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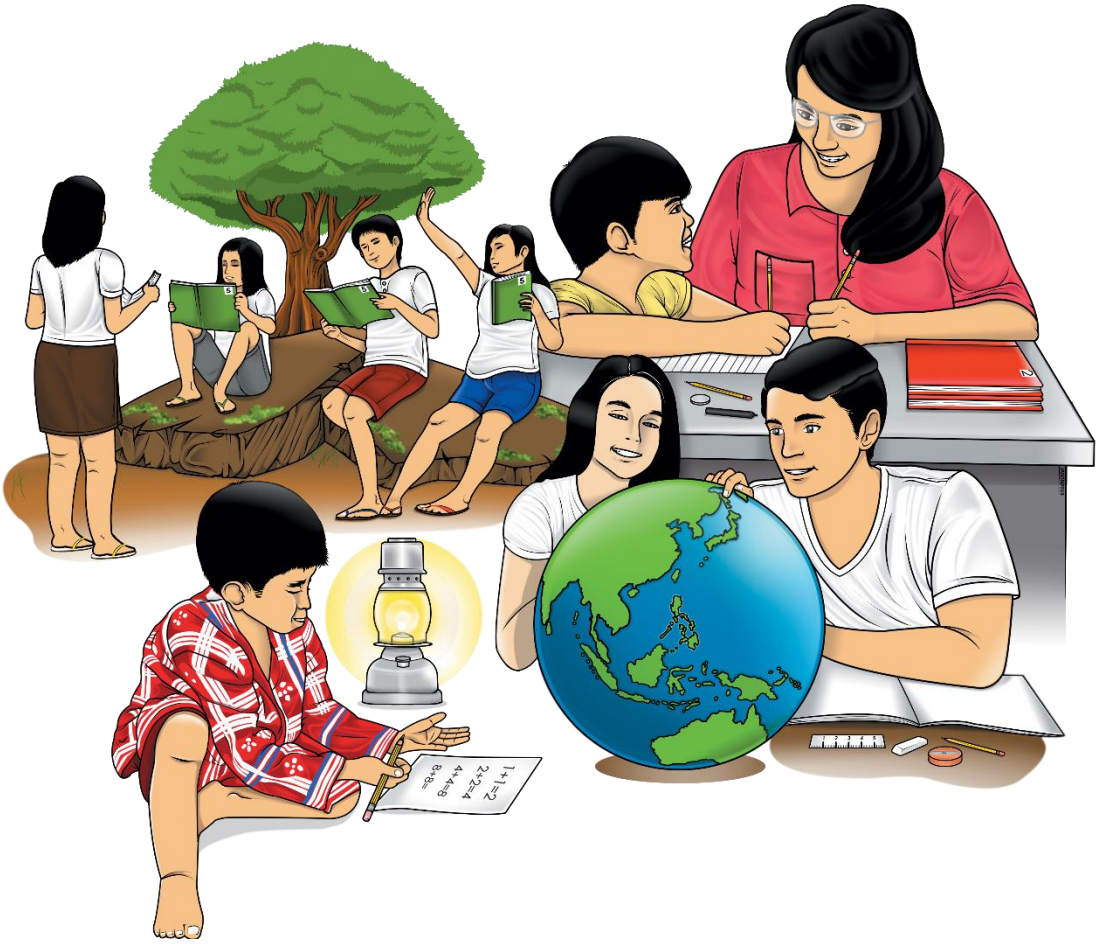
Mathematics

Quarter 3 – Module 1

Conditions that Make a Quadrilateral a Parallelogram



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Quarter 3 – Module 1: Conditions that Make a Quadrilateral a Parallelogram
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Mathematics

Quarter 3 – Module 1

Conditions that Make a Quadrilateral a Parallelogram

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Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

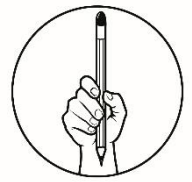
If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

What I Need to Know

This module contains:

Lesson 1 - determine the conditions that make a quadrilateral a parallelogram.



What I Know

Part I. Read and answer each question carefully and write the letter that corresponds to the correct answer on a sheet of paper.

1. How do you describe any two opposite angles in a parallelogram?
    a. They are congruent.
    b. They are supplementary.
    c. They are complementary.
    d. All of the above.
2. What can you say about two consecutive angles in a parallelogram?
    a. They are always congruent.
    b. They are always supplementary.
    c. They are sometimes complementary.
    d. They are never congruent.
3. Which of the following conditions is not sufficient to prove that a quadrilateral is a parallelogram?
    a. Two pairs of sides are parallel.
    b. Two pairs of opposite sides are congruent.
    c. Two angles are supplementary.
    d. Two diagonals bisect each other.

4. Which of the following quadrilaterals has diagonals that do not bisect each other?

a. Square

b. Rhombus

c. Rectangle

d. Trapezoid

5. What condition will make parallelogram WXYZ a rectangle?

a. $(\overline{WX}) \cong (\overline{YZ})$

b. $(\overline{WX}) \parallel (\overline{YZ})$

c. $\angle X$ is a right angle.

d. $(\overline{WX})$ and $(\overline{YZ})$ bisect each other.

Part II. Does each set of given information guarantee that quadrilateral JKLM is a parallelogram? Write YES or NO.

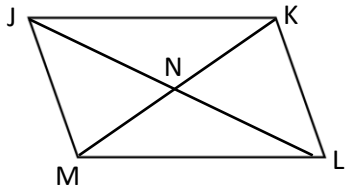
6. $|JN| = 25$ cm, $|JL| = 50$ cm, $|KN| = 13$ cm, $|KM| = 26$ cm

7. $\angle MJL \cong \angle KLJ$, $\overline{JM} \cong \overline{LK}$

8. $\overline{JM} \cong \overline{JK}$, $\overline{KL} \cong \overline{LM}$

9. $\angle MJL \cong \angle MLJ$, $\angle KJL \cong \angle KLJ$

10. $\triangle JKN \cong \triangle LMN$



Lesson 1

CONDITIONS THAT MAKE A QUADRILATERAL A PARALLELOGRAM

In the previous lesson, you learned that a parallelogram is a quadrilateral with two pairs of opposite sides parallel to each other. In this module, you will determine the conditions that guarantee a quadrilateral is a parallelogram.

What's In

Do you still remember what quadrilaterals are?

The following activities will help you recall significant concepts on quadrilaterals.

Activity 1: Quadrilaterals Properties

Put an X in the box if the shape always exhibits the property.

Property	Parallelogram	Rectangle	Rhombus	Square	Trapezoid	Isosceles Trapezoid	Kite
1. Both pairs of opposite sides are congruent. .							
2. Diagonals are congruent.							
3. Diagonals are perpendicular.							
4. Diagonals bisect each other.							
5. Consecutive angles are supplementary .							
6. Both pairs of opposite angles are congruent.							

What's New

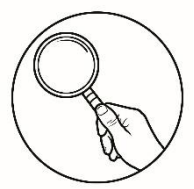
Read the selection below.

QUADRILATERALS: ALL AROUND THE WORLD

Around the world, people have something very much in common - sports. Every sport uses quadrilaterals in some ways, be it in the shape of the field, court, or the materials used. Take for example, a basketball court. It is rectangular in shape and has right angles. It has all the properties of a rectangle.

The emergence of COVID-19 has upended the sporting calendar, with professional leagues everywhere suspending their activities to prevent the spread of the virus. Even the Summer Olympics, typically one of the world's most-watched sporting broadcasts, has been pushed back a year.

In these difficult times, the ability of sports to bring people together is missed more than ever. The industry itself may be in new territory, but with the right strategies it can surface from the crisis stronger and more popular than ever.



What is It

Let us explore on the following activities to prove the different theorems on the conditions that guarantee a quadrilateral is a parallelogram.

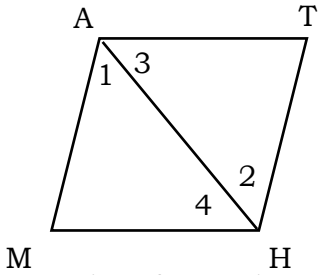
ACTIVITY 1 :

1. Draw quadrilateral MATH with $\overline{MA} \cong \overline{TH}$ and $\overline{AT} \cong \overline{MH}$
2. Draw diagonal $\overline{AH}$.

   Why is $\Delta MAH \cong \Delta THA$?

   Is $\angle 1 \cong \angle 2$? Why?

   Is $\angle 3 \cong \angle 4$? Why?



3. Besides being congruent, what can be said about the two pairs of opposite sides, $\overline{MA}$ and $\overline{TH}$, $\overline{AT}$ and $\overline{MH}$?
4. What condition makes quadrilateral MATH a parallelogram?

THEOREM

A quadrilateral is a parallelogram if it's opposite sides are congruent.

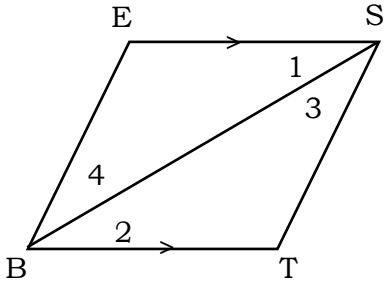
Given: Quadrilateral MATH with $\overline{MA} \cong \overline{HT}$ and $\overline{MH} \cong \overline{AT}$

Prove: Quadrilateral MATH is a parallelogram.

Proof:

STATEMENT	REASON
1. Draw $\overline{AH}$	1. Two different points determine a line.
2. $\overline{MA} \cong \overline{HT}$, $\overline{MH} \cong \overline{AT}$	2. Given
3. $\overline{AH} \cong \overline{HA}$	3. Reflexive Property
4. $\Delta MAH \cong \Delta THA$	4. SSS Congruence Postulate
5. $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$	5. CPCTC
6. $\overline{MA} \parallel \overline{HT}$, $\overline{MH} \parallel \overline{AT}$	6. If two lines are cut by a transversal so that alternate interior angles are congruent, then the lines are parallel.
7. Quadrilateral MATH is a parallelogram.	7. Definition of a parallelogram.

ACTIVITY 2:



1. Draw quadrilateral BEST with $\overline{ES} \cong \overline{TB}$ and $\overline{ES} \parallel \overline{TB}$.
2. Draw diagonal $\overline{BS}$. Is $\angle 1 \cong \angle 2$? Why?
3. Is $\Delta BES \cong \Delta STB$? Why? Is $\overline{BE} \cong \overline{ST}$? Why?
4. What condition makes quadrilateral BEST a parallelogram?

THEOREM

A quadrilateral is a parallelogram if a pair of opposite sides are parallel and congruent.

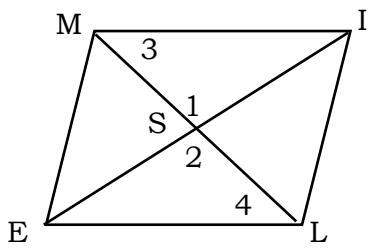
Given: Quadrilateral BEST with $\overline{ES} \cong \overline{TB}$ and $\overline{ES} \parallel \overline{TB}$

Prove: Quadrilateral BEST is a parallelogram.

Proof:

STATEMENT	REASON
1. Draw $\overline{BS}$	1. Two distinct points determine a line.
2. $\overline{ES} \cong \overline{TB}$, $\overline{ES} \parallel \overline{TB}$	2. Given
3. $\angle 1 \cong \angle 2$	3. If two parallel lines are cut by a transversal, then alternate interior angles are congruent.
4. $\overline{BS} \cong \overline{SB}$	4. Reflexive Property
5. $\Delta BES \cong \Delta STB$	5. SAS Congruence Postulate
6. $\angle 3 \cong \angle 4$	6. CPCTC
7. $\overline{EB} \parallel \overline{ST}$	7. If two lines are cut by a transversal so that alternate interior angles are congruent, then the lines are parallel.
8. Quadrilateral BEST is a parallelogram.	8. Definition of parallelogram.

ACTIVITY 3:



1. Draw quadrilateral MILE with diagonals $\overline{ML}$ and $\overline{IE}$ bisecting each other.
2. Mark the intersection of the diagonals as S.
   Is $\angle 1 \cong \angle 2$? Why? Why is $\Delta MSI \cong \Delta LSE$?
   Is $\angle 3 \cong \angle 4$? Why?
3. What can be said about $\overline{MI}$ and $\overline{LE}$?
4. What condition makes quadrilateral MILE a parallelogram?

THEOREM

A quadrilateral is a parallelogram if its diagonals bisect each other.

Given: $\overline{EI}$ and $\overline{ML}$ bisect each other at S.

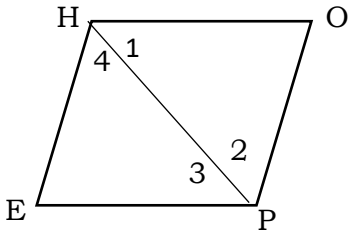
Prove: Quadrilateral MILE is a parallelogram.

Proof:

STATEMENT	REASON
1. $\overline{EI}$ bisects $\overline{ML}$ $\overline{ML}$ bisects $\overline{EI}$	1. Given
2. $\overline{ES} \cong \overline{IS}$, $\overline{MS} \cong \overline{LS}$	2. A segment bisects another segment at its midpoint. Definition of midpoint.
3. $\angle 1 \cong \angle 2$	3. Vertical Angle Theorem
4. $\Delta MSI \cong \Delta LSE$	4. SAS Congruence Postulate
5. $\overline{MI} \cong \overline{LE}$, $\angle 3 \cong \angle 4$	5. CPCTC
6. $\overline{MI} \parallel \overline{LE}$	6. If two lines are cut by a transversal so that alternate interior angles are congruent, then the lines are parallel.
7. Quadrilateral MILE is a parallelogram.	7. If a pair of opposite sides of a quadrilateral are parallel and congruent, then the quadrilateral is a parallelogram.

ACTIVITY 4:

1. Draw quadrilateral HOPE with $\angle H \cong \angle P$ and $\angle O \cong \angle E$.
2. Draw diagonal $\overline{HP}$.
3. Is $\angle 2 \cong \angle 4$? Why?
4. Is $\overline{HE} \parallel \overline{PO}$? Is $\overline{HO} \parallel \overline{PE}$? Why?



THEOREM

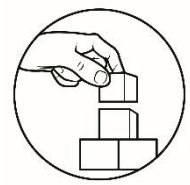
A quadrilateral is a parallelogram if its opposite angles are congruent.

Given: Quadrilateral HOPE with $\angle EHO \cong \angle OPE$, $\angle O \cong \angle E$

Prove: Quadrilateral HOPE is a parallelogram.

Proof:

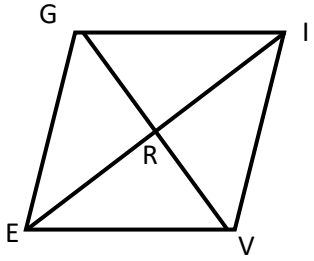
STATEMENT	REASON
1. Quadrilateral HOPE with ∠EHO ≅ ∠OPE, ∠ 0 ≅ ∠E	1. Given
2. m∠EHO = m∠OPE m∠O = m∠E	2. Definition of congruent angles
3. Construct $\overline{HP}$	3. Two distinct points determine a line.
4. m∠1 + m∠2 + m∠O = 180° m∠3 + m∠4 + m∠E = 180°	4. The sum of the measures of the angles of a triangle is 180°.
5. m∠1 + m∠2 + m∠O = m∠3 + m∠4 + m∠E	5. Transitive Property
6. m∠1 + m∠2 = m∠3 + m∠4	6. Subtraction Property of Equality
7. m∠EHO = m∠1 + m∠4 m∠OPE = m∠2 + m∠3	7. Angle Addition Postulate
8. m∠1 + m∠4 = m∠2 + m∠3	8. Transitive Property
9. m∠2 - m∠4 = m∠4 - m∠2	9. Subtraction Property of Equality, Statements 6 and 8
10. 2(m∠2) = 2(m∠4) or m∠2 = m∠4	10. Simplification
11. $\overline{HE} \parallel \overline{OP}$	11. If two lines are cut by a transversal so that alternate interior angles are congruent, then the lines are parallel.
12. m∠1 = m∠3	12. Subtraction Property of Equality, Statements 8 and 10
13. $\overline{HO} \parallel \overline{EP}$	13. If two lines are cut by a transversal so that alternate interior angles are congruent, then the lines are parallel.
14. Quadrilateral HOPE is a parallelogram.	14. Definition of a parallelogram.



What's More

Activity 1:

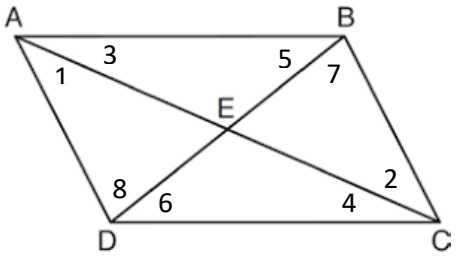
For each of the following, state the condition that supports why quadrilateral GIVE is a parallelogram.



1. $\overline{RG} \cong \overline{RV}$, $\overline{RE} \cong \overline{RI}$
2. $\overline{GI} \cong \overline{VE}$, $\overline{GE} \cong \overline{VI}$
3. $\overline{GI} \cong \overline{VE}$, $\overline{GI} \parallel \overline{VE}$
4. $\angle EGI \cong \angle IVE$, $\angle VEG \cong \angle GIV$
5. $\overline{GE} \parallel \overline{VI}$, $\overline{GE} \cong \overline{VI}$
6. $|GR| = \frac{1}{2} |GV|$, $\overline{IR} \cong \overline{ER}$

Activity 2:

Given the indicated measures, tell why quadrilateral ABCD is a parallelogram.



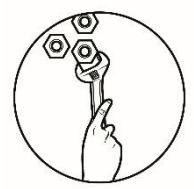
1. |AD| = 3 cm, |BC| = 3 cm
   |AB| = 6 cm, |DC| = 6 cm
2. |DE| = 4 cm, |BE| = 4 cm
   |AE| = 6 cm, |CE| = 6 cm
3. m∠ABC = 115^0, m∠ADC = 115^0
   m∠BAD = 65^0, m∠BCD = 65^0
4. |AB| = 10 cm, |DC| = 10 cm
   $\overline{AB} \parallel \overline{DC}$
5. |AB| = 5 cm, |CD| = 5 cm
   |BC| = 5 cm, |AD| = 5 cm
6. m∠1 + m∠3 = m∠2 + m∠4
   m∠5 + m∠7 = m∠6 + m∠8

What I Have Learned

CONDITIONS WHICH GUARANTEE THAT A QUADRILTERAL IS A PARALLELOGRAM

THEOREMS:

1. A quadrilateral is a parallelogram if it's opposite sides are congruent.
2. A quadrilateral is a parallelogram if a pair of opposite sides are parallel and congruent.
3. A quadrilateral is a parallelogram if it's diagonals bisect each other.
4. A quadrilateral is a parallelogram if it's opposite angles are congruent.
5. A quadrilateral is a parallelogram if any two consecutive angles are supplementary.



What I Can Do

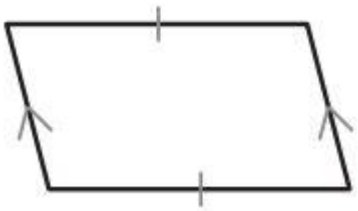
Task 1:

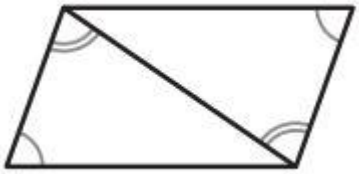
Quadrilateral DEFG is a parallelogram. Kaye uses its properties in completing the table.

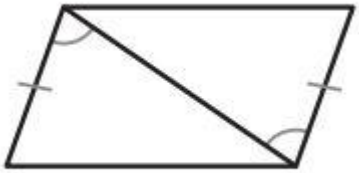
STATEMENT	REASON
1. $\overline{EF} \cong$ _____	a.
2. $\angle E \cong$ _____	b.
3. $\overline{FG} \cong$ _____	c.
4. $\Delta DEF \cong$ _____	d.
5. $m\angle D + m\angle G =$ _____	e.

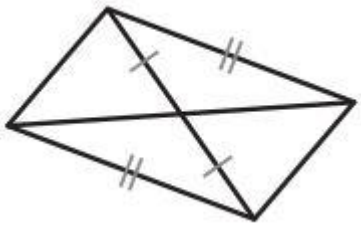
Task 2:

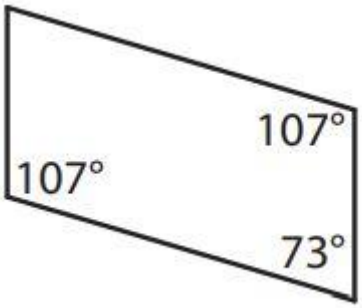
Determine if each quadrilateral is a parallelogram. Justify your answer.

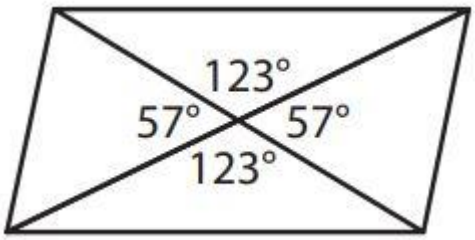
1. 

2. 

3. 

4. 

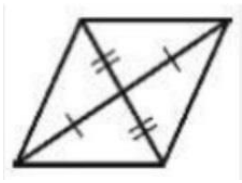
5. 

6. 

Assessment

Directions: Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

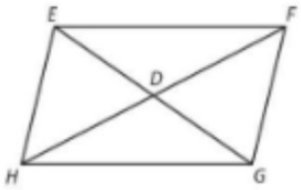
1. Determine whether the figure at the right is a parallelogram. If so, state the reason.



   A. Yes, it is a quadrilateral with 2 pairs of opposite sides that are congruent.
   B. Yes, it is a quadrilateral with 2 pairs of opposite angles that are congruent.
   C. Yes, it is a quadrilateral with diagonals that bisect each other.
   D. It is not a parallelogram.

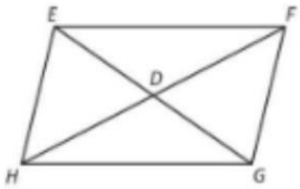
2. Which statement below can be used to prove that quadrilateral EFGH is a parallelogram?

   A. $\overline{EG}$ and $\overline{FH}$ bisect each other
   B. $\overline{EG}$ and $\overline{HF}$ are congruent
   C. $\overline{EF}$ and $\overline{FG}$ are congruent
   D. $\overline{EG}$ and $\overline{DF}$ are congruent

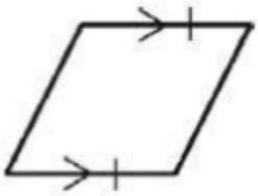


3. Given m∠HEF = 100^0. What must be m∠FGH to prove that quadrilateral EFGH is a parallelogram?

   A. 20
   B. 80
   C. 90
   D. 100

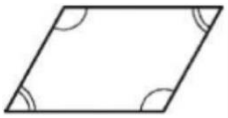


4. Determine whether the figure below is a parallelogram or not. If so, state the reason.



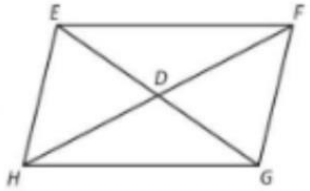
   A. Yes, it is a quadrilateral with 2 pairs of opposite sides that are congruent.
   B. Yes, it is a quadrilateral with any two consecutive angles that are supplementary.
   C. Yes, it is a quadrilateral with one pair of opposite sides that are parallel and congruent.
   D. It is not a parallelogram.

5. Determine whether the figure at the right is a parallelogram or not. If so, state the reason.



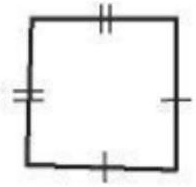
   A. Yes, it is a quadrilateral with 2 pairs of opposite sides that are congruent.
   B. Yes, it is a quadrilateral with 2 pairs of opposite angles that are congruent.
   C. Yes, it is a quadrilateral with one pair of opposite sides that are parallel and congruent.
   D. It is not a parallelogram

6. Which statement below can be used to prove that quadrilateral EFGH is a parallelogram?



   A. $\overline{EF} \cong \overline{FG}$ and $\overline{GH} \cong \overline{HE}$
   B. $\overline{EF} \cong \overline{EH}$ and $\overline{HG} \cong \overline{FG}$
   C. $\overline{EF} \cong \overline{HG}$ and $\overline{EH} \cong \overline{FG}$
   D. $\overline{EG} \cong \overline{FH}$ and $\overline{EH} \cong \overline{FG}$

7. Determine whether the figure below is a parallelogram or not. If so, state the reason.



   A. Yes, it is a quadrilateral with 2 pairs of opposite sides that are congruent.
   B. Yes, it is a quadrilateral with any two consecutive angles that are supplementary .
   C. Yes, it is a quadrilateral with one pair of opposite sides that is parallel and congruent.
   D. It is not a parallelogram.

8. Which polygon below is NOT a parallelogram?

   A. Figure 1
   B. Figure 2
   C. Figure 3
   D. Figure 4

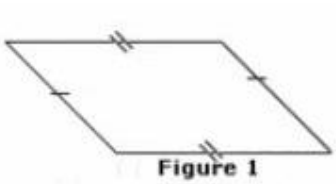


Figure 1

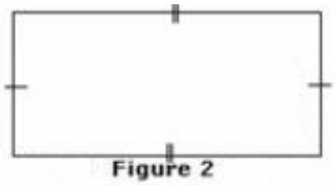


Figure 2

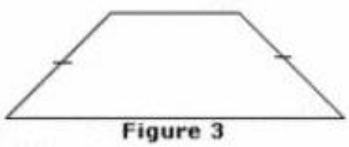


Figure 3

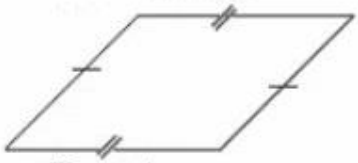
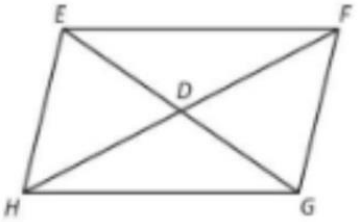


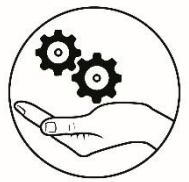
Figure 4

9. Given m∠HEF = 100^0. What must be m∠EFG and m∠GHE to prove that quadrilateral EFGH is a parallelogram?



   A. $20^0, 20^0$
   B. $80^0, 80^0$
   C. $90^0, 90^0$
   D. $100^0, 100^0$

10. Every parallelogram has _____ sides.
   A. 2 B. 4 C. 6 D. 8



Additional Activities

A lot of quadrilaterals are used in our daily life. In fact, many of the things we see in our daily life resemble geometric shapes like circle, quadrilateral, triangle, etc. Buildings, transportations, tools, toys, clothing, and even food, take some of these shapes for aesthetic appeal or for some practical reasons.

Quadrilaterals are used in electronic devices like mobile phones, laptops, computers, TVs, etc. Why do you think these devices take the shape of a quadrilateral? How can these devices help in facing the challenges of the "new normal"?

_____
_____
_____
_____

Problem-Based Learning Worksheet

The railing braces form the diagonals of a parallelogram.

Let's Analyze!

1. The opposite sides of a parallelogram are _____.
2. The diagonals of a parallelogram always _____
3. What can you say about the consecutive angles of a parallelogram?
4. Which is not a property of a parallelogram?
   a. Diagonals bisect each other.
   b. Both pairs of opposite sides are parallel.
   c. Diagonals are always congruent.
   d. Both pairs of opposite angles are congruent.
5. If one angle of a parallelogram measures 100^0, what is the measure of the angle consecutive to it?

E-Sites

To further explore the concept learned today and if it is possible to connect to the internet, you may visit the following links:

https://www.mathplanet.com/education/geometry/quadrilaterals/properties-of-parallelograms

https://www.onlinemath4all.com/proving-quadrilaterals-are-parallelograms-worksheet.html

https://www.slideshare.net/rina0812/l2-parallelogram

Answer Key

Assessment
1. C
2. A
3. D
4. C
5. B
6. C
7. D
8. C
9. B
10. B

Problem-Based Learning Worksheet

1. Congruent
2. Bisect each other
3. They are supplementary
4. C
5. 80°

What I Know

1. A
2. B
3. C
4. D
5. C
6. Yes
7. Yes
8. No
9. No
10. Yes

ACTIVITY 2

1. A quadrilateral is a parallelogram if its opposite sides are congruent.
2. A quadrilateral is a parallelogram if it's diagonal bisect each other.
3. A quadrilateral is a parallelogram if both pairs of opposite angles are congruent.
4. A quadrilateral is a parallelogram if a pair of opposite sides is parallel and congruent.
5. A quadrilateral is a parallelogram if its opposite sides are congruent.
6. A quadrilateral if both pairs of opposite angles are congruent.

What I Can Do

Task 1

STATEMENT	REASON
1. $\overline{EF} \cong \overline{DG}$	a. Opposite sides are congruent.
2. $\angle E \cong \angle G$	b. Opposite angles are congruent.
3. $\overline{FG} \cong \overline{ED}$	c. Opposite sides are congruent.
4. $\triangle DEF \cong \triangle FGD$	d. SAS Congruence
5. $m\angle D + m\angle G = 180^{\circ}$	e. Consecutive angles are supplementary.

Task 2

1. No. One pair of opposite sides are parallel. A different pair of opposite sides are congruent.
2. Yes. The third pair of angles in the triangles are congruent. So, both pairs of opposite angles are congruent.
3. Yes. A pair of alternate interior angles are congruent so a pair of opposite sides are parallel. The same pair of opposite sides are congruent.
4. No. One pair of opposite sides are congruent and one diagonal is bisected by the other.
5. Yes . The 73° angle is supplementary to both of the 107° angles. This shows that both pairs of opposite sides are parallel by the Converse of the same-side interior angles Postulate.
6. No. Your are only given the measure of the four angles formed by intersecting diagonals of the quadrilateral.

References

Soledad Jose-Dilao, Ed.D., Julieta G. Bernabe (2009) Geometry Textbook for Third Year. pp 127 – 133, SD Publication, Inc. Araneta Avenue, Quezon City

Priscila C. De Sagun, Jisela M. Naz-Ulpina (1999) Dynamic Math II. pp. 196 – 200, Diwa Scholastic Press Inc. Gil Puyat Avenue, Makati City

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