CA|}{|C'A'|} = k$. Find the ratio of the lengths of the altitudes $\frac{BH}{B'H'}$ of the two triangles.



Solution:

If the two triangles are similar, their corresponding angles are congruent. Hence, \angle BAH and \angle B'A'H' are congruent. We now examine the triangles, \triangle BAH and \triangle B'A'H'. These triangles have two pairs of corresponding congruent angles: \angle BAH and \angle B'A'H' and the right angles, \angle BHA and \angle B'H'A'. The triangles are similar and therefore:

$$\frac{AB}{A'B'} = \frac{BH}{B'H'} = \mathbf{k}$$

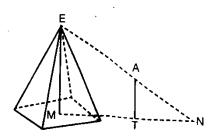


Indirect Measurement

It is believed that the first person to determine the difficult-to-obtain heights without the aid of a measuring tool lived from 624 - 547 BC. The Greek mathematician, *Thales* determined the heights of pyramids in Egypt by the method called **shadow reckoning.** The activity that follows is a version of how Thales may have done it.



Image by Dreamstime https://www.dreamstime.com



The sun shines from the western part of the pyramid and casts a shadow on the opposite side.

Analyze the figure and answer the following questions.

- 1.) |ME| is the _____ of the pyramid.
- 2.) |MN| is the length of the shadow of _____.
- 3.) _____ is the height of a vertical post.
- 4.) |TN| is the length of the _____ of the vertical post.
- 5.) Which of the following can be measured directly with the use of a measuring tool? If it can be measured directly, write YES, otherwise, write NO.

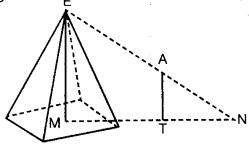
Lengths	Answer	Lengths	Answer
ME		MN	
AT		TN	

- 6.) Why is the height of the pyramid difficult to measure using a measuring tool?
- 7.) Like the post, the height of the pyramid is also vertical. What can you conclude about ME and AT?
- 8.) If $\overline{\text{ME}} \parallel \overline{\text{AT}}$, what can you say about ΔEMN and ΔATN ?
- 9.) What theorem justifies your answer?

10.) The figure is not drawn to scale. Which of the following situations is true or false?

If the length of the shadow of the pyramid is greater than the height of the pyramid, the possibility is that the measurement of the shadow was done	True or False?
early in the morning	
early in the afternoon	
late in the morning	
late in the afternoon	

11.) If |MN| = 80 ft., |NT| = 8 ft., and |AT| = 6 ft., what is the height of the pyramid in this activity?





What I Have Learned

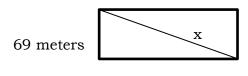
In solving problems involving triangle similarity and right triangles, you may apply your knowledge about the theorems in triangle similarity and right triangles.



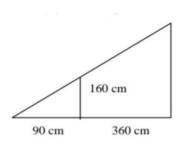
What I Can Do

Solve the following problems.

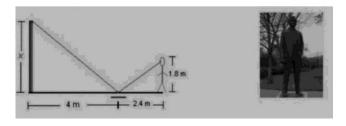
1.) What is the length of the diagonal of a rectangle 100 meters long and 69 meters wide?



100 meters

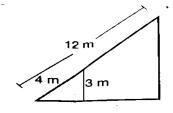


- 2.) A girl 160 cm tall, stands 360 cm from a lamp post one night. Her shadow is 90 cm long. How high is the lamp post?
- 3.) A statue honoring Ray Hnatyshyn (1934 2002), can be found on Spading Crescent East, near the University Bridge in Saskatoon. Use the information below to determine the unknown height x of the statue.

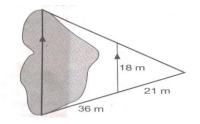


http://khippenmeyer.weebly.com/

4.) A 12 – meter fire truck ladder leaning on the vertical fence also leans on the vertical wall of a burning three – story building as shown in the figure. How high does the ladder reach?



5.) Solve for the indicated distance across the lake.





Solve each of the following items accurately. Make your computations clear on your answer sheet and encircle your final answer.

- 1.) A tree 24 feet high casts a shadow 12 feet long. Brad is 6 feet tall. How long is Brad's shadow at the same time of the day?
- 2.) Triangles EFG and QRS are similar. The lengths of the sides of Δ EFG are 144 cm, 128 cm, and 112 cm. If the length of the shortest side of triangle QRS is 280 cm, what is the length of its longest side?
- 3.) A 40-foot flagpole casts a 25-foot shadow. Find the length of the shadow cast by a nearby building 200 feet tall at the same time of the day.
- 4.) Triangles IJK and TUV are similar. The lengths of the sides of Δ IJK are 40 cm, 50 cm, and 24 cm. If the length of the longest side of Δ TUV is 275 cm, what is its perimeter?
- 5.) A triangle with side lengths 5 cm, 11 cm, and 15 cm is similar to another triangle with longest side of length 24 cm. What is perimeter of the larger triangle?
- 6.) Two vertical poles 20 feet and 80 feet high are 100 feet apart. Ropes connect the top of each pole with the base of the other pole. At what height, in feet, do these ropes intersect?
- 7.) Peter takes a picture of Wendy during vacation in Neverland. Wendy is 158 cm tall and is 2.4 meters away from the camera lens. The film is 4 cm from the lens. How tall is her image on the film to the nearest tenth of a centimeter?
- 8.) A 5-foot student casts a 3-foot shadow. At the same time of the day, if the length of the shadow cast by a flagpole is 12 feet, what is the height of the flagpole?
- 9.) On a sunny day, Bill wants to find the height of a tree. He walks 25 feet along the shadow that the tree casts until his shadow ends at the same point as the tree's shadow. Bill is 6 feet tall and the length of his shadow is 9 feet. How many inches taller is the tree than Bill?
- 10.) If the base of an isosceles triangle is 12 cm and each leg measures 10 cm, what is its altitude?

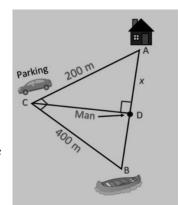


Additional Activities

Consider the given situation and answer the following questions below.

Let's say, a man has to check the schedule of the boat trips at the information center, A. The 200-m path to the information center and the 400-m path to the boat rental dock, B, intersect at the parking lot, C, forming a right angle. He walks straight from the parking lot to the lake D as shown, where a sign tells him that he is approximately 357.77 m from the dock.

- a.) How far is the man from the information center?
- b.) What is the equation to represent the situation?
- c.) If the man will go back to the parking lot from the information center. What is total distance he walked?



E-Search

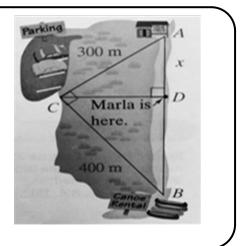
You may also check the following link for your reference and further learnings on solving quadratic equation using completing square.

- https://www.analyzemath.com/Geometry/similar-trianglesexamples-and-problems-with-solutions.html
- <u>https://www.khanacademy.org/math/geometry/hs-geo-</u> <u>similarity/hs-geo-solving-similar-triangles/v/similarity-example-</u> <u>problems</u>
- <u>https://www.khanacademy.org/math/geometry/hs-geo-</u> <u>similarity/hs-geo-solving-similar-</u> <u>triangles/e/solving_similar_triangles_1</u>
- <u>https://www.khanacademy.org/math/geometry/hs-geo-similarity/hs-geo-solving-similar-triangles/e/solving_similar_triangles_2</u>

PROBLEM – BASED LEARNING WORKSHEET

Extended ECQ

Enhanced Community Quarantine strictly prohibits clustering in every grocery store in recreational parks. Marla visits a lake nearby and wants to try canoe but she has to fill up a health declaration and contact tracing form at the reception (A). The 300-m path to the reception and the 400-m path to the canoe rental dock (B) intersect at the parking lot (C) forming a right angle. Marla walks straight from the parking lot to the lake (D) as shown, where a sign tells her that she is 320 m from the dock. How far is Marla from the reception?



Let's Analyze

- 1. What was strictly prohibited during ECQ?
- 2. How far is the reception from the canoe rental dock?
- 3. How many meters is Marla away from the dock?
- 4. What equation could be used to find how far Marla is from the reception?

5. How far is Marla from the reception?

4 B ЯΙ

NI S'TAHW

WONN I TAHW

12' B	15. D	Э [.] С	9 [.] D	3' D
14. C	A.II	8' D	2. C	5' D
A.EI	10 [.] B	A .7	4' B	1. B

SAS .7	AAA .5
SSS [.] 9	2. AA
SAS .3	I. SSS

8. Right Triangle Similarity Theorem	A. SAS

WHAT'S MORE

. 60 ft.	II	ε∍Y – NT	s = Y – TA
.True, False, False, True	JC	səY – NM	5. ME – No
Triangle Proportionality Theorem	.6		4. Shadow
Similar triangles	.8		7A .E
Parallel	·7		5. EM
Because pyramid is 3-dimensional figure	.9		1. Height

WHAT I HAVE LEARNED

m 98.84 .2	m 9.4m E.E	2. 800 cm	m 8.121.1

ASSESSENT

	9. 200 in	.11 ð1 .ð	3. 125 ft.
	.11 02 .8	5. 49.6	5 . 360
10.8 cm	тэ д.2 .7	4.627	1. 3 ft.

ADDITIONAL ACTIVITIES

- A. 89.44meters
- B. x = |AB| |BD|
- C. 468.33 meters





References

- Oronce, O.,Mendoza, M. (2015).E- Math 9 Worktext in Mathematics. REX Printing Company Inc.
- Bernabe, Julieta G. (2002). Geometry Textbook for Third Year.JTW Corporation

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