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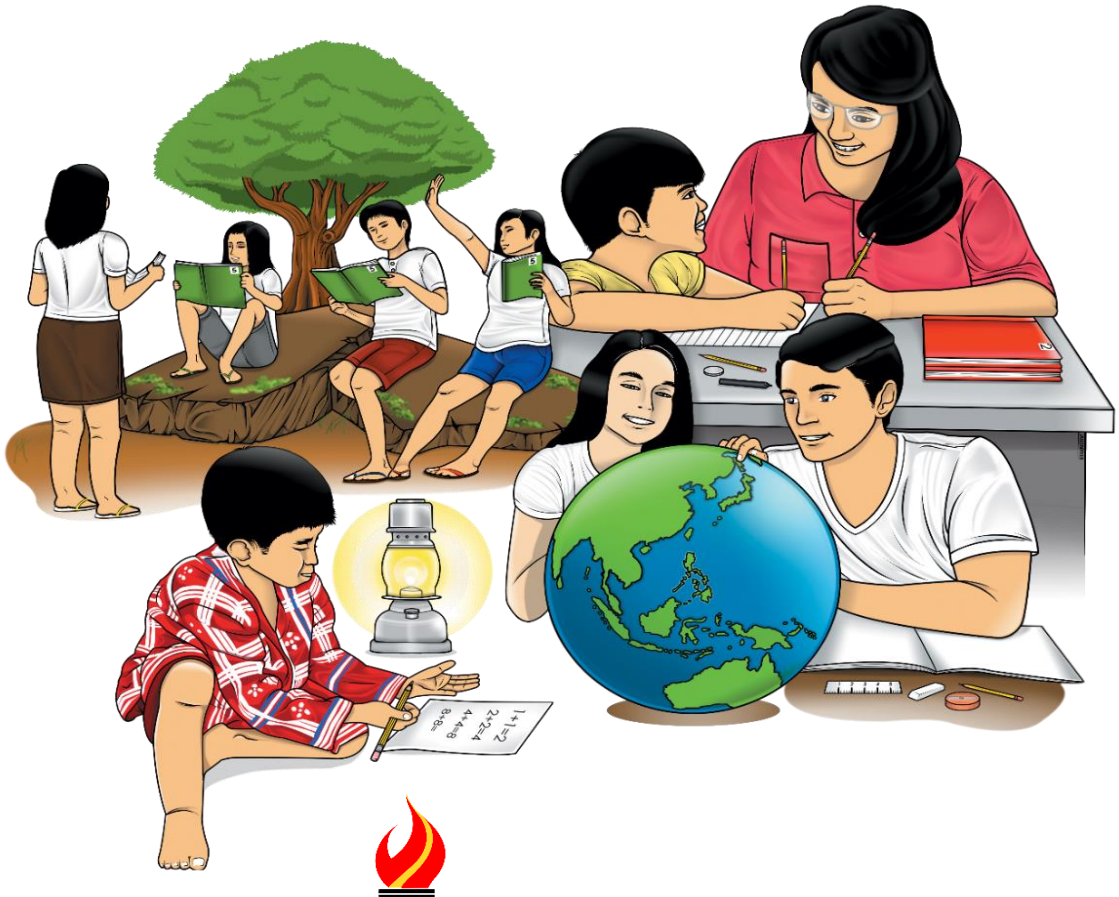
DEPARTMENT OF EDUCATION

Mathematics

Quarter 3 – Module 7
Proving Statements on Triangle Congruence



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Mathematics

Quarter 3 – Module 7
Proving Statements on Triangle Congruence

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Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

What I Need to Know

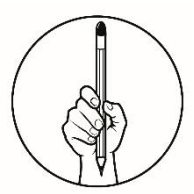
This module is designed for you to understand how to prove statements on triangle congruence. You will be able to know how to prove that the corresponding parts of congruent triangles are congruent. The scope of this module enables you to use it in many different learning situations. The lesson is arranged to follow the standard sequence of the lesson. But the order in which you read them can be changed to correspond with the textbook you are now using.

This module contains:

Lesson 1: Proving Statements on Triangle Congruence

After going through this module, you are expected to:

1. identify statements on triangle congruence;
2. apply the postulates and theorems on triangle congruence to prove statements involving (a) multiple angles, (b) isosceles triangle, (c) overlapping triangles; and
3. relate the importance of proving statements on triangle congruence in real life situations.



What I Know

Pre-Assessment

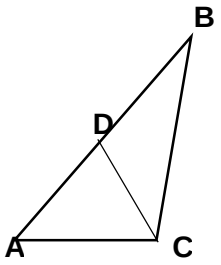
Directions: Choose the letter of the correct answer and write on a separate sheet of paper.

1. Which of the following refers to the triangles that have common region?
   A. Multiple triangles
   B. Isosceles triangles
   C. Congruent triangles
   D. Overlapping triangles

2. Which of the following theorems states that, "if two sides of a triangle are congruent, then the angles opposite them are congruent."?
   A. Right Angle Theorem
   B. Isosceles Triangle Theorem
   C. Interior Angles of a Triangle Theorem
   D. Exterior Angles of a Triangle Theorem

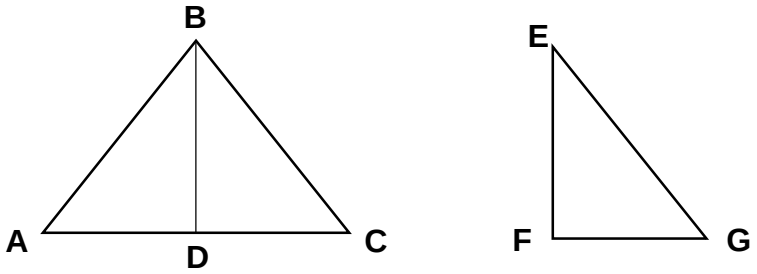
For items 3 and 4, use the figure at the right.

3. In an isosceles ΔABC, let $\overline{CD}$ be an angle bisector of $\angle BCA$. What theorem or postulate can justify $\Delta DCA \cong \Delta DCB$?
   A. Angle-Angle-Side
   B. Hypotenuse-Acute Angle Side
   C. Hypotenuse-Leg
   D. Side-Angle-



4. In item number 3, $\Delta DCA \cong \Delta DCB$. What other corresponding sides are congruent by CPCTC aside from $\overline{CD} \cong \overline{CD}$?
   A. $\overline{CD} \cong \overline{CA}$
   B. $\overline{CD} \cong \overline{CB}$
   C. $\overline{DA} \cong \overline{DB}$
   D. $\overline{DA} \cong \overline{AC}$

For items 5 to 7, use the figure below.

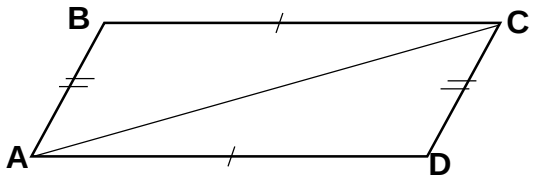


5. Which of the following properties justifies the statement "$\overline{BD} \cong \overline{BD}$"?
   A. Equivalence
   B. Reflexive
   C. Symmetric
   D. Transitive

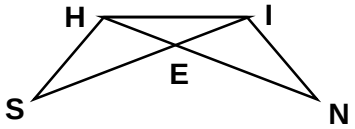
6. Which of the following properties justifies the statement "If $\overline{AD} \cong \overline{DC}$ and $\overline{DC} \cong \overline{FG}$, then $\overline{AD} \cong \overline{FG}$"?
   A. Equivalence
   B. Reflexive
   C. Symmetric
   D. Transitive

7. If $\overline{AB} \cong \overline{BC}$, which angles are guaranteed congruent by the Isosceles Triangle Theorem?
   A. $\angle ABD$ and $\angle CBD$
   B. $\angle BAD$ and $\angle BCD$
   C. $\angle EFG$ and $\angle BDC$
   D. $\angle FEG$ and $\angle EGF$

8. In the figure, $\Delta CBA \cong \Delta ADC$. Which of the following can justify the statement $\angle B \cong \angle D$?
   A. CPCTC
   B. Reflexive Property
   C. Definition of Midpoint
   D. Isosceles Triangle Theorem



9. The figure at the right are overlapping triangles where $\angle SIH \cong \angle NHI$ and $\overline{SI} \cong \overline{NH}$. Which of the following relations is true?
   A. $\Delta SHI \cong \Delta HIN$
   B. $\Delta SHI \cong \Delta HNI$
   C. $\Delta SHI \cong \Delta NHI$
   D. $\Delta SHI \cong \Delta NIH$



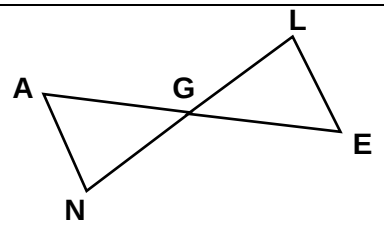
10. In ΔGEM, let $\overline{ES}$ be a perpendicular bisector of $\overline{GM}$. If $\Delta GES \cong \Delta MES$, which side of the triangles is congruent by CPCTC?
   A. $\overline{ES} \cong \overline{EM}$
   B. $\overline{ES} \cong \overline{ES}$
   C. $\overline{GS} \cong \overline{ES}$
   D. $\overline{GE} \cong \overline{ME}$

For items 11 to 15.
Complete the proof by filling in the blanks with the correct statements and reasons. Options are found in the box.

A. CPCTC	D. SAS Congruence Postulate
B. Definition of Vertical Angles	E. $\Delta AGN \cong \Delta EGL$
C. Given	F. Vertical Angle Theorem

Given: $\overline{GA} \cong \overline{GE}$; $\overline{GN} \cong \overline{GL}$

Prove: $\overline{AN} \cong \overline{EL}$



Proof:

Statements	Reasons
1. $\overline{GA} \cong \overline{GE}$	1. Given
2. $\overline{GN} \cong \overline{GL}$	2. (11)
3. $\angle AGN$ *and* $\angle EGL$ are vertical angles.	3. (12)
4. $\angle AGN \cong \angle EGL$	4. (13)
5. (14)	5. SAS Congruence Postulate
6. $\overline{AN} \cong \overline{EL}$	6. (15)

Lesson 1 Proving Statements on Triangle Congruence

This lesson is designed for you to prove statements on triangle congruence. You will prove statements on triangle congruence by justifying it through CPCTC. Activities are provided for you to understand following the step-by-step process of proving statements on triangle congruence.

What's In

Find My Match!

Directions: Match each postulate or theorem in Column A with its corresponding illustration in Column B. Write the letter of the correct answer on a separate sheet of paper.

Column A

_____ 1. (SSS Congruence)
If three sides of one triangle are congruent to corresponding three sides of another triangle, then the triangles are congruent.

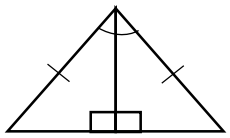
_____ 2. (SAS Congruence)
If two sides and an included angle of one triangle are congruent to corresponding two sides and an included angle of another triangle, then the triangles are congruent.

_____ 3. (ASA Congruence)
If two angles and an included side of one triangle are congruent to corresponding two angles and an included side of another triangle, then the triangles are congruent.

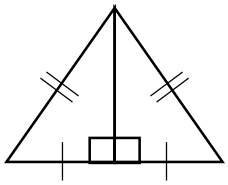
_____ 4. (AAS Congruence)
If two angles and non-included side of one triangle are congruent to corresponding two angles and non-included side of another triangle, then the triangles are congruent.

Column B

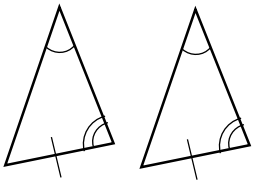
A.



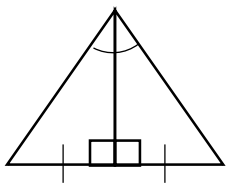
B.



C.

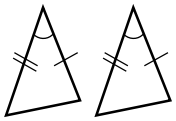


D.



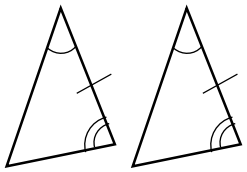
_____ 5. (LL Congruence)
If the legs of a right triangle are congruent to the corresponding legs of another triangle, then the triangles are congruent.

E.



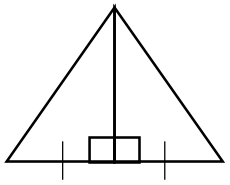
_____ 6. (LA Congruence)
If the leg and the acute angle of a right triangle are congruent to the corresponding leg and acute angle of another right triangle, then the triangles are congruent.

F.



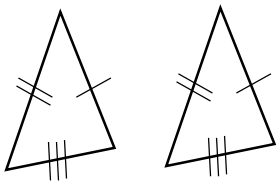
_____ 7. (HyA Congruence)
If the hypotenuse and an acute angle of a right triangle are congruent to the corresponding hypotenuse and acute angle of another right triangle, then the triangles are congruent.

G.



_____ 8. (HyL Congruence)
If the hypotenuse and the leg of a right triangle are congruent to the corresponding hypotenuse and leg of another right triangle, then the triangles are congruent.

H.



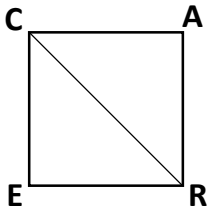
Questions:

1. Were you able to identify the illustrations of the different postulates and theorems?
2. How did you identify the illustration of the given theorem or postulate?

What's New

Directions: Fill in the missing statements and reasons in the two-column proof below. Answer the questions that follow. Write your answer on a separate sheet.

1. Given: ■CARE is a square and $\overline{CR}$ is its diagonal



Prove: $\Delta CER \cong \Delta CAR$

Proof:

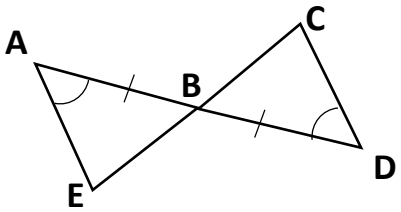
Statements	Reasons
1. _____ *(What is given?)*	1. Given
2. _____ *(What can you say about the sides of the square?)*	2. Definition of Square
3. _____ *(What statement can show the triangle congruence as justified by the reason?)*	3. Reflexive Property
4. $\Delta CER \cong \Delta CAR$	4. _____ *(What theorem or postulate justifies $\Delta CER \cong \Delta CAR$?*

Proving is a whole process of thinking, in which, from the start you must have a direct way of constructing the statement to prove. In this case, we want to justify $\Delta CER \cong \Delta CAR$. First, based on the given information, which congruence theorem can be used? We are given with a square, which is by definition all of its sides are congruent. Thus we can use this to justify the congruence of two pairs of corresponding sides of the triangles. Looking further, the other pair of corresponding sides of these two triangles is actually a common side which is the diagonal CR. Now based on this we can use the SSS congruence theorem to justify the congruence of the triangles.

Questions:

1. Were you able to supply the statement or reason needed in the proof?
2. What mathematical concept did you apply to complete the proof?

2. Given: B is the midpoint of $\overline{AD}$.
   $\angle A \cong \angle D$
   Prove: $\Delta ABE \cong \Delta DBC$



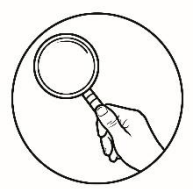
Proof:

Statements	Reasons
1. $\angle A \cong \angle D$	1. _____ *(What is the reason?)*
2. B is the midpoint of $\overline{AD}$.	2. _____ *(What is the reason?)*
3. $\overline{AB} \cong \overline{BD}$	3. _____ *(Based on 2nd statement, what mathematical concept justifies the 3rd statement?)*
4. $\angle ABE$ and $\angle DBC$ are vertical angles	4. _____ *(What mathematical concept that supports the 4th statement?)*
5. $\angle ABE \cong \angle DBC$	5. _____ *(What theorem justifies the 5th statement?)*
6. $\Delta ABE \cong \Delta DBC$	6. _____ *(What postulate or theorem justifies that $\Delta ABE \cong \Delta DBC$?)*

We want to justify that $\Delta ABE \cong \Delta DBC$. First, based on the given information, which congruence theorem can be used. We are given with B as the midpoint of $\overline{AD}$, which is by definition of midpoint, two segments where the midpoint lies is divided into two congruent segments. We are provided also with the given that $\angle A \cong \angle D$. Thus, we can use this to justify the congruence of the corresponding side and angle of the triangles. Looking further, the other pair of corresponding angles of these two triangles is actually vertical angles and the vertical angle theorem states that vertical angles are congruent. Now based on this we can use the ASA congruence theorem to justify the congruence of the triangles.

Questions:

1. Were you able to supply the reasons needed in the proof?
2. What mathematical concept did you apply to complete the proof?
3. Can we also prove that the other corresponding parts of congruent triangles are congruent? How are we going to do it?



What is It

Triangles are classified according to sides and angles. Some triangles have characteristics that come from both of these classifications like **scalene right triangle** and **isosceles right triangle**. Figure 1 is an example of scalene right triangle. Aside from having a right angle, no two sides are congruent while i**sosceles right triangle** as shown in Figure 2 has congruent legs. First, let us review the parts of the scalene right and isosceles right triangle.

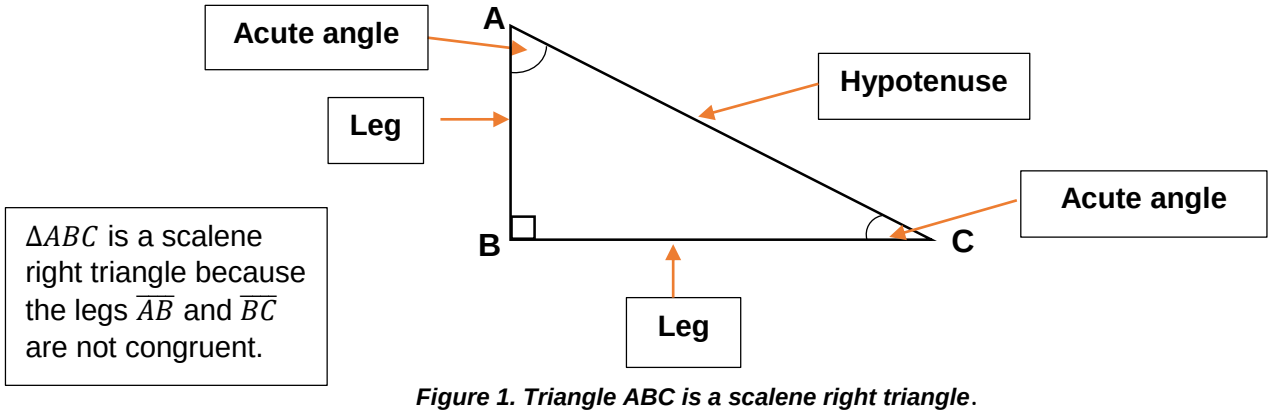


Figure 1. Triangle ABC is a scalene right triangle.

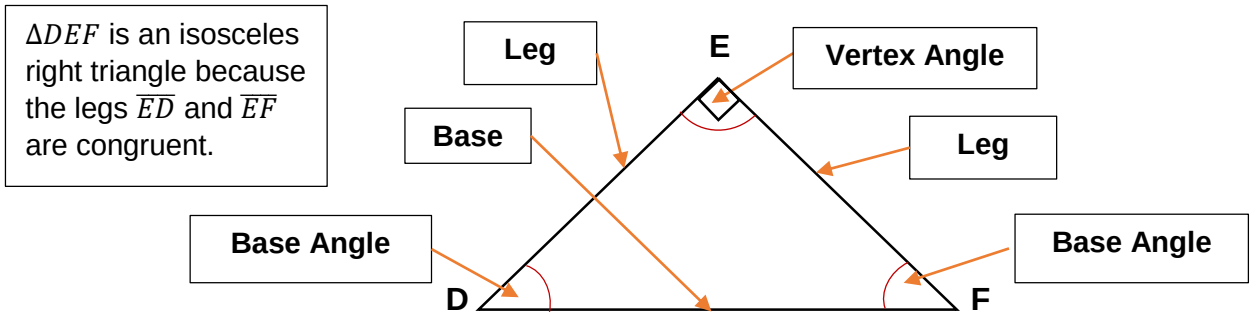
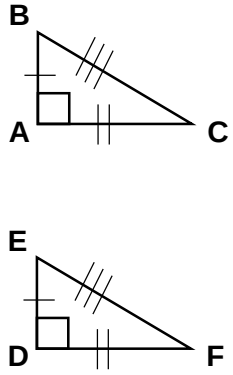
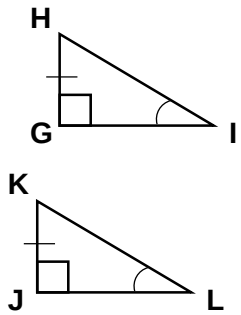
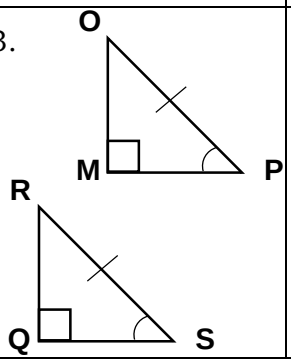


Figure 2. Triangle DEF is an isosceles right triangle.

The table shows the use of corresponding parts of scalene right triangle and isosceles right triangle in proving that two triangles are congruent. After proving that the two triangles are congruent, we can also say that the other **corresponding parts of the congruent triangles are congruent (CPCTC)**. Here are some examples.

Congruent Triangles	**Description**	**Other corresponding parts of congruent triangles that can be justified as congruent by CPCTC**

Congruent Triangles	Description	Other corresponding parts of congruent triangles that can be justified as congruent by CPCTC
1. 	The two scalene right triangles, $\Delta ABC \cong \Delta DEF$ by LL Congruence or HyL Congruence based on the marks in the figure. LL Congruence: $\overline{AB} \cong \overline{DE}$ $\overline{AC} \cong \overline{DF}$ HyL Congruence: $\overline{BC} \cong \overline{EF}$ $\overline{AB} \cong \overline{DE}$ or $\overline{BC} \cong \overline{EF}$ $\overline{AC} \cong \overline{DF}$	$\angle B \cong \angle E$ $\angle C \cong \angle F$ $\overline{BC} \cong \overline{EF}$ $\angle B \cong \angle E$ $\angle C \cong \angle F$ $\overline{AC} \cong \overline{DF}$ $\angle B \cong \angle E$ $\angle C \cong \angle F$ $\overline{AB} \cong \overline{DE}$
2. 	The two scalene right triangles, $\Delta GHI \cong \Delta JKL$ by LA Congruence based on the figure. LA Congruence: $\overline{GH} \cong \overline{JK}$ $\angle I \cong \angle L$	$\angle H \cong \angle K$ $\overline{HI} \cong \overline{KL}$ $\overline{GI} \cong \overline{JL}$
3. 	The two triangles ΔMOP and ΔQRS are isosceles right triangle. $\Delta MOP \cong \Delta QRS$ by HyA Congruence Theorem. HyA Congruence: $\overline{OP} \cong \overline{RS}$ $\angle P \cong \angle S$	$\angle O \cong \angle R$ $\overline{MO} \cong \overline{RQ}$ $\overline{MP} \cong \overline{QS}$

The pairs of triangles shown in the table are congruent. However, not all problems involving congruent triangles looked like the examples above. We also have overlapping triangles that can also be proven congruent. Other problems may be presented like the figures below.

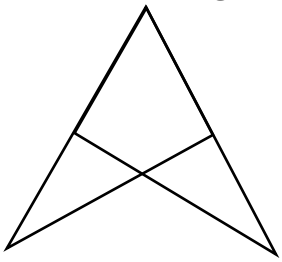


Figure 3

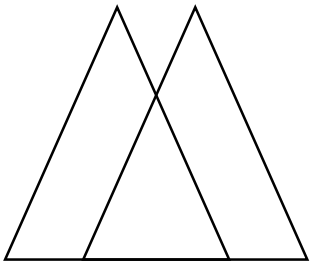


Figure 4

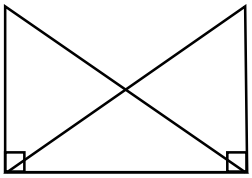


Figure 5

Let us try to prove the congruence of overlapping triangles.

Illustrative Example 1

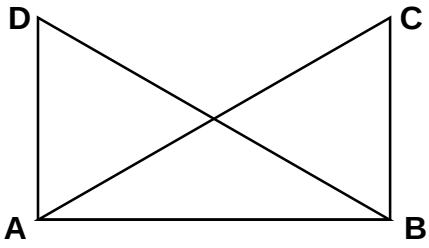
ΔDAB and ΔCBA are overlapping scalene right triangles. In this example, we will prove that the other corresponding parts of congruent triangles are also congruent.

Direction: Complete the proof below by answering the guide questions provided in each missing statements and reasons.

1. Consider the given overlapping scalene right triangles.

Given: $\overline{DA} \perp \overline{AB}$;
$\overline{CB} \perp \overline{AB}$;
$\overline{DB} \cong \overline{CA}$

Prove: $\angle ADB \cong \angle BCA$



Proof:

Statements	Reasons
1. _____ *(What is the first **Given** statement?*	1. Given
2. _____ *(If $\overline{DA} \perp \overline{AB}$, then what kind of angle is formed? What is the name of the angle formed?)*	2. Definition of Perpendicular Lines
3. $\overline{CB} \perp \overline{AB}$	*3.* Given
4. $\angle CBA$ is a right angle	*4.* _____ *(Based on the3rd statement, what do you think is the reason for the 4th statement?)*
5. _____ *(In the 2nd and 4th statements, formulate a statement based on the right angle theorem)*	5. Right Angle Theorem
6. _____	6. Definition of right triangle

Statements	Reasons
(If $\angle DAB$ and $\angle CBA$ are right angles, what kind of triangle is ΔDAB and ΔCBA?)	
7. $\overline{AB} \cong \overline{AB}$	7. Reflexive Property
8. _____ *(What is the third given statement?*	8. Given
9. $\Delta DAB \cong \Delta CBA$	*9.* _____ *(Based from statements 1 to 8, what theorem can be used to prove that the two triangles are congruent?)*
10. $\angle$ADB $\cong \angle$BCA	*10.*_____ *(After proving the two triangles are congruent, what statement can justify that the other corresponding parts are congruent?*

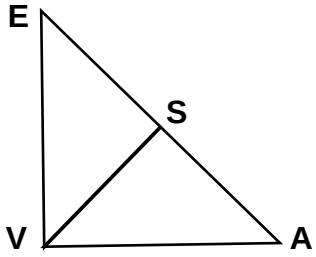
Solution: (Complete Proof)

Statements	Reasons
1. $\boldsymbol{\overline{DA} \perp \overline{AB}}$	1. Given
2. $\boldsymbol{\angle DAB}$ ***is a right angle***	2. Definition of Perpendicular Lines
3. $\overline{CB} \perp \overline{AB}$	*3.* Given
4. $\angle CBA$ is a right angle	*4.* ***Definition of Perpendicular Lines***
5. $\boldsymbol{\angle DAB \cong \angle CBA}$	5. Right Angle Theorem
6. $\boldsymbol{\Delta DAB \cong \Delta CBA}$	6. Definition of right triangle
7. $\overline{AB} \cong \overline{AB}$	7. Reflexive Property
8. $\boldsymbol{\overline{DB} \cong \overline{CA}}$	8. Given
9. $\Delta DAB \cong \Delta CBA$	*9.* ***HyL Congruence Theorem***
10. $\angle$ADB $\cong \angle$BCA	*10.* ***CPCTC***

In this example, we want to justify the congruence of $\angle$ADB $\cong \angle$BCA. Based on the given information, which congruence theorem can be used? We are given two pair of perpendicular segments $\overline{DA} \perp \overline{AB}$ and $\overline{CB} \perp \overline{AB}$, which by definition, these pair of segments formed right angles, hence, the two triangles are right triangles. We are provided also with the given that $\overline{DB} \cong \overline{CA}$. Since the two triangles are overlapping, the two triangles shared the congruent segments by reflexive property. Thus, we can use this to justify the congruence of the corresponding hypotenuses and legs of the triangles. Now based on this, we can use the HyL congruence theorem to justify the congruence of the triangles.

The reason for numbers 1, 3 and 8 is ***given*** since it is a starting point or connector points in formulating other statements in proving triangle congruence. Once the congruence of two triangles is established, then, you can say that the other corresponding parts of congruent triangles are also congruent or also known as CPCTC. Thus, we can conclude that the last statement $\angle ADB \cong \angle BCA$ is proven by CPCTC.

Illustrative Example 2



Directions: Complete the proof below by considering the given clue in each missing statements and reasons.

Consider the figure at the right, ΔAVE is a right triangle and $\overline{VS} \perp \overline{AE}$. S is the midpoint of $\overline{AE}$. Prove that ΔAVE is an isosceles triangle.

Proof:

Statements	**Reasons**
1. $\overline{VS} \perp \overline{AE}$	1. Given
2. _____ *(What kind of angles are formed when $\overline{VS} \perp \overline{AE}$)*	*2.* Definition of Perpendicular lines
3. $\angle ASV \cong \angle ESV$	3. _____ *(Theorem that states: All right angles are congruent)*
4. _____ *(This statement is found in the given.)*	4. Given
5. $\overline{ES} \cong \overline{AS}$	5. Definition of Midpoint
6. $\overline{VS} \cong \overline{VS}$	6. _____ *(What property of congruence that states any figure is congruent to itself?)*
7. _____ *(Name the two congruent triangles based on the previous statements.)*	7. SAS Congruence Postulate
8. $\overline{AV} \cong \overline{EV}$	*8.* CPCTC
9. ΔAVE is an isosceles triangle	9. _____ *(Based on the 9th statement, state the reason that justifies that ΔAVE is an isosceles triangle.)*

Remember that when two rays, lines or line segments are perpendicular, there is a right angle formed. Moreover, when two legs of a right triangle are congruent, then the triangle is an isosceles right triangle.

Statements	Reasons
1. $\overline{VS} \perp \overline{AE}$	1. Given
2. $\angle ASV$ and $\angle ESV$ are right angles	*2.* Definition of Perpendicular lines
3. $\angle ASV \cong \angle ESV$	**3. Right Angle Theorem**
4. S is a midpoint of $\overline{AE}$	4. Given
5. $\overline{ES} \cong \overline{AS}$	5. Definition of Midpoint
6. $\overline{VS} \cong \overline{VS}$	**6. Reflexive Property**
7. $\Delta AVS \cong \Delta EVS$	7. SAS Congruence Postulate or LL Congruence Theorem
8. $\overline{AV} \cong \overline{EV}$	*8.* CPCTC
9. ΔAVE is an isosceles triangle	**9. Definition of Isosceles Triangle**

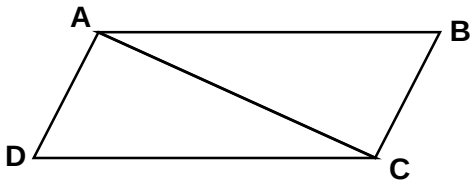
Solution: (Complete Proof)

To justify that ΔAVE is an isosceles triangle, we are given with $\overline{VS} \perp \overline{AE}$ which is by definition the angles formed by the two segments are right angles which are $\angle ASV$ and $\angle ESV$. We are provided also with the given that S is the midpoint of $\overline{AE}$ which is by definition the segment is divided into two congruent segments. Since the two triangles has common side and by reflexive property that says the segment is congruent to itself. Thus, we can use this to justify the congruence of the corresponding sides of the triangles. Now based on this we can use the SAS congruence theorem or LL Congruence to justify the congruence of the triangles. When two triangles are congruent, corresponding parts of congruent triangles are also congruent. Thus, another pair of corresponding sides are congruent that makes the triangle isosceles by definition.

We had just proven that ΔAVE is an isosceles triangle since we have established that $\overline{AV}$ and $\overline{EV}$ are two congruent sides of ΔAVE by CPCTC.

Illustrative Examples 3:

1. Given: $\angle D \cong \angle B$
   $\angle DAC \cong \angle BCA$
   Prove: $\angle ACD \cong \angle CAB$
   Proof:



Statements	Reasons
1. _____ *(What are the given statements?)*	1. Given
2. $\overline{AC} \cong \overline{AC}$	2. Reflexive Property
3. $\Delta ADC \cong \Delta CBA$	3. _____ *(Based on the statements, what theorem can justify $\Delta ADC \cong \Delta CBA$?)*
4. $\angle ACD \cong \angle CAB$	4. _____ *(What can justify the congruence of the corresponding parts of congruent triangles?)*

ΔACD and ΔCAB are adjacent triangles since they have a common side.

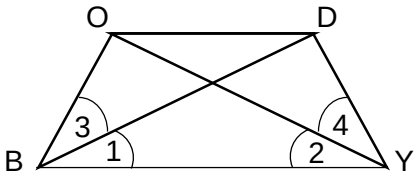
Solution: (Complete Proof)

Statements	Reasons
1. $\boldsymbol{\angle D \cong \angle B}$ $\boldsymbol{\angle DAC \cong \angle BCA}$	1. Given
2. $\overline{AC} \cong \overline{AC}$	2. Reflexive Property
3. $\Delta ADC \cong \Delta CBA$	**3. AAS Congruence Theorem**
4. $\angle ACD \cong \angle CAB$	**4. CPCTC**

To justify that $\angle ACD \cong \angle CAB$, we are given with $\boldsymbol{\angle D \cong \angle B}$ and $\boldsymbol{\angle DAC \cong \angle BCA}$. Since the two triangles have a common side, by reflexive property that says the segment is congruent to itself. Thus, we can use this to justify the congruence of the corresponding sides and the congruence of the corresponding angles of the triangles. Now based on this we can use the AAS congruence theorem to justify the congruence of the triangles. When two triangles are congruent, corresponding parts of congruent triangles are also congruent. Thus, another pair of corresponding angles are congruent by CPCTC.

Illustrative Examples 4:

Given: $\overline{DB} \cong \overline{OY}$
$\angle 1 \cong \angle 2$
Prove: $\overline{OB} \cong \overline{DY}$



Proof:

Statements	Reasons
1. _____ *(What are the given statements?)*	1. Given
2. _____ *(Which side of ΔBOY and ΔYDB is common?)*	2. Reflexive Property
3. _____ *(What are the congruent triangles based on the previous statements?)*	3. SAS Congruence Postulate
4. _____ *(What is to be proven?)*	4. CPCTC

Solution: (Complete Proof)

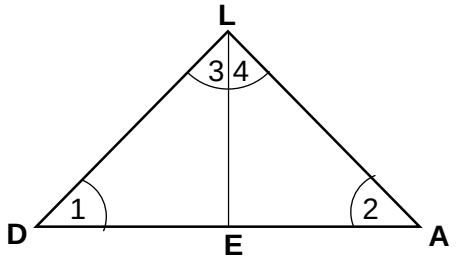
Statements	Reasons
1. $\overline{DB} \cong \overline{OY}$, $\angle 1 \cong \angle 2$	1. Given
2. $\overline{BY} \cong \overline{BY}$	2. Reflexive Property
3. $\Delta BOY \cong \Delta YDB$	3. SAS Congruence Postulate
4. $\overline{OB} \cong \overline{DY}$	4. CPCTC

ΔBOY and ΔYDB are congruent by SAS Congruence Postulate and by CPCTC, $\overline{OB} \cong \overline{DY}$. We could also prove the congruency of multiple angles such as $\angle BOY \cong \angle YDB$ and $\angle OBY \cong \angle DYB$.

Illustrative Examples 5:

Given: $\overline{DL} \cong \overline{AL}$
E is a midpoint at $\overline{DA}$
Prove: $\angle 3 = \angle 4$

Proof:



Statements	Reasons
1. _____ *(What is the first given?)*	1. Given
2. ΔDAL is an isosceles triangle	2. Definition of Isosceles Triangle
3. $\angle 1 \cong \angle 2$	3. _____ *(Based on the previous statements,*

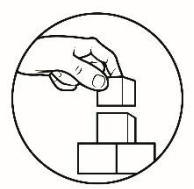
Statements	Reasons
	what theorem is used to deduce this statement?)
4. E is a midpoint at $\overline{DA}$	4. Given
5. _____ *(If E is a midpoint of $\overline{DA}$, what happens to $\overline{DA}$?)*	5. Definition of Midpoint
6. $\Delta DEL \cong \Delta AEL$	6. _____ *(What postulate or theorem can justify the congruency of the two triangles based on the proved statements?)*
7. $\angle 3 = \angle 4$	7. _____ *(What statement can justify the other corresponding parts of congruent triangles are also congruent?)*

Solution: (Complete Proof)

Statements	Reasons
1. $\overline{DL} \cong \overline{AL}$	1. Given
2. ΔDAL is an isosceles triangle	2. Definition of Isosceles Triangle
3. $\angle 1 \cong \angle 2$	3. I**sosceles Triangle Theorem**
4. E is a midpoint at $\overline{DA}$	4. Given
5. $\overline{DE} \cong \overline{AE}$	5. Definition of Midpoint
6. $\Delta DEL \cong \Delta AEL$	**6. SAS Congruence Postulate**
7. $\angle 3 = \angle 4$	**7. CPCTC**

To justify that $\angle 3 = \angle 4$, we are given with $\overline{DL} \cong \overline{AL}$ by definition ΔDAL is an isosceles triangle and if two sides of an isosceles triangle are congruent, then the angles opposite them are also congruent. Since the two triangles has common side, by reflexive property, the segment is congruent to itself. We are also given that E is a midpoint at $\overline{DA}$. By definition, the segment is divided into two congruent segments. Thus, we can use this to justify the congruence of the corresponding sides and the congruence of the corresponding angles of the triangles. Now based on this, we can use the SAS congruence theorem to justify the congruence of the triangles. When two triangles are congruent, corresponding parts of congruent triangles are also congruent. Thus, another pair of corresponding angles are congruent by CPCTC.

It is really possible to go further after proving the congruence of two triangles like proving the other corresponding parts are also congruent.

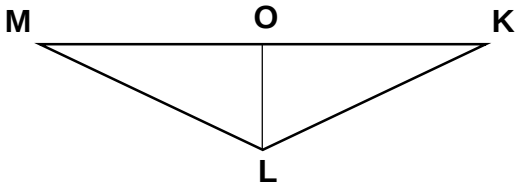


What's More

Activity 1: Complete Me!

Given: $\overline{LO}$ bisects $\overline{KM}$
$\angle KOL \cong \angle MOL$

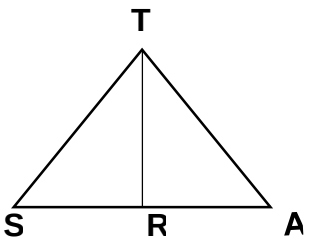
Prove: $\angle KLO \cong \angle MLO$



Proof:

Statements	**Reasons**
1. _____ *(Which of the given statement will help the next statements)*	1. Given
2. $\overline{KO} \cong \overline{MO}$	2. Definition of Segment Bisector
3. _____ *(Based on the reason given on the right, which statement will lead to the congruence of the triangles in the statement 5?)*	3. Reflexive Property
4. $\angle KOL \cong \angle MOL$	4. _____ *(Based on statement number 4, what will be the next reason?)*
5. $\Delta KOL \cong \Delta MOL$	5. _____ *(What postulate or theorem that justifies the congruency of the two triangles?)*
6. $\angle KLO \cong \angle MLO$	6. _____ *(After proving that the two triangles are congruent, what will happen to the other parts of the triangle? What is the appropriate reason for it?)*

Activity 2: Fill Me!



Given: Isosceles ΔSTA with respect to the vertex $\angle T$, $\overline{AR} \cong \overline{SR}$

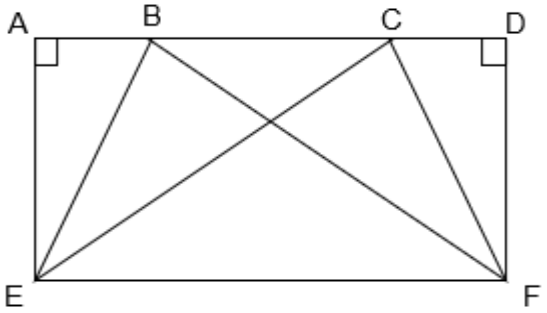
Prove: $\angle ATR \cong \angle STR$

Proof:

Statements	Reasons
1. _____ *(Which of the given statement will help the next statements)*	1. Given
2. _____ *(Which parts of the given triangle are congruent as defined by the isosceles triangle?)*	2. Definition of Isosceles Triangle
3. _____ *(Which side of ΔATR and ΔSTR is common?)*	3. Reflexive Property
4. _____ *(What other statement is given?)*	4. Given
5. _____ *(What are the congruent triangles based on the previous statements)*	5. SSS Congruence Postulate
6. _____ *(What other corresponding parts of the two triangles are to be proven congruent?)*	6. CPCTC

Activity 3: Prove Me!

Given: $\overline{EA} \perp \overline{AC}$
$\overline{FD} \perp \overline{DB}$
$\overline{EC} \cong \overline{FB}$
$\overline{AB} \cong \overline{CD}$
Prove: $\overline{AE} \cong \overline{DF}$



Proof:

Statements	Reasons
1. $\overline{EA} \perp \overline{AC}$ and $\overline{FD} \perp \overline{DB}$	1. Given
2. _____ *(What angle will be formed if $\overline{EA} \perp \overline{AC}$ and $\overline{FD} \perp \overline{DB}$? Name the angles)*	2. Definition of Perpendicular Lines
3. $\angle EAC \cong \angle FDB$	3. _____ *(What theorem that states that all right triangles are congruent?)*
4. ΔEAC and ΔFDB are right triangles	4. Definition of Right Triangle
5. $\overline{EC} \cong \overline{FB}$ and $\overline{AB} \cong \overline{CD}$	5. Given

6. $\|AB\| = \|CD\|$	6. Definition of congruent line segments
7. $\overline{BC} \cong \overline{BC}$	7. _____ *(What property states that a segment is always congruent to itself?)*
8. $\|AB\| + \|BC\| = \|AC\|$ and $\|BC\| + \|CD\| = \|BD\|$	8. Segment Addition Postulate
9. $\|CD\| + \|BC\| = \|AC\|$ and $\|BC\| + \|CD\| = \|BD\|$	9. Substitution Property
10. $\|AC\| = \|BD\|$	10. Transitivity or Transitive Property
11. $\overline{AC} \cong \overline{BD}$	11. Definition of Congruent line segments
12. $\Delta EAC \cong \Delta FDB$	12. _____ *(What theorem justifies that the two triangles are congruent?)*
13. $\overline{AE} \cong \overline{DF}$	13. CPCTC

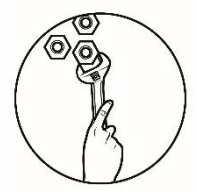
What I Have Learned

Directions: Fill in the blank with the correct word or phrase. Write your answer on a separate sheet of paper.

I have learned that there are two classifications of triangles; that is according to 1. _____ and according to 2. _____. Some triangles have characteristics that come from both of these classifications. Examples of them are 3. _____ and 4. _____.

I also learned that after proving two triangles are congruent, we can prove other corresponding parts of congruent triangles like proving congruence of 5. _____ and congruence of other 6. _____ of triangles by CPCTC which is also known as 7. _____.

We can also prove congruence of triangles that are 8. _____ or triangles that have common parts.



What I Can Do

Directions: Name and illustrate the objects found at home that show congruent triangles. Justify why the parts of the triangles are congruent.

Questions:

1. Were you able to name and identify objects/things that show triangle congruence?
2. Identify concepts/principles that you use in naming and illustrating the congruence of objects/things. Where do you usually apply these concepts/principles?

Rubrics:

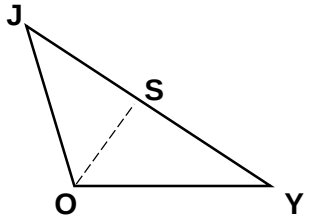
10	8	6	4	2
Able to identify at least 5 objects showing congruent triangles and able to justify their congruence	Able to identify 4 objects showing congruent triangles and able to justify their congruence	Able to identify 3 objects showing congruent triangles and able to justify their congruence	Able to identify 2 objects showing congruent triangles and able to justify their congruence	Able to identify at least 1 object showing congruent triangles but unable to justify their congruence

Assessment

Directions: Answer each of the following items accurately. Write the letter of the correct answer on your answer sheet.

1. ΔABC and ΔDEF are isosceles right triangles. $\overline{AB} \cong \overline{DE}$ and $\overline{AC} \cong \overline{DF}$, which of the following statements is true by CPCTC?
   A. $\overline{AC} \cong \overline{EF}$ C. $\overline{CA} \cong \overline{EF}$
   B. $\overline{BC} \cong \overline{EF}$ D. $\overline{CB} \cong \overline{FD}$

For item 2 and 3, use the figure at the right.



2. In ΔJOY, let S be a midpoint of $\overline{JY}$ and $\overline{OS} \perp \overline{JY}$. What theorem or postulate can justify that $\Delta JOS \cong \Delta YOS$?
   A. AAS C. LL
   B. ASA D. HyL

3. Refer to item number 2, what are the other corresponding angles that are congruent by CPCTC?

I. $\angle JOS \cong \angle OYS$ II. $\angle JOS \cong \angle YOS$ III. $\angle OJS \cong \angle OYS$

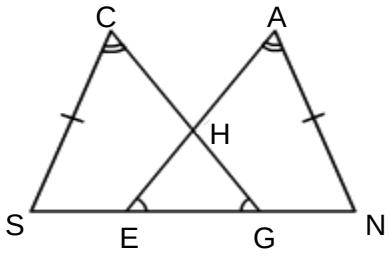
A. I only B. II only C. I and II only D. II and III only

4. In ΔWET and ΔDRY, $\overline{WE} \cong \overline{DR}$, $\overline{ET} \cong \overline{RY}$ and $\overline{WT} \cong \overline{DY}$. By SSS Congruence Postulate, we can say that $\Delta WET \cong \Delta DRY$. Which congruence statement/s of the other corresponding parts of the two triangles is/are true?

I. $\angle W \cong \angle D$ II. $\angle E \cong \angle R$ III. $\angle T \cong \angle Y$

A. I only B. II only C. II and III only D. I, II and III

5. The figure at the right is formed by overlapping triangles. Which of the following congruence statements is true?

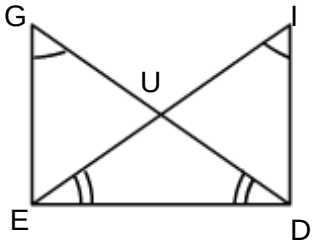


A. $\Delta CGS \cong \Delta AEN$
B. $\Delta CGS \cong \Delta HEN$
C. $\Delta SCG \cong \Delta AEN$
D. $\Delta SCG \cong \Delta NEH$

6. Refer to item number 5, $\angle CSG \cong \angle ANE$ can be justified by which of the following mathematical concept?

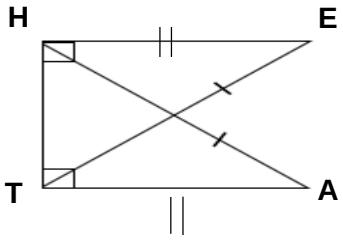
A. CPCTC C. Vertical Angle Theorem
B. Right Angle Theorem D. ASA Congruence Postulate

7. Which theorem or postulate can be used to show that $\Delta GED \cong \Delta IDE$?



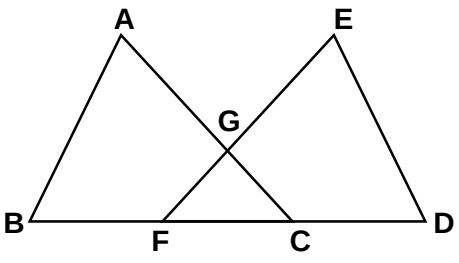
A. AAS Congruence Theorem
B. ASA Congruence Postulate
C. HL Congruence Theorem
D. SAS Congruence Postulate

8. If $\Delta EHT \cong \Delta ATH$, which congruence statement of the corresponding parts of the two triangles is true?



A. $\angle HET \cong \angle AHT$
B. $\angle HET \cong \angle TAH$
C. $\angle TEH \cong \angle ETA$
D. $\angle THE \cong \angle THA$

For items 9 to 12, use the figure at the right. Compete the proof by filling in the missing statements or reasons. Choose the letter of the correct answer from the box.



Given: $\overline{AB} \cong \overline{ED}, \overline{AC} \cong \overline{EF}, \overline{BF} \cong \overline{DC}$

Prove: $\angle ACB \cong \angle EFD$

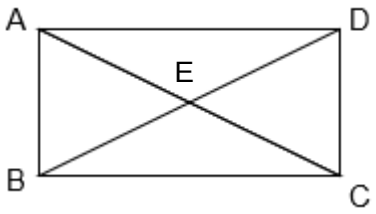
A. $\|BF\| + \|FC\| = \|DC\| + \|CF\|$
B. $\overline{GF} \cong \overline{GC}$
C. Reflexive Property of Equality
D. Transitive Property of Equality
E. $\Delta ABC \cong \Delta EFD$
F. $\Delta ABC \cong \Delta EDF$

Proof:

Statements	**Reasons**
1. $\overline{AB} \cong \overline{ED}$ $\overline{AC} \cong \overline{EF}$ $\overline{BF} \cong \overline{DC}$	1. Given
2. $\overline{FC} \cong \overline{FC}$	2. (9)
3. $\overline{FC} \cong \overline{CF}$	3. Symmetric Property
4. (10)	4. Addition Property of Equality
5. $\|BF\| + \|FC\| = \|BC\|$ $\|DC\| + \|CF\| = \|DF\|$	5. Definition of Betweenness
6. $\|BC\| = \|DF\|$	6. (11)
7. $\overline{BC} \cong \overline{DF}$	7. Definition of Congruent Segments
8. (12)	8. SSS Congruence Postulate
9. $\angle ACB \cong \angle EFD$	9. CPCTC

For items 13 to 15, use the figure at the right. Compete the proof by filling in the missing statements or reasons. Choose the letter of the correct answer form the box.

A. $\overline{BE} \cong \overline{CE}$
B. CPCTC
C. Converse of Isosceles Triangle Theorem
D. SAS Congruence Postulate
E. Definition of Isosceles Triangle
F. Isosceles Triangle Theorem



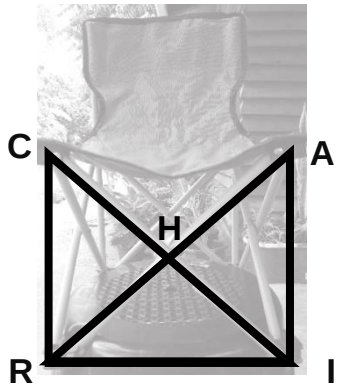
Given: $\overline{AC} \cong \overline{DB}$
$\overline{AB} \cong \overline{DC}$
$\angle BAC \cong \angle CDB$
Prove: ΔBEC is an isosceles triangle

Proof:

Statements	Reasons
1. $\overline{AC} \cong \overline{DB}$ $\overline{AB} \cong \overline{DC}$ $\angle BAC \cong \angle CDB$	1. Given
2. $\Delta BAC \cong \Delta CDB$	2. (13)
3. $\angle ACB \cong \angle DBC$	3. CPCTC
4. $\overline{BE} \cong \overline{CE}$	4. (14)
5. ΔBEC is an isosceles triangle	5. (15)

Additional Activities

Direction: Make a two-column proof based on the situation below.



The front diagonal legs of the fordable chair are congruent and are joined at their midpoints, $\overline{CI} \cong \overline{AR}$. If we draw a line from the corner of the chair to both of the front foot caps, we will have $\overline{CR}$ and $\overline{AI}$ perpendicular to $\overline{RI}$. Prove that $\angle RCI \cong \angle IAR$.

Proof:

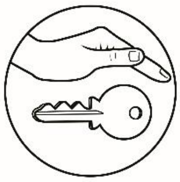
Statements	**Reasons**
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7	7
8.	8.

Question:

1. What are other corresponding parts of ΔCRI and ΔAIR that can be proven congruent also by CPCTC?

Rubrics:

10	8	6	4	2
Able to provide correct , logical, 2 column proof	Able to provide statements and reasons but with 1 erroneous statements or reasons	Able to provide statements and reasons but with 2 erroneous statements or reasons	Able to provide statements and reasons but with at least 3 erroneous statements or reasons	Able to provide only the given statements



Answer Key

What I Know

1. D
2. B
3. D
4. C
5. B
6. D
7. B
8. A
9. D
10. D
11. C
12. B
13. F
14. E
15. A

What's In

1. H	5. G
2. E	6. D
3. F	7. A
4. C	8. B

What's New

I.

Statements	Reasons
1. Square CARE and its diagonal $\overline{CR}$	1.
2. $\overline{CE} \cong \overline{ER} \cong \overline{RA} \cong \overline{AC}$	2.
3. $\overline{CR} \cong \overline{CR}$	3.
4.	4. SSS Congruence Postulate

II.

Statements	Reasons
1.	1. Given
2.	2. Given
3.	3. Definition of Midpoint
4.	4. Definition of Vertical angles
5.	5. Vertical Angle Theorem
6.	6. ASA Congruence Postulate

What's More: Act 1

Statements	Reasons
1. $\overline{LO}$ bisects $\overline{KM}$	1.
2.	2.
3. $\overline{LO} \cong \overline{LO}$	3.
4.	4. Given
5.	5. SAS Congruence Postulate
6.	6. CPCTC

What's More: Act 2

Statements	Reasons
1. Isosceles ΔSTA with vertex $\angle T$	1.
2. $\overline{AT} \cong \overline{ST}$	2.
3. $\overline{TR} \cong \overline{TR}$	3.
4. $\overline{AR} \cong \overline{SR}$	4.
5. $\Delta ATR \cong \Delta STR$	5.
6. $\angle ATR \cong \angle STR$	6.

What's More: Act 3

Statements	Reasons
1.	1.
2. $\angle EAD$ and $\angle FDA$ are right angles	2.
3.	3. Right Angle Theorem
4.	4.
5.	5.
6.	6. Reflexive Property
7.	7.
8.	8.
9.	9.
10.	10. HyL Congruence
11.	11.

What I Have Learned

I have learned that there are two classifications of triangles; that is according to **sides** and according to **sides**. There are some triangles contain characteristics that come from both of these classifications. Few of them are **scalene right triangle** and **isosceles right triangle**.

I also learned that in proving two triangles are congruent, we can also prove the other **corresponding** parts as congruent by **CPCTC**. CPCTC is also known as **corresponding parts of congruent triangles are congruent**. We can prove statements from the congruent triangles like proving **multiple** angles and other sides of triangles.

We can also prove triangles that **overlapped**. **Overlapping** triangles are triangles that have common parts; that is, two triangles that has common sides or angles.

What I Can Do

(Students answer varies.)

Assessment

1. B	6. A	11. D
2. C	7. A	12. F
3. D	8. B	13. D
4. D	9. C	14. C
5. A	10. A	15. E

Additional Activities

Statements	Reasons
1. The front diagonal legs of the fordable chair are congruent and are joined at their midpoints, $\overline{CI} \cong \overline{AR}$	1. Given
2. $\overline{CR}$ and $\overline{AI}$ perpendicular to $\overline{RI}$	2. Given
3. $\angle CRI$ and $\angle AIR$ are right angles	3. Definition of Perpendicular Lines
4. $\angle CRI \cong \angle AIR$	4. Right Angle Theorem
5. ΔCRI and ΔAIR are right triangles	5. Definition of Right Theorem
6. $\overline{RI} \cong \overline{RI}$	6. Reflexive Property
7. $\Delta CRI \cong \Delta AIR$	7. HyL Congruence Theorem
8. $\angle RCI \cong \angle IAR$	8. CPCTC

References

A. Book

1. Emmanuel Abunzo, et. al., *Mathematics 8 Learner's Module* (Pasig City, Philippines:

Book Media Press, Inc. and Prinwell, Inc., 2013).

2. A Baccay, *Geometry for Secondary Schools* (Philippines: Phoenix Publishing House), 140-190.

3. Ricardo Crisostomo, Priscila de Sagun, and Alicia Padua, *Our World of Math* (Quezon City, Philippines: Vibal Publishing House, Inc., 2013).

4. Soledad Dilao and Julieta Bernabe, *Geometry* (Quezon City, Philippines: SD Publication, Inc., 2010).

5. Robert Kalin and Mary Kay Corbitt, Geometry, *Prentice Hall (*Jurong, Singapore: Pearson Education, Inc., 1993).

6. Karl Romero and Freidrich Jose, *Geometry in the Real World: Explorations and Applications (*Makati City, Philippines: Salesiana Publishers, Inc., 2003).

B. Electronic Resources

1. www.mathbitnotebook.com/Geometry/BasicTerms/BTauxiliary.html

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