## Mathematics Quarter 3 - Module 11: Conditions for Proving Triangles Similar



## Mathematics - Grade 9

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## 9

# Mathematics 

# Quarter 3 - Module 11: Conditions for Proving Triangles Similar 

## Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-bystep as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to selfcheck your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.


## What I Need to Know

- Prove the conditions for similarity of triangles. (M9GE-IIIg-h-1)
- Solve problems and prove statements by means of the AA Similarity Postulate, SAS Similarity Theorem and SSS Similarity Theorem.



## What I Know

Find out how much you already know about the module. Write the letter that you think is the best answer to each question on a sheet of paper. Answer all items. After taking and checking this short test, take note of the items that you were not able to answer correctly and look for the right answers as you go through this module.

1. $\triangle A B C \sim \triangle P Q R$ based from the figure below. Which of the following will support the similarity statement?
a. AA Similarity Postulate
b. SAS Similarity Theorem
c. SSS Similarity Theorem

d. ASA Similarity Postulate
2. Which theorem or postulate proves that the triangles below are similar?
a. AA Similarity Postulate
b. SSS Similarity Theorem
c. SAS Similarity Theorem

d. ASA Similarity Postulate
3. Which theorem proves that $\triangle E D C \sim \triangle E A B$ ?
a. SAS ~ Theorem
b. SSS ~ Theorem
c. ASA ~ Theorem
d. AA ~ Postulate

4. If two angles of one triangle are congruent to 2 angles of another triangle, then the triangles are similar by $\qquad$
a. SSS Theorem
c. ASA Postulate
b. AA Postulate
d. SAS Theorem
5. Which pairs of triangles below are similar?
a. $d$ and $f$
b. d and g
c. f and g

(a)


(f)

(g)
d. No triangles are similar.
6. Which of the conditions below, prove that the two triangles are similar?
a. SAS ~ Theorem
b. SSS~Theorem
c. ASA~ Postulate
d. AA~ Postulate

7. The lengths of the corresponding sides of two similar triangles are 5 cm and 20 cm . Which of the following is the ratio of the corresponding sides?
a. 1: 2
b. 1: 4
c. $3: 4$
d. $2: 3$

For items $8 \& 9$, let $\triangle A B C$ and $\triangle R S T$ be similar triangles.
8. Find the value of $x$.
a. 20
b. 18
c. 16
d. 15

9. What must be the value of $y$ ?
a. 20
b. 18
c. 16
d. 15
10.In $\triangle A B C, \overline{D E} \| \overline{A C}$. If $|B D|=12 \mathrm{~cm},|B C|=30 \mathrm{~cm},|A C|=35 \mathrm{~cm}$ and $|A D|=$ 3 cm , then which of the following must be the length of $\overline{D E}$ ?
a. 16 cm
b. 20 cm
c. 24 cm
d. 28 cm

11.Are the triangles at the right similar? Then which of the following will support your statement?
a. Yes, $\triangle P Q R \sim \triangle A B C$ by SSS
b. Yes, $\triangle P Q R \sim \triangle A B C$ by AAA
c. No, corresponding sides are not proportional.
d. No, corresponding angles are not congruent.

12. If $\triangle P Q R$ is similar to $\triangle X Y Z$, then what is the perimeter of $\triangle X Y Z$ ?
a) 21
b) 63
c) 105
d) 126

13. Which of the following pairs of similar triangles illustrates the SAS Similarity Theorem?
a.


c.


b.

d.


14. Which of the following theorems proves that the two triangles are similar?
a. SSS
b. SAS

c. ASA
d. AA
15. Which of the following conditions can be used to prove the statement; "Two right triangles are similar, if the legs of the first right triangle are proportional to the corresponding legs of another right triangle"?
a. ASA~Theorem
b. SAS~Theorem
c. SSS~Theorem
d. AA~Theorem

## Lesson

1

## Proves the Conditions for Similarity of a Triangle - SAS, SSS and AA Similarity Theorem

In the previous lessons, you have learned about similarity between two triangles including right triangles. In this module, you will apply the right triangle similarity theorem to prove another theorem that we use to find the measures of the sides of a right triangle.


## What's In

A. Name the postulate or theorem that can be used to prove that the triangles are congruent.
1.

4.

2.

5.

3.

6.

B. Solve the following proportion.
7. $\frac{30}{7}=\frac{60}{x}$
8. $\frac{8}{24}=\frac{k}{48}$
9. $\frac{18}{y}=\frac{3}{5}$
10. $\frac{12}{9}=\frac{z}{7}$


## What's New



In Metro Manila, city planners are also called urban planners. They determine the best use of a community's land and resources for homes, business, and recreation. They also work on community problems such as traffic congestion and air pollution. The effects of the proposed changes in the enhanced community quarantine ( ECQ ) to general community quarantine ( GCQ ) in the metro are also being studied by them. These Planners use mathematical analysis to evaluate the different conditions and to predict the impact of each condition on a community.

Planners wanted to construct and justify statements about the proportionality between the cause and effect of their planning and its implementation to the community. They use different signs that using equilateral triangles which are similar to inform the public on the direction and danger.

These strategies are comparable to the Triangle Similarity Theorems such as AA (angle-angle) Similarity Postulate, SAS (side-angle-side) Similarity Theorem, and SSS (side-side-side) Similarity Theorem. Using these conditions, the city planner can easily determine the impact of their plan to the community by comparing it to the scenario on the field.

In the previous year, you learned that polygons having the same size and shape are congruent. In this module, you will study triangles that have the same shape but are not necessarily of the same size. These triangles are said to be similar. How do we define similar triangles? What are the conditions to say that two triangles are similar?


## What is It

## Definition:

Two triangles are similar if and only if their corresponding angles are congruent and their corresponding sides are proportional.

Given: $\triangle \mathrm{ABC} \sim \triangle \mathrm{DEF}$


Congruence of Corresponding angles:

$$
\angle A \cong \angle D, \angle B \cong \angle E, \angle C \cong \angle F
$$

Proportionality of Corresponding sides:

$$
\frac{\overline{A B}}{\overline{D E}}=\frac{\overline{B C}}{\overline{E F}}=\frac{\overline{A C}}{\overline{D F}}
$$

Since triangle similarity is a correspondence between triangles, the angles and the sides must be listed in corresponding order. Hence, $\triangle A B C \sim \triangle D E F$ is not the same as $\triangle B C A \sim \triangle D E F$.

The AA Similarity Postulate: If the corresponding angles of two triangles are congruent, then the two triangles are similar.

Condition: $\angle A \cong \angle D$ $\angle B \cong \angle E$, $\angle C \cong \angle E$
Conclusion: $\triangle A B C \sim \triangle D E F$


The AA Similarity Theorem is derived from this postulate.

## Theorems on triangle similarity

1. The AA Similarity Theorem: If two angles of a triangle are congruent to two angles of another triangle then the two triangles are similar.
Given: $\angle A \cong \angle D$

$$
\angle B \cong \angle E
$$

Prove: $\triangle A B C \sim \triangle D E F$


Proof:

| Statements | Reasons |
| :---: | :---: |
| 1. $\angle A \cong \angle D$ | 1. Given |
| $\angle B \cong \angle E$ | 2. Definition of congruent |
| angles |  |
| 2. $m \angle A=\mathrm{m} \angle D$ |  |
|  | $m \angle B=\mathrm{m} \angle E$ |


| 3. $m \angle A+m \angle B=\mathrm{m} \angle D+m \angle \mathrm{E}$ | 3. APE |
| :---: | :---: |
| $\begin{aligned} & \text { 4. } m \angle A+m \angle B+m \angle \mathrm{C}=180^{\circ} \\ & \mathrm{m} \angle D+m \angle \mathrm{E}+m \angle \mathrm{~F}=180^{\circ} \end{aligned}$ | 4. In any triangle, the sum of the measures of its angle is $180^{\circ}$. |
| 5. $m \angle A+m \angle B+m \angle \mathrm{C}=\mathrm{m} \angle D+m \angle \mathrm{E}+m \angle \mathrm{~F}$ | 5. Transitive property |
| 6. $m \angle \mathrm{C}=m \angle \mathrm{~F}$ | 6. Subtraction Property of equality |
| 7. $\angle \mathrm{C} \cong \angle \mathrm{F}$ | 7. Definition of congruent angles |
| 8. $\triangle A B C \sim \triangle D E F$ | 8. AAA Similarity Postulate |

## Example 1:

Determine if the two triangles are similar.

Since $\mathrm{m} \angle \mathrm{M}=\mathrm{m} \angle \mathrm{Q}=120^{\circ}$ and
$\mathrm{m} \angle \mathrm{O}=\mathrm{m} \angle \mathrm{R}=40^{\circ}$,

$\angle M \cong \angle Q$ and $\angle O \cong \angle R$.

By the AA Similarity Postulate,
$\Delta \mathrm{MNO} \sim \Delta \mathrm{QPR}$.

## Example 2.

Explain why the triangles below are similar. Then write a similarity statement.


## Solution:

- The two triangles are similar because of AA Similarity Postulate ;

$$
\mathrm{m} \angle \mathrm{R}=\mathrm{m} \angle \mathrm{~V}=45^{\circ} \text {, therefore } \angle \mathrm{R} \cong \angle \mathrm{~V} \text {. }
$$

Also, $\angle \mathrm{WSR} \cong \angle \mathrm{BSV}$ because of Vertical Angle Theorem.

- The similarity statement for the above triangles is $\triangle W S R \sim \Delta B S V$.

2. SAS Similarity Theorem. If an angle of one triangle is congruent to an angle of another triangle, and the lengths of the sides forming those angles are in proportion, then the triangles are similar.

Given: $\frac{\overline{A B}}{\overline{Q R}}=\frac{\overline{A C}}{\overline{Q S}}, \angle \mathrm{~A} \cong \angle \mathrm{Q}$

Prove: $\triangle A B C \sim \Delta Q R S$


Proof:

| Statements | Reasons |
| :---: | :---: |
| 1. Let X be a point on $\overline{Q R}$ such that $\overline{A B} \cong \overline{Q X}$ | 1. By construction |
| 2. Draw a line through $X$ parallel to $\overline{R S}$. $(\overline{X Y} \\| \overline{R S})$ | 2. Parallel postulate |
| 3. $\angle \mathrm{QXY} \cong \angle \mathrm{R}$ | 3. If two lines are cut by a transversal then corresponding angles are congruent. |
| 4. $\angle \mathrm{Q} \cong \angle \mathrm{Q}$ | 4. Reflexive Property |
| 5. $\angle \mathrm{A} \cong \angle \mathrm{Q}$ | 5.Given |
| 6. $\triangle Q X Y \sim \triangle Q R S$ | 6. AA Similarity Theorem |
| 7. $\frac{\overline{Q X}}{\overline{Q R}}=\frac{\overline{Q Y}}{\overline{Q S}}$ | 7. Definition of similar triangles |
| 8. $\frac{\overline{A B}}{\overline{Q R}}=\frac{\overline{Q Y}}{\overline{Q S}}$ | 8. Substitution (from statement 1) |
| 9. $\frac{\overline{A B}}{\overline{Q R}}=\frac{\overline{A C}}{\overline{Q S}}$ | 9. Given |
| 10. $\frac{\overline{Q Y}}{\overline{Q S}}=\frac{\overline{A C}}{\overline{Q S}}$ | 10. Transitive property (statement 8 and 9) |
| 11. $\overline{Q Y} \cong \overline{A C}$ | 11. Multiplication Property |
| 12. $\|Q Y\|=\|A C\|$ | 12. Definition of Congruent Segments |
| 13. $\angle \mathrm{A} \cong \angle \mathrm{Q}$ | 13. Given |
| 14. $\triangle A B C \cong \triangle Q X Y$ | 14. SAS Congruence |
| 15. $\angle \mathrm{B} \cong \angle \mathrm{QXY}$ | 15. CPCTC |
| 16. $\angle \mathrm{B} \cong \angle \mathrm{R}$ | 16. Transitive property (statement 3 and 15) |
| 17. $\triangle A B C \sim \triangle Q R S$ | 17. AA Similarity Theorem |

Example 1: Given $\triangle P Q R$ and $\triangle D E F$, if $\angle Q \cong \angle E$ write a proportion so that $\triangle P Q R \sim$ $\triangle D E F$ by SAS Similarity theorem.

Solution: Draw $\triangle P Q R$ and $\triangle D E F$. The sides that form $\angle Q$ are $\overline{P Q}$ and $\overline{Q R}$. The sides that form $\angle E$ are $\overline{D E}$ and $\overline{E F}$.

Therefore, the proportion is $\frac{|P Q|}{|Q R|}=\frac{|D E|}{|E F|}$


Example 2. Find the length of $\overline{D E}$ in the figure at the right.
A
3. SSS Similarity Theorem: If the corresponding sides of two triangles are

## Solution:

$\angle A B C \cong \angle E B D$ because vertical angles are congruent.

$$
\frac{|A B|}{|E B|}=\frac{12}{18}=\frac{2}{3} \text { and } \frac{|C B|}{|D B|}=\frac{16}{24}=\frac{2}{3}, \frac{|A B|}{|E B|}=\frac{|C B|}{|D B|}
$$

Therefore, $\triangle A B C \sim \triangle E B D$ by the SAS $\sim$ Theorem
 $\frac{|C A|}{|D E|}=\frac{|A B|}{|E B|}=\frac{|C B|}{|D B|}=\frac{2}{3} \quad$ Corresponding sides of similar triangles are proportional.

$$
\begin{aligned}
\frac{10}{|D E|} & =\frac{2}{3} \quad \text { Substitution } \\
2|D E| & =30 \\
|D E| & =15 \mathrm{~cm}
\end{aligned}
$$

## Proof:

| Statements | Reasons |
| :--- | :--- |
| 1. Let $\mathrm{X} \overline{\mathrm{be} \text { a point on } \overline{Q R} \text { such }}$that $\overline{A B} \cong \overline{Q X}$ <br> 2. Draw a line through X <br> parallel to $\overline{R S} .(\overline{X Y} \\| \overline{R S})$ <br> 3. $\angle \mathrm{QXY} \cong \angle \mathrm{R}$ <br> 4. $\angle \mathrm{Q} \cong \angle \mathrm{Q}$ <br> 5. $\Delta Q X Y \sim \Delta Q R S$ <br> 6. $\frac{\overline{Q X}}{\overline{Q R}}=\frac{\overline{Q Y}}{\overline{Q S}}=\frac{\overline{X Y}}{\overline{R S}}$ <br> 3. If two lines are cut by a <br> transversal then corresponding <br> angles are congruent. <br> 7. $\frac{\overline{A B}}{\overline{Q R}}=\frac{\overline{Q Y}}{\overline{Q S}}=\frac{\overline{X Y}}{\overline{R S}}$ | 4. Reflexive property |
| 8. $\frac{\overline{A B}}{\overline{Q R}}=\frac{\overline{B C}}{\overline{R S}}=\frac{\overline{A C}}{\overline{Q S}}$ | 5. AA Similarity Theorem |


| 9. $\frac{\overline{Q Y}}{\overline{Q S}}=\frac{\overline{A C}}{\overline{Q S}}, \frac{\overline{X Y}}{\overline{R S}}=\frac{\overline{B C}}{\overline{R S}}$ | 9. Transitive property (statement 7 <br> and 8) |
| :--- | :--- |
| $10 .\|Q Y\|=\|A C\|,\|X Y\|=\|B C\|$ | 10. Multiplication Property |
| $11 . \overline{Q Y} \cong \overline{A C} ; \overline{X Y} \cong \overline{B C}$ | 1. Definition of Congruent <br> Segments |
| $12 . \Delta A B C \cong \triangle Q X Y$ | 12. SSS Congruence |
| $13 . \angle B \cong \angle Q X Y, \angle \mathrm{~A} \cong \angle \mathrm{Q}$ | 13. CPCTC |
| $14 . \angle \mathrm{B} \cong \angle \mathrm{R}$ | 14. Transitive property (statement 3 <br> and 12 ) |
| $15 . \triangle A B C \sim \triangle Q R S$ | 15. AA Similarity Theorem |

Example 1. Which of the following triangles are similar?


## Solutions:

Compare the sides of three triangles by checking if the sides are in proportion.

| $\triangle A B C$ and $\triangle R S T$ | $\triangle A B C$ and $\triangle Q P R$ | $\triangle R S T$ and $\triangle Q P R$ |
| :---: | :---: | :---: |
| $\frac{\|A B\|}{\|S R\|}=\frac{\|B C\|}{\mid S T}=\frac{\|A C\|}{\mid R T}$ | $\frac{\|A B\|}{\|Q P\|}=\frac{\|B C\|}{\|P R\|}=\frac{\|A C\|}{\|Q R\|}$ | $\frac{\|R S\|}{\|Q P\|}=\frac{\|S T\|}{\mid P R}=\frac{\|R T\|}{\|Q R\|}$ |
| $\overline{\|S R\|}=\frac{B C \mid}{\|S T\|}=\frac{\|R T\|}{\mid R T}$ | $\overline{\|Q P\|}=\frac{\|P R\|}{\|P R\|}=\frac{\|Q R\|}{\|c\|}$ | $\overline{\|Q P\|}=\frac{\|P R\|}{\|P R\|}=\frac{\|Q R\|}{}$ |
| $\frac{8}{6} \neq \frac{6}{4.5} \neq \frac{4}{2.5}$ | $\frac{8}{4}=\frac{6}{3}=\frac{4}{2}=\frac{2}{1}$ | $\frac{6}{4}=\frac{4.5}{3} \neq \frac{2.5}{2}$ |
| ce the lengths of the es are not proportional, $B$ and $\triangle R S T$ are not ilar. | Because the lengths of the sides are proportional, $\triangle A B C$ and $\triangle Q P R$ are similar or $\triangle A B C \sim \triangle Q P R$ by SSS Similarity theorem. | Since the lengths of the sides are not proportional, $\triangle A B C$ and $\triangle R S T$ are not similar |

Example 2. $\triangle A B C$ and $\triangle R S T$ are equilateral triangles. Prove that they are similar.

## Solution:

To write a proof, it is necessary to draw the figures to easily understand and visualize the statement.

To prove that the two equilateral triangles are similar, show that their sides are proportional. But how? Here is the formal proof.


Given: $\triangle A B C$ and $\triangle R S T$ are equilateral.
Prove: $\triangle A B C \sim \triangle R S T$

## Proof:

| Statements | Reasons |
| :--- | :--- |
| 1. $\Delta A B C$ and $\triangle R S T$ are equilateral. | 1. Given |
| 2. $\|A B\|=\|B C\|=\|A C\|$ | 2. Definition of equilateral <br> triangles |
| $\|R S\|=\|S T\|=\|R T\|$ | 3. MPE (multiply all terms by $\frac{1}{R S}$ ) |
| 3. $\frac{\|A B\|}{\|R S\|}=\frac{\|B C\|}{\|R S\|}=\frac{\|A C\|}{\|R S\|}$ | 4. Substitution |
| 4. $\frac{\|R\|}{\|R S\|}=\frac{\|B C\|}{\|S T\|}=\frac{\|A C\|}{\|R T\|}$ | 5. SSS Similarity Theorem |
| 5. $\Delta A B C \sim \Delta R S T$ |  |

Example 3. Victor wants to determine the height of a pole using a mirror that he places on the ground facing upward. He is 1 m away from the mirror and the distance from his eyes to the ground is 1.5 m . He sees the top of the pole in a mirror that is 5 m from the pole. How high is the pole?

## Solution:

In solving a worded problem like this, it is always helpful to illustrate the situation. The height of the pole and the height of the man are proportional to the distance of the mirror from the pole and from the man, meaning the two heights are proportional to the two horizontal distances.


Let $\mathbf{y}$ represent the height of the pole.

$$
\begin{gathered}
\frac{y}{1.5}=\frac{5}{1} \\
\mathrm{y}=\frac{5(1.5)}{1} \\
\mathrm{y}=7.5 \mathrm{~m}
\end{gathered}
$$

Hence, the pole is 7.5 meters.


## What's More

## ACTIVITY 1

A. Are the triangles similar? Analyze the figures below then complete the statements that follow. Write your answer on the space provided


1. $|A B|=|B C|=$ $\qquad$ and $|X Y|=$ $\qquad$ $=12 \mathrm{~cm}$.
2. The two sides of $\triangle A B C$ are congruent, so $\triangle A B C$ is $\qquad$ .

The two sides of $\triangle X Y Z$ are congruent, so $\triangle X Y Z$ is $\qquad$ .
3. Complete the proportions: $\frac{|A B|}{|X Y|}=\frac{?}{12}$ and $\frac{|B C|}{|Y Z|}=$ ?
4. By transitive property, $\frac{|A B|}{|X Y|}=$ ?
5. Two sides of $\triangle A B C$ are $\qquad$ to two sides of $\triangle X Y Z$.
6. $\angle B$ is the included angle of sides $\qquad$ and $\qquad$ .
$\angle Y$ is the included angle of sides $\qquad$ and $\qquad$ .
7. $\triangle A B C \sim \triangle X Y Z$ by $\qquad$ similarity theorem.
B. Solve for the values of x and y given that the given triangles in each item are similar. Write your answer below the given figures
1.

2.

3.

4.

5.

6.
.

C. Which of the following pairs of triangles are similar? Write the theorem/ postulate that makes them similar. Write your answers below each figures

2.

3.


D. Solve as indicated. Write your solutions after each given problem.

1. In the $\triangle A B C,|A B|=12 \mathrm{~cm},|\mathrm{AX}|=5 \mathrm{~cm},|\mathrm{AC}|=14 \mathrm{~cm}$ and $\overline{X Y} \| \overline{B C}$. Find the length of $\overline{A Y}$ and $\overline{Y C}$

2. If $\triangle P Q R \sim \triangle T U V,|P Q|=36 \mathrm{~cm},|Q R|=24 \mathrm{~cm},|P R|=30 \mathrm{~cm},|T U|=48 \mathrm{~cm}$, then what is the length of $\overline{T V}$ ?
3. Given $\triangle A B C \sim \triangle D E F,|A B|=11 \mathrm{~cm},|D E|=22 \mathrm{~cm},|B C|=24 \mathrm{~cm},|D F|=$ 60 cm , find the length of $\overline{A C}$
4. Given $\triangle A B C \sim \triangle D E F,|D F|=25 \mathrm{~cm},|A C|=15 \mathrm{~cm},|E F|=65 \mathrm{~cm},|D E|=70 \mathrm{~cm}$, find the length of $\overline{A B}$.
5. Given $\triangle P R C$ with right angle at $\angle P R C$ and that $\overline{R T} \perp \overline{P C}$. If the length of $\overline{P T}$ is one-half of the length of $\overline{P R}$ and the length of $\overline{R T}=5 \mathrm{~cm}$, then what is the length of $\overline{P C}$ ?



## What I Have Learned

## Side-Angle-Side Similarity (SAS) Theorem

If an angle of one triangle is congruent to an angle of a second triangle, and the lengths of the sides forming the two angles are proportional, then the triangles are similar.

## Side-Side-Side Similarity (SSS) Theorem

If the corresponding sides of two triangles are proportional, then the triangles are similar.

## Angle - Angle Similarity (AA ~ Postulate)

If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.


## What I Can Do

A. Prove that "Triangles similar to the same triangles are similar" by supplying the missing statements and reasons of the proof below.

Given: $\triangle \mathrm{ABC} \sim \Delta \mathrm{XYZ}$ and $\triangle \mathrm{DEF} \sim \Delta \mathrm{XYZ}$.
Prove: $\triangle \mathrm{ABC} \sim D E F$

Proof:
figure


| Statements | Reasons |
| :--- | :--- |
| 1. $\triangle \mathrm{ABC} \sim \Delta \mathrm{XYZ}$ and $\triangle \mathrm{DEF} \sim \Delta \mathrm{XYZ}$. | 1. |
| 2. $\angle A \cong \angle X, \angle B \cong \angle Y, \angle C \cong \angle Z$ | 2. |
| $\angle D \cong \angle X, \angle E \cong \angle Y, \angle F \cong \angle Z$ |  |
| 3. | 3. Transitive property |
| 4. $\triangle \mathrm{ABC} \sim \triangle D E F$ | 4. |

B. Prove and solve the following.

1. If $\overline{A B} \| \overline{D E}$,
a. Prove $\triangle \mathrm{ABC} \sim \triangle \mathrm{EDC}$.

b. Find $|C D|$ and $|\mathrm{DE}|$.
2. If $\overline{S T} \| \overline{A C}$, show that $\triangle B S T \sim \triangle B A C$.

3. Prove that if two isosceles triangles have congruent vertex angles, then they are similar.
4. Given: $\frac{|A C|}{|P R|}=\frac{|B C|}{|Q R|}$


Prove: $\triangle \mathrm{ABC} \sim \triangle \mathrm{PQR}$
C. Solve. Show your solution below each given problem.

1. In sunlight, a cactus casts a 9 -ft shadow. At the same time, a person 6 ft . tall casts a 4 -ft shadow. Use similar triangles to find the height of the cactus.

2. A choreographer places a mirror on the ground 40.5 ft . from the base of a building. He walks backwards until he can see the top of the building in the middle part of the mirror. At that point, the choreographer's eyes are 6 ft . above the ground and he is 7 ft . from the image in the mirror. Find the height of the building using similar triangles.


## Directions: Read and answer each question carefully. Write the letter of the correct answer on your paper.

1. What do you call the equality between two ratios?
a. portion
b. ratio
c. proportion
d. expression
2. If $\triangle M N Q \sim \Delta S T V$ then $|M N|:|S T|=|M Q|$ :
a. $|S T|$
b. $|N Q|$
c. $|S V|$
d. $|T V|$
3. Find the value of $x$ so that $\triangle A B C \sim \triangle D E F$.

a. 115
b. 72
c. 65
d. 55
4. Are $\triangle R T S$ and $\triangle L P M$ at the right similar?
a. Yes, by AA Postulate
b. Yes, by SAS Theorem
c. Yes, by ASA Postulate
d. No

5. $\triangle \mathrm{ABC}$ is similar to $\triangle \mathrm{XYZ}$. What is the length of segment $\overline{\mathrm{BC}}$ ?
a. 5 cm
b. 7.5 cm
c. 8 cm
d. 9 cm

6. $\triangle W A V \sim \Delta S A T$, then which of the following will support the similarity statement?
a. SSS~Theorem
b. AA~Postulate
c. SAS~Theorem
d. ASA~Postulate

7. Which group contains triangles that are all similar?
a.

c.

b.

d.

8. In the diagram, $\triangle \mathrm{ABC} \sim \Delta \mathrm{LMN}$. Find the value of x .
a. 8
b. 10
c. 12
d. 14
9. $\triangle A B C \sim \triangle P Q R$. Which of the following will support the similarity statement?
a. AA Similarity theorem
b. SAS Similarity theorem
c. SSS Similarity theorem
d. AAA Similarity postulate

10.Two triangles are similar. The lengths of the sides of the smaller triangle are $12 \mathrm{~cm}, 17 \mathrm{~cm}, 21 \mathrm{~cm}$. The shortest side of the larger triangle measures 24 cm . What is the perimeter of the larger triangle?
a. 48 cm
b. 54 cm
c. 96 cm
d. 100 cm
10. Which of the similarity conditions can be used to prove that $\triangle A B C \sim \triangle F E D$ ?
a. SAS Similarity theorem
b. SSS Similarity theorem
c. ASA Similarity theorem
d. Not enough information

11. In $\triangle M E A$, if $\overline{S H} \| \overline{M E}$ then the two triangles are similar by AA similarity theorem. Which of the following is the correct similarity statement?
a. $\triangle S A H \sim \triangle M E A$
b. $\triangle S A H \sim \triangle M A E$
c. $\triangle S A H \sim \triangle E M A$
d. $\triangle S A H \sim \triangle E A M$

12. What condition can be used to prove that an isosceles triangle with base angles each measures $50^{\circ}$ is similar to another isosceles triangle with the same measurement of base angles?
a. ASA~ Theorem
b. SAS~Theorem
c. $\mathrm{SSS} \sim$ Theorem
d. AA~Theorem
13. Three of the four triangles below are similar. Which one does not belong to the group?
a.

b.

c.

d.

14. Given $\triangle Q U S$ and $\triangle W A X$, with $\frac{|Q U|}{|W A|}=\frac{|U S|}{|A X|}$. Which of the following is always true if $\triangle Q U S \sim \triangle W A X$ ?
a. $\angle A \cong \angle X$
b. $\angle A \cong \angle U$
c. $\angle S \cong \angle W$
d. $\angle Q \cong \angle S$

## Additional Activities

## REFLECTIVE LEARNING SHEET



Directions:-Write a reflective learning about the city planners narrated in the introduction of this module by answering the questions below. You may express your answers in more critical and creative presentation.

1. What are the expertise of the city planners based on the story?
2. Explain briefly the mathematical analysis that these planners use.
3. How is the idea of similarity of triangles utilized by the city planners as stated in the story?


Answer Key

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## References

1. https://mathbitsnotebook.com/Geometry/Similarity/SMProofs.html
2. https://tutors.com/math-tutors/geometry-help/similar-triangles
3. https://www.ck12.org/geometry/Similarity-by-AA-SSS-SAS/lesson/AA-Similarity-BSC-GEOM/
4. https://www.khanacademy.org/math/geometry/hs-geo-similarity/hs-geo-triangle-similarity-intro/v/similarity-postulates
5. https://www.youtube.com/watch?v=VXIFEilh-cw
6. https://www.azquotes.com/quote/799337

- Ponsones R., Remoto- Ocampo S., Math Ideas and Life Applications Second Edition
- Coronel A., Growing Up with Math Copyright 14
- Mathematics Learner's Material First Edition
- Wilcutt R., Essentials for Algebra Concepts and Skills Houghton Mifflin Company
- Tom Robbins.,azquotes.com/quote/799337

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