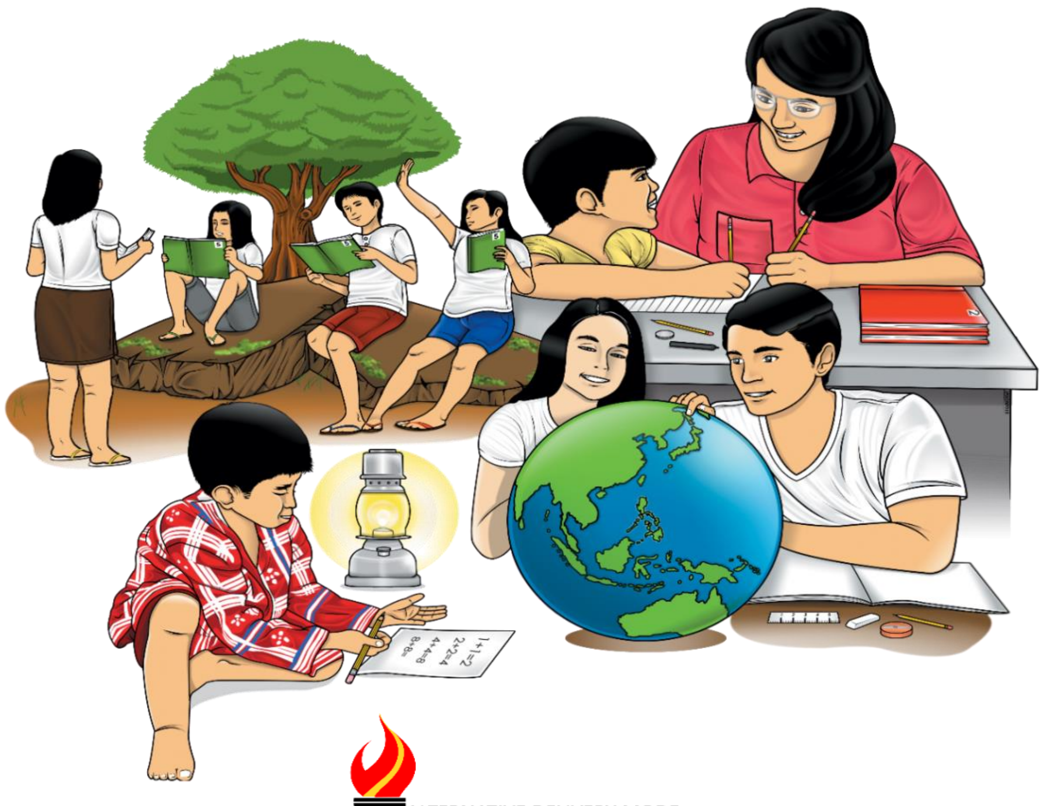
9

Mathematics

Quarter 3 – Module 14:
Applying Triangle Similarity Theorems



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Mathematics

Quarter 3 – Module 14:
Applying Triangle Similarity Theorems

DEPARTMENT OF EDUCATION

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

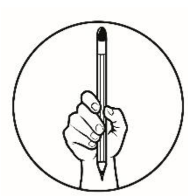
If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

What I Need to Know

Two triangles are similar if their corresponding angles are congruent. In effect, the corresponding sides of similar triangles are proportional. You have also learned that there are several theorems that can show similarity between two or more triangles. These statements prove our assumptions and can also further be used in determining the measures of the remaining sides of the said triangles.

After going through with this module, you are expected to be able to apply the theorems to show that given triangles are similar.

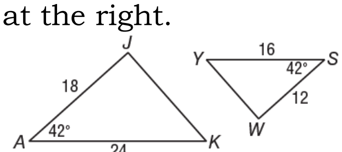


What I Know

Find out how much you already know about the module. Write the letter that you think corresponds to the best answer to each question on a sheet of paper. Answer all items. After taking and checking this short test, take note of the items that you were not able to answer correctly and look for the right answers as you go through this module.

1. If $\Delta ABC \sim \Delta DEF$, which of the following is true?
   A. $\angle A \cong \angle B$ B. $\angle A \cong \angle F$ C. $\frac{|AB|}{|BC|} = \frac{|DE|}{|EF|}$ D. $\frac{|AB|}{|BC|} = \frac{|EF|}{|DE|}$

For numbers 2 to 5, base your answers on the figures at the right.



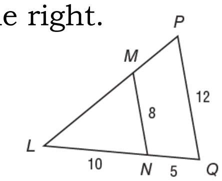
2. Complete the statement: $\Delta JKA \sim$______?
   A. ΔSYW B. ΔWYS C. ΔSWY D. ΔWSY

3. Which theorem proves the similarity of the two triangles?
   A. AA B. SAS C. SSS D. RTST

4. What is the ratio of the lengths of the corresponding sides of ΔAJK and ΔSWY?
   A. 1:3 B. 2:3 C. 3:1 D. 3:2

5. What is the ratio of the areas of ΔAJK and ΔSWY?
   A. 2:3 B. 4:9 C. 3:2 D. 9:4

For numbers 6 to 9, base your answers on the figure at the right.



6. Which of the following is true if $\Delta LMN \sim \Delta LPQ$?
   A. $\overline{LM} \parallel \overline{LN}$ B. $\overline{LP} \parallel \overline{LQ}$ C. $\overline{MN} \parallel \overline{PQ}$ D. $\overline{MP} \parallel \overline{NQ}$

7. What is the ratio of the lengths of the corresponding sides of ΔLMN and ΔLPQ?
   A. 2:3 B. 3:5 C. 4:5 D. 5:6

8. If $|MP| = 7\ cm$, what is $|LP|$?
   A. 7 cm B. 14 cm C. 21 cm D. 28 cm

9. What is the ratio of the areas of ΔLMN and ΔLPQ?
   A. 4:9 B. 9:25 C. 16:25 D. 25:36

10. Which theorem can prove the similarity of the triangles?
    A. Triangle Probability Theorem
    B. Triangle Inequality Theorem
    C. Triangle Proportionality Theorem
    D. Triangle Isosceles Theorem

11. The ratio of the lengths of the sides of two similar triangles is 5:4. What is the ratio of their areas?
    A. 8:10 B. 16:25 C. 10:8 D. 25:16

12. What is the ratio of the lengths of the corresponding sides of similar triangles, if the ratio of their areas is 1:16?
    A. 1:2 B. 1:4 C. 1:8 D. 1:16

For numbers 13 to 15. At 2:00 pm, a student whose height is 5 feet casts a 3-foot shadow.

13. At the same time, how long is the shadow cast by a 35-foot flagpole?
    A. 15 ft B. 21 ft C. 27 ft D. 33 ft

14. Which similarity theorem is best applicable for this situation?
    A. AA B. SAS C. SSS D. RTST

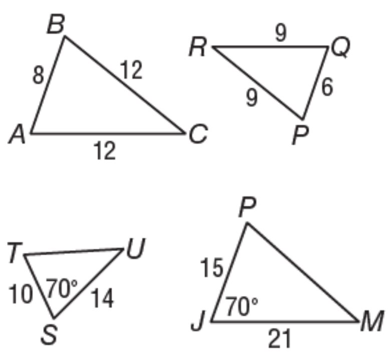
15. What is the ratio of the shadow lengths of the student and the flagpole?
    A. 1:3 B. 1:5 C. 1:7 D. 1:9

Lesson 1 Applying Triangle Similarity Theorems

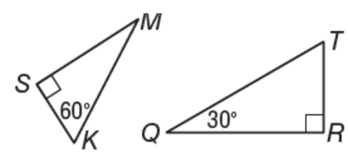
What's In

Determine whether the triangles in each pair are similar or not. If so, write a similarity statement. If not, what would be sufficient to prove the triangles similar? Explain your reasoning.

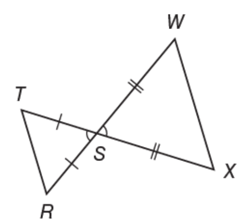
1.



2.



3.

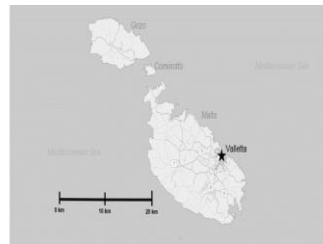


What's New

Directions: Read the selection below.

SAILING IN BALANGAYS

SCALE DRAWING

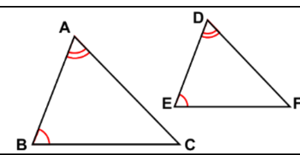
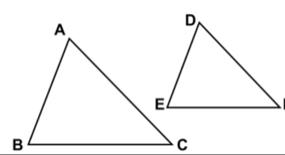
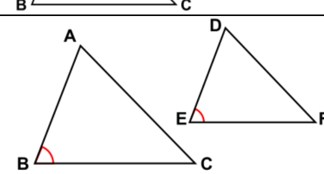
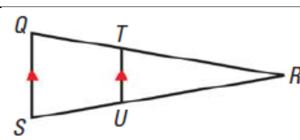


Drawing a thing to reduce or enlarge it from its original size is called scale drawing. The scale used is the ratio of the size of the drawing to the size of the original or reference object being drawn. This measurement being followed is called scale ratio. These are usually seen on different maps like world map. This is also used in blueprints made when constructing houses or buildings.

Some scale drawing of figures like triangles were made easier through theorems and postulates proved by mathematicians. They do not need to know the measure of all dimensions of a triangle to do scale drawing.

What are the only parts of triangles that would be needed to make sure that the scale drawing will be similar to the reference figure? How would you know the other measure if you're given sufficient conditions to prove their similarity?

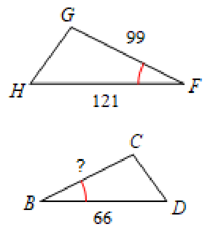
By definition, two triangles are said to be similar if their corresponding angles are congruent and their corresponding sides are proportional. The following are illustrations of the similarity theorems that we have discussed in the previous modules. We can use these theorems to show similarity between triangles.

AA Similarity Corollary If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.	
SSS Similarity Theorem If the lengths of the corresponding sides of two triangles are proportional, then the triangles are similar.	
SAS Similarity Theorem If two sides of a triangle are proportional to two sides of a second triangle, and their included angles are congruent, then the triangles are similar.	
Triangle Proportionality Theorem If a line parallel to one side of a triangle intersects the other two sides, then it divides the two sides proportionally.	

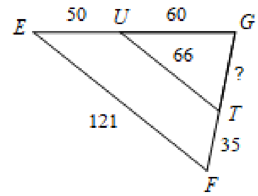
Aside from showing or proving similarities between two triangles, we can find the lengths of their sides by applying the theorems and solving the proportions.

Example 1 – Give the theorem that supports the similarity of the given pairs of triangles. Then, find the length of the indicated side.

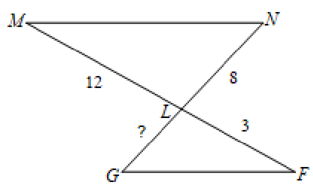
a.



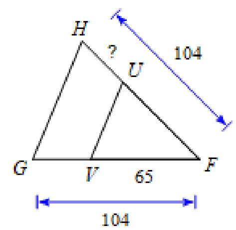
b.



c.



d.



Solution:

a. We can show that $\Delta FGH \sim \Delta BCD$ by SAS Similarity Theorem.

$$\frac{|BC|}{|BD|} = \frac{|FG|}{|FH|}$$ Write the observable proportion and solve for the unknown

$$\frac{|BC|}{66} = \frac{99}{121}$$ Use the known values

$$|BC| = \frac{99 \cdot 66}{121}$$ Solve for $|BC|$ and simplify

$$|BC| = \mathbf{54}$$

b. We can say that $\Delta EGF \sim \Delta UGT$ by **SSS Similarity Theorem or SAS Similarity Theorem.**

$$\frac{|GT|}{|UT|} = \frac{|GF|}{|EF|}$$ Write the observable proportion and solve for the unknown

$$\frac{|GT|}{66} = \frac{|GT| + 35}{121}$$ Use the known values

$$121|GT| = 66(|GT| + 35)$$ Solve for $|GT|$ and simplify

$$11|GT| = 6(|GT| + 35)$$

$$11|GT| - 6|GT| = 6(35)$$

$$5|GT| = 210$$

$$|GT| = 42$$

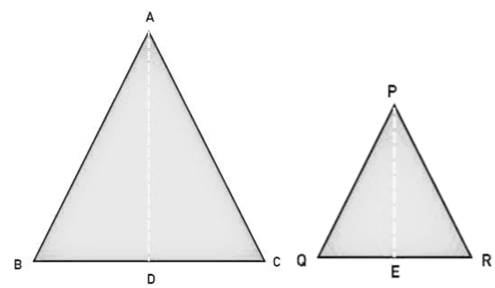
This problem can also be solved using the triangle proportionality theorem provided that $\overline{UT}$ and $\overline{EF}$ are parallel.

c. We can say that $\Delta LMN \sim \Delta LFG$ by SAS.

$\dfrac{\lvert LG\rvert}{\lvert LF\rvert} = \dfrac{\lvert LN\rvert}{\lvert LM\rvert}$	Write the observable proportion and solve for the unknown
$\dfrac{\lvert LG\rvert}{3} = \dfrac{8}{12}$	Use the known values
$\lvert LG\rvert = \dfrac{8 \cdot 3}{12}$	Solve for $\lvert LG\rvert$ and simplify
$\lvert LG\rvert = \mathbf{2}$	

d. We can show that $\Delta UVF \sim \Delta HGF$ by SAS. Observe that ΔHGF is an isosceles triangle since $|HF| = |GF|$. So we can also assume that ΔUVF is an isosceles triangle with $|UF| = |VF|$. With this, we can say that $|HU| = |GV| = 104 - 65$. Hence $|\boldsymbol{HU}| = \mathbf{39}$.

What is It



If two triangles are similar, then not only their angles and sides are related but also their perimeters, altitudes, angle bisectors, and areas.

In the figures at the right, $\Delta ABC \sim \Delta PQR$, and $\overline{AD}$ and $\overline{PE}$ are the altitudes of the triangles, respectively. Since the triangles are similar, their corresponding sides are proportional. Thus, $\frac{|AB|}{|PQ|} = \frac{|BC|}{|QR|} = \frac{|AC|}{|PR|}$. Since $\overline{AD}$ and $\overline{PE}$ are altitudes, then $m\angle ADB = m\angle PEQ = 90^0$ and $m\angle B = m\angle Q$ since corresponding angles of similar triangles are congruent. By AA(Angle) similarity postulate, ΔADB ~ ΔPEQ. Therefore, $\frac{|BC|}{|QR|} = \frac{|AD|}{|PE|}$ since corresponding sides of similar triangles are proportional. Let the ratio of the length of bases and heights between the triangles be $x{:}y$ or $\frac{x}{y}$, Now, find the ratio between the areas of the triangles.

$$\frac{A_{\Delta ABC}}{A_{\Delta PQR}} = \frac{\frac{1}{2}|BC| \cdot |AD|}{\frac{1}{2}|QR| \cdot |PE|} = \frac{|BC| \cdot |AD|}{|QR| \cdot |PE|}$$

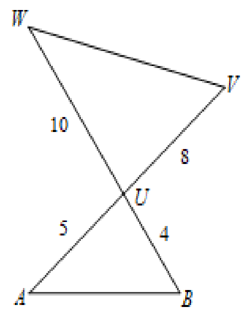
Since $\frac{|BC|}{|QR|} = \frac{|AD|}{|PE|} = \frac{x}{y}$

$$\frac{A_{\Delta ABC}}{A_{\Delta PQR}} = \frac{x \cdot x}{y \cdot y} = \frac{x^2}{y^2} = \left(\frac{x}{y}\right)^2$$

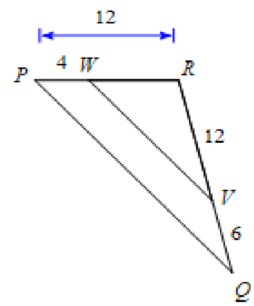
Therefore, the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Example 2 – Find the ratio of the areas of the given pairs of similar triangles.

a.



b.



Solution:

a. By SAS, $\Delta UVW \sim \Delta UBA$

$$\frac{|UV|}{|UB|} = \frac{|UW|}{|UA|}$$

$$\frac{8}{4} = \frac{10}{5} = \frac{2}{1} \; or \; 2:1$$

$$\frac{A_{\Delta \mathrm{UVW}}}{A_{\Delta UBA}} = \left(\frac{2}{1}\right)^2 = \frac{2^2}{1^2} = \frac{4}{1}$$

Therefore, the ratio of their areas is $\mathbf{4:1}$.

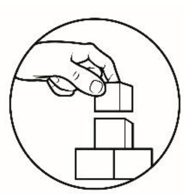
b. By SAS, $\Delta WRV \sim \Delta PRQ$

$$\frac{|WR|}{|PR|} = \frac{|RV|}{|RQ|}$$

$$\frac{8}{12} = \frac{12}{18} = \frac{2}{3} \, or \; 2:3$$

$$\frac{A_{\Delta \mathrm{WRV}}}{A_{\Delta \mathrm{PRQ}}} = \left(\frac{2}{3}\right)^2 = \frac{2^2}{3^2} = \frac{4}{9}$$

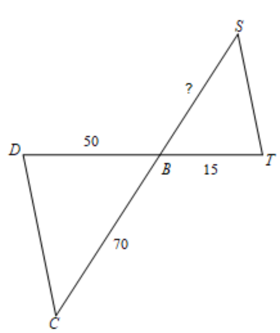
Therefore, the ratio of their areas is $\mathbf{4:9}$.



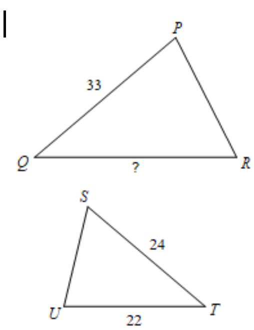
What's More

Activity 1: Given that the triangles in each pair shown below are similar, find the length of the missing side.

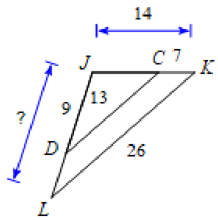
1. $|SB|$



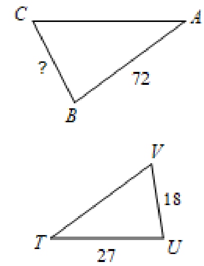
2. $|QR|$



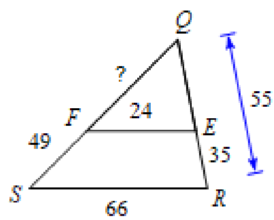
3. $|JL|$



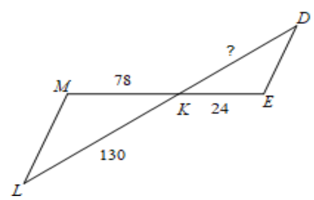
4. $|BC|$



5. $|QF|$

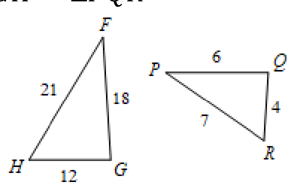


6. $|DK|$

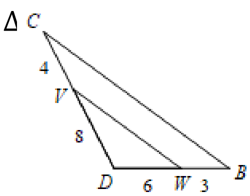


Activity 2: Find the ratio of the lengths of the corresponding sides, and the ratio of the areas of the given pairs of similar triangles.

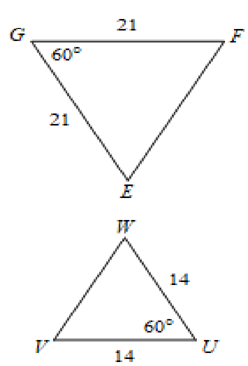
1. $\Delta FGH \sim \Delta PQR$



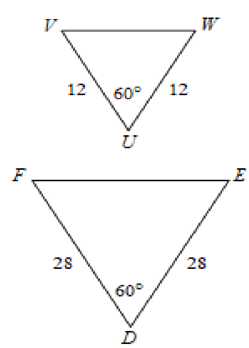
2. $\Delta DVW \sim \Delta$



3. $\Delta EFG \sim \Delta WUV$

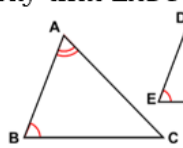
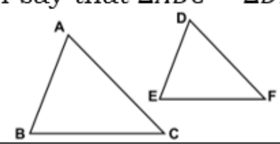
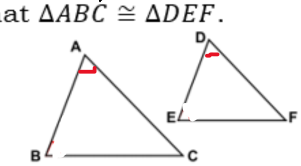
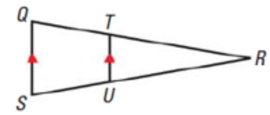


4. $\Delta VWU \sim \Delta FED$



What I Have Learned

The following are some of the triangle similarity theorems that you may use in proving similarity between triangles.

AA Similarity Corollary Given $\angle A \cong \angle D$ and $\angle B \cong \angle E$, we can say that $\Delta ABC \sim \Delta DEF$.  	**SSS Similarity Theorem** Given $\frac{AB}{\lvert AC} = \frac{\lvert DE\rvert}{\lvert DF\rvert}, \frac{AB}{BC\rvert} = \frac{DE\rvert}{EF}$, and $\frac{AC}{BC\rvert} = \frac{\lvert DF\rvert}{\lvert EF\rvert}$ we can say that $\Delta ABC \sim \Delta DEF$. 
SAS Similarity Theorem Given $\frac{\lvert AB}{AC} = \frac{DE\rvert}{\lvert DF\rvert}$ and $\angle B \cong \angle E$, we can say that $\Delta ABC \cong \Delta DEF$.  	**Triangle Proportionality Theorem** Given triangles ΔRTU and ΔRQS such that $\overline{TU}$ and $\overline{QS}$ are parallel, then $\frac{\lvert RT\rvert}{TQ} = \frac{\lvert RU}{US}$. 

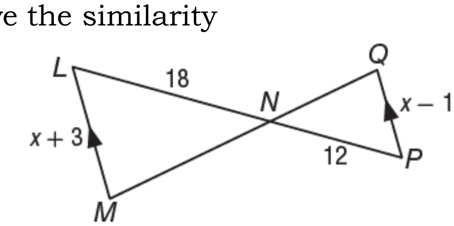
If the ratio of the lengths of the sides of two similar triangles is $a{:}b$, then the ratio of their areas is $a^2{:}b^2$

What I Can Do

Answer the given questions.

For #1 to #3, base your answers on the figure at the right.

1. Identify the pair of similar triangles and give the similarity theorem that proves it.
2. What is the value of x in the figure?
3. Find $|LM|$ and $|PQ|$.



For #4 and #5, consider this situation. "A lighthouse casts a 128-foot shadow. A nearby lamppost that measures 5 feet casts an 8-foot shadow."

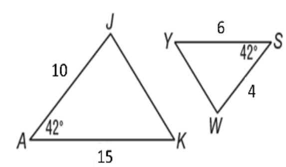
4. Write a proportion that can be used to determine the height of the lighthouse.
5. What is the height of the lighthouse?

Assessment

Read and answer each of the following items accurately. Write the letter of the correct answer on your answer sheet.

1. If $\Delta HIJ \sim \Delta EFG$, which of the following is true?
   A. $\angle H \cong \angle I$ B. $\angle H \cong \angle F$ C. $\frac{|IJ|}{|JH|} = \frac{|EF|}{|FG|}$ D. $\frac{|HI|}{|IJ|} = \frac{|EF|}{|FG|}$

For numbers 2 to 5, base your answers on the figure at the right.



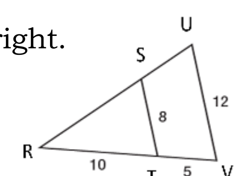
2. Complete the statement: $\Delta JAK \sim$______?
   A. ΔSYW B. ΔWYS C. ΔSWY D. ΔWSY

3. Which theorem proves the similarity?
   A. AA B. SSS C. SAS D. RTST

4. What is the ratio of the lengths of the corresponding sides of ΔAJK and ΔSWY?
   A. 2:5 B. 4:5 C. 5:2 D. 5:4

5. What is the ratio of the areas of ΔAJK and ΔSWY?
   A. 4:25 B. 16:25 C. 25:4 D. 25:16

For numbers 6 to 9, base your answers on the figure at the right.



6. Which of the following is true if $\Delta RST \sim \Delta RUV$?
   A. $\overline{RS} \parallel \overline{RT}$ B. $\overline{RU} \parallel \overline{RV}$ C. $\overline{ST} \parallel \overline{UV}$ D. $\overline{SU} \parallel \overline{TV}$

7. What is the ratio of the lengths of the sides of $\Delta RST$and ΔRUV?
   A. 2:3 B. 3:5 C. 4:5 D. 5:6

8. If $|SU| = 7\ cm$, what is $|RU|$?
   A. $7\ cm$ B. $14\ cm$ C. $21\ cm$ D. $28\ cm$

9. What is the ratio of the areas of ΔRST and ΔRUV?
   A. 4:9 B. 9:25 C. 16:25 D. 25:36

10. Which theorem can prove the similarity between two triangles?
   A. Triangle Probability Theorem
   B. Triangle Inequality Theorem
   C. Triangle Isosceles Theorem
   D. Triangle Proportionality Theorem

11. The ratio of the lengths of the corresponding sides of similar triangles is 5:4, what is the ratio of their areas?
   A. 8:10
   B. 16:25
   C. 10:8
   D. 25:16

12. What is the ratio of the lengths of the corresponding sides of similar triangles, if the ratio of their areas is 1:16?
   A. 1:2
   B. 1:4
   C. 1:8
   D. 1:16

For numbers 13 to 15, consider this. At 3:00pm, a man, 4 feet tall, casts a 6-foot shadow.

13. If a flagpole is 28 ft high, how long is its shadow at the same time?
   A. 12 ft
   B. 36 ft
   C. 42 ft
   D. 48 ft

14. Which similarity theorem is best applicable for this situation?
   A. AA
   B. SAS
   C. SSS
   D. RTST

15. What is the ratio of the shadow lengths of the student and the flagpole?
   A. 1:3
   B. 1:5
   C. 1:7
   D. 1:9

Additional Activities

Consider the given situation and answer the following questions below.

People dream of a house they would like to own someday. Architects can be hired to do the process and the product of designing, planning, and constructing this dream house. In the process, designs are drawn which use smaller measurements in imagining the final output or product. This is only one of the many applications of similarity between triangles and other figures.

a. How can you represent your idea for a building?
b. How does the concept of similarity apply on this?
c. What other professions use concepts of similarity in their work?
d. Write a word problem involving triangle similarities and let your classmate answer your problem and vice versa.

E-Search

You may also check the following link for your reference and further learnings on solving quadratic equation using completing square.

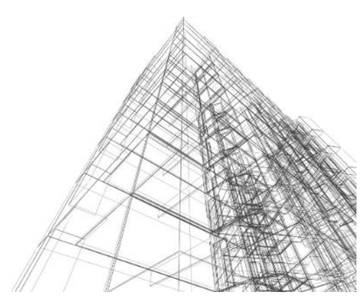
- Similar Triangle Theorems, Mathwarehouse.com
  https://www.mathwarehouse.com/geometry/similar/triangles/similar-triangle-theorems.php
- Geometry: Congruence, Proving Similarity of Triangles, sparknotes
  https://www.sparknotes.com/math/geometry2/congruence/section5/
- Similar Triangles / Similarity Theorems, by lvallar, Quizizz,
  https://quizizz.com/admin/quiz/5c5418240a6052001a2e1d6b/similar-triangles-similarity-theorems
- Similarity Theorems, by shupea, Quizziz,
  https://quizizz.com/admin/quiz/583d9f572615fe565a2884c5/similarity-theorems

PROBLEM – BASED LEARNING WORKSHEET

Paper Designs of Life-Sized Figures

People dream of a house, that they would like to own someday. There are people that can be hired to do both the process and the product of designing, planning, and constructing buildings or other structures. They are the ones who studied architecture.

A client gave an instruction to create a plan of a pyramid-like structure. He set a triangular face to measure10 meters on its base and 15 meters on the other sides.



Are these information enough to be able to create a scale plan of the structure? What theorem could be used to support the similarity of the triangles drawn on the plan and the structure indicated by the client? If the base is drawn in the plan with a measure of 3 inches, what would be the measure of the other sides?

Let's Analyze

1. What is the title of the story? ____________________
2. What do they study in architecture?

   ____
3. Are the information given by the client enough to be able to create a scale plan of the triangle? ____________________
4. What theorem could be used to support the similarity of the triangles drawn on the plan and the structure indicated by the client?

   ____
5. If the base is drawn in the plan with a measure of 3 inches, what would be the measure of the other sides?

   ____

Answer Key

WHAT I KNOW

1. C	6. C	11. D
2. B	7. A	12. B
3. B	8. C	13. B
4. D	9. A	14. B
5. D	10. C	15. C

WHAT'S IN

1. $\Delta ABC \sim \Delta PQR$ by SSS Similarity Theorem
2. $\Delta MSK \sim \Delta QRT$ by AA Similarity Postulate
3. $\Delta STU \sim \Delta JPM$ by SAS Similarity Theorem
4. $\Delta RST \sim \Delta WSX$ by SAS Similarity Theorem

WHAT'S MORE

Activity 1

1. 21
2. 36
3. 18
4. 48
5. 28
6. 40

Activity 2

1. Sides → 3:1 Area → 9:1
2. Sides → 2:3 Area → 4:9
3. Sides → 3:2 Area → 9:4
4. Sides → 3:7 Area → 9:49

WHAT I CAN DO

1. $\Delta LMN \sim \Delta PQN$ by AA Similarity Corollary
2. $x = 9$
3. $LM = 12 \quad PQ = 8$
4. $\frac{Height_{lighthouse}}{Height_{lamppost}} = \frac{Shadow_{lighthouse}}{Shadow_{lamppost}}$
5. $Height_{lamppost} = 80\ ft$

ASSESSMENT

1. D	6. C	11. D
2. D	7. A	12. B
3. C	8. C	13. C
4. C	9. A	14. B
5. C	10. D	15. C
		16.

PROBLEM-BASED WORKSHEET

1. Paper Design of Life-Sized Figures
2. Designing, planning, and construction of buildings and structures
3. Yes
4. SSS Similarity Theorem
5. $4.5\ inches$

References:

A. Book

1. Larson, R, Boswell, L., et al. Geometry (Student Edition). McDougal Littell, 2007.
2. Oronce, O and Mendoza, M. *E-Math 9 Worktext in Mathematics.* Philippines: Vibal Group Inc., 2016.

B. Electronic Resources

1. Scott, M. Peck Quotes. July 3, 2020. http://brainyquotes.com

For inquiries or feedback, please write or call:

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