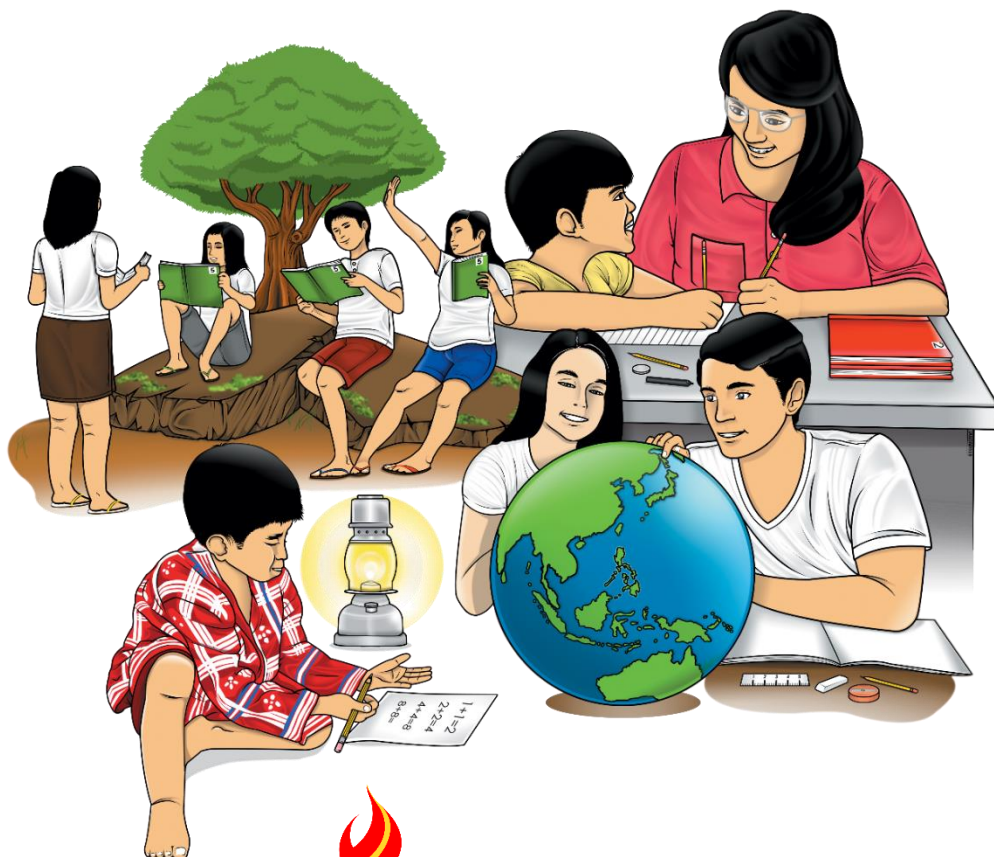


Mathematics

Quarter 3 – Module 6

Proving Two Triangles are Congruent



Mathematics – Grade 8
Alternative Delivery Mode
Quarter 3 – Module 6: Proving Two Triangles are Congruent
First Edition, 2021

Republic Act 8293, section 176 states that: No copyright shall subsist in any work of the Government of the Philippines. However, prior approval of the government agency or office wherein the work is created shall be necessary for exploitation of such work for profit. Such agency or office may, among other things, impose as a condition the payment of royalties.

Borrowed materials (i.e., songs, stories, poems, pictures, photos, brand names, trademarks, etc.) included in this book are owned by their respective copyright holders. Every effort has been exerted to locate and seek permission to use these materials from their respective copyright owners. The publisher and authors do not represent nor claim ownership over them.

Published by the Department of Education
Secretary: Leonor Magtolis Briones
Undersecretary: Diosdado M. San Antonio

Development Team of the Module

Authors:	: Michelle R. Alipao, Fe Gagarin A. Lilangan
Editor:	: Joel B. Asonto
Reviewers	: Ruel C. Alaan. Vicente P. Balbuena
Illustrators	: Mercedita N. Pangilinan, Rachelle T. Tranquilo
Layout Artists	: Jeru Salem O. Cuarteron, Ivin Mae N. Ambos, Darven G. Cinchez
Management Team	: Francis Cesar B. Bringas Isidro M. Biol, Jr. Maripaz F. Magno Josephine Chonie M. Obseñares Josita B. Carmen Celsa A. Casa Regina Euann A. Puerto Bryan L. Arreo Elnie Anthony P. Barcenas Leopardo P. Cortes, Jr. Claire Ann P. Gonzaga

Printed in the Philippines by _____

Department of Education – Caraga Region

Office Address: Learning Resource Management Section (LRMS)
J.P. Rosales Avenue, Butuan City, Philippines 8600
Telefax Nos.: (085) 342-8207 / (085) 342-5969
E-mail Address: caraga@deped.gov.ph

Mathematics

Quarter 3 – Module 6

Proving Two Triangles are Congruent

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.



What I Need to Know

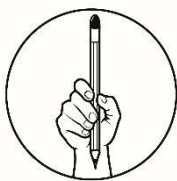
This module is designed for you to understand what it means for two triangles to be congruent and the ways to prove that these triangles are congruent using the theorems and postulates on triangle congruence. This will help you also learn how to prove some theorems of triangle congruence including the right triangles. You will be guided on how to make statements step-by-step and how to make reasons in each corresponding statement. The scope of this module enables you to use it in many different learning situations. The lessons are arranged to follow the standard sequence of the course. But the order in which you read them can be changed to correspond with the textbook you are now using.

This module contains:

Lesson 1: Proving Two Triangles Are Congruent

After going through this module, you are expected to:

1. identify conditions for triangle congruence;
2. use triangle congruence postulates and theorems to prove that two triangles are congruent;
3. use two-column proof in proving that two triangles are congruent; and
4. recognize real-life applications of congruent triangles.



What I Know

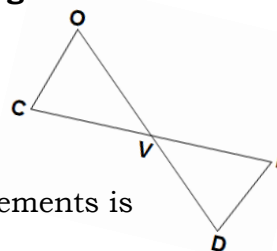
Pre- Assessment:

Directions: Choose the letter of the correct answer. Write your answer on a separate sheet of paper.

- “If two angles and the included side of one triangle are congruent to the corresponding two angles and an included side of another triangle, then the triangles are congruent”. Which postulate proves this statement?
 - AAS Congruence
 - ASA Congruence
 - SAS Congruence
 - SSS Congruence
- Which statement is NOT sufficient to prove the congruence of two triangles?
 - Three angles of one triangle are congruent respectively to three angles of another triangle.
 - Three sides of one triangle are congruent respectively to the three sides of another triangle.
 - Two angles and the included side of one triangle are congruent respectively to the two angles and the included side of another triangle.
 - Two angles and the non-included side of one triangle are congruent respectively to the two angles and the non-included side of another triangle.

For item numbers 3, 4 and 5 refer to the figure at the right.

- \overline{CI} bisects \overline{DO} at V. Which of the following is TRUE?
 - $\overline{OV} \cong \overline{VI}$
 - $\overline{CV} \cong \overline{VD}$
 - $\overline{OV} \cong \overline{VD}$
 - $\angle COV \cong \angle VID$



- \overline{OD} and \overline{CI} bisect each other. Which set of congruence statements is correct?
 - $\overline{VI} \cong \overline{VD}, \overline{CV} \cong \overline{OV}$
 - $\overline{DV} \cong \overline{OV}, \overline{CV} \cong \overline{IV}$
 - $\overline{VC} \cong \overline{VO}, \overline{VD} \cong \overline{VI}$
 - $\overline{VO} \cong \overline{VI}, \overline{VC} \cong \overline{VD}$
- If \overline{OD} and \overline{CI} bisect each other, which set of congruence statements is correct?
 - $\angle CVO \cong \angle IVD, \overline{OV} \cong \overline{VI}, \overline{CV} \cong \overline{VD}$
 - $\angle CVO \cong \angle IVD, \overline{OV} \cong \overline{VD}, \overline{CV} \cong \overline{VI}$
 - $\angle CVO \cong \angle IVD, \angle VCO \cong \angle VID, \overline{CV} \cong \overline{VD}$
 - $\angle CVO \cong \angle IVD, \angle VOC \cong \angle VDI, \overline{OV} \cong \overline{VI}$

6. Refer to the figure at the right, $\overline{OV} \perp \overline{DE}$ at V. Which set of congruence statements can be used to prove that $\triangle DVO \cong \triangle EVO$ by HyA Congruence Theorem?

A. $\overline{OE} \cong \overline{OD}$, $\overline{DV} \cong \overline{EV}$
 B. $\overline{OE} \cong \overline{OD}$, $\overline{OV} \cong \overline{OV}$
 C. $\overline{OD} \cong \overline{OE}$, $\angle ODV \cong \angle OEV$
 D. $\overline{OD} \cong \overline{OE}$, $\angle DVO \cong \angle EVO$

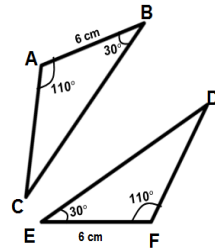
For item numbers 7 and 8, consider the figures at the right.

7. Which one states the congruence of the two triangles?

A. $\triangle ABC \cong \triangle EFD$ C. $\triangle ABC \cong \triangle FED$
 B. $\triangle ABC \cong \triangle EDF$ D. $\triangle ABC \cong \triangle FDE$

8. Which theorem or postulate can prove the congruence of the two triangles?

A. AAS Congruence C. SAS Congruence
 B. ASA Congruence D. SSS Congruence



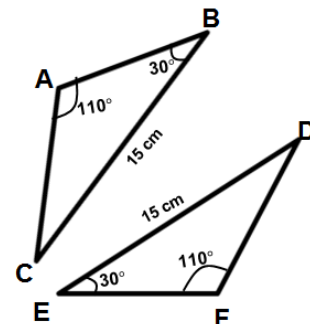
9. It is given that $\triangle CAN \cong \triangle FED$. Which of the following statement/s is/are true?

i. $\angle CAN \cong \angle FED$ ii. $\angle ANC \cong \angle EDF$ iii. $\overline{CA} \cong \overline{FE}$

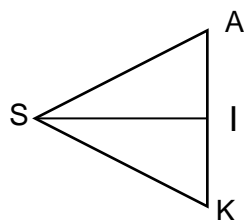
A. i and ii C. ii and iii
 B. i and iii D. i, ii, and iii

10. The figure at the right shows that $\triangle ABC \cong \triangle FED$. Which theorem or postulate supports this statement?

A. AAS Congruence
 B. ASA Congruence
 C. SAS Congruence
 D. SSS Congruence



For item numbers 11 and 12, use the figure below.



11. Which of the following statements is NOT correct if I is the midpoint of \overline{AK} and $\overline{SI} \perp \overline{AK}$ at I?

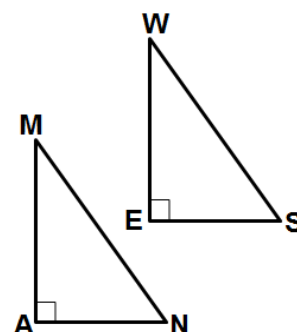
A. $\angle SAK$ and $\angle SKA$ are right angles C. $\overline{SI} \cong \overline{SI}$
 B. $\angle SIA$ and $\angle SIK$ are right angles D. $\overline{AI} \cong \overline{KI}$

12. Given: \overline{SI} bisects $\angle ASK$, $\overline{SI} \perp \overline{AK}$ at I , which of the following statements about triangle congruence is true?

- A. $\triangle ISA \cong \triangle ISK$ by ASA Congruence
- B. $\triangle ISA \cong \triangle IKS$ by SAS Congruence
- C. $\triangle SIA \cong \triangle SKI$ by ASA Congruence
- D. $\triangle SIA \cong \triangle SKI$ by SAS Congruence

13. Triangles MAN and SEW are right triangles. If $\overline{AN} \cong \overline{ES}$ and $\overline{NM} \cong \overline{SW}$, what special right triangle theorem will prove that $\triangle MAN \cong \triangle WES$?

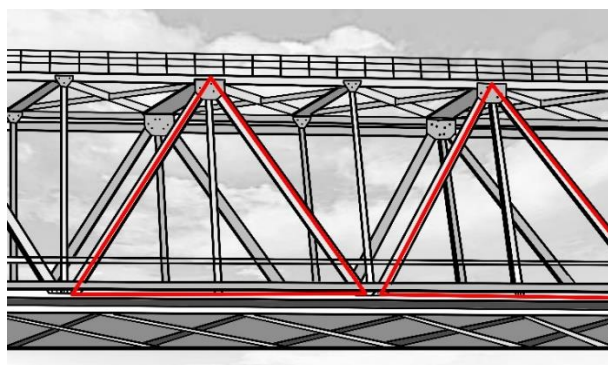
- A. HyA Congruence Theorem
- B. HyL Congruence Theorem
- C. LA Congruence Theorem
- D. LL Congruence Theorem



Congruent triangles are commonly seen in buildings and other structures such as bridges, towers and the like. The figure below is a bridge showing congruent triangles.

For items 14 and 15, refer to the image at the right. Complete the proof below by choosing the letter of the correct answer.

- A. ASA Congruence Postulate
- B. SAS Congruence Postulate
- C. SSS Congruence Postulate
- D. $\overline{RA} \cong \overline{TA}$
- E. $\overline{SA} \cong \overline{SA}$
- F. $\overline{ST} \cong \overline{ST}$



Given: $\overline{SR} \cong \overline{ST}$
 \overline{SA} bisects $\angle RST$

Prove: $\triangle SRA \cong \triangle STA$

Proof:

Statements	Reasons
1. $\overline{SR} \cong \overline{ST}$	1. Given
2. \overline{SA} bisects $\angle RST$	2. Given
3. $\angle RSA \cong \angle TSA$	3. Definition of Angle Bisector
4. (14)	4. Reflexive Property
5. $\triangle SRA \cong \triangle STA$	5. (15)

Lesson

1

Proving Two Triangles are Congruent

A new tree park site will be built in your school in which it will contain three triangular gardens. The design for the tree park is shown on the graphing paper.

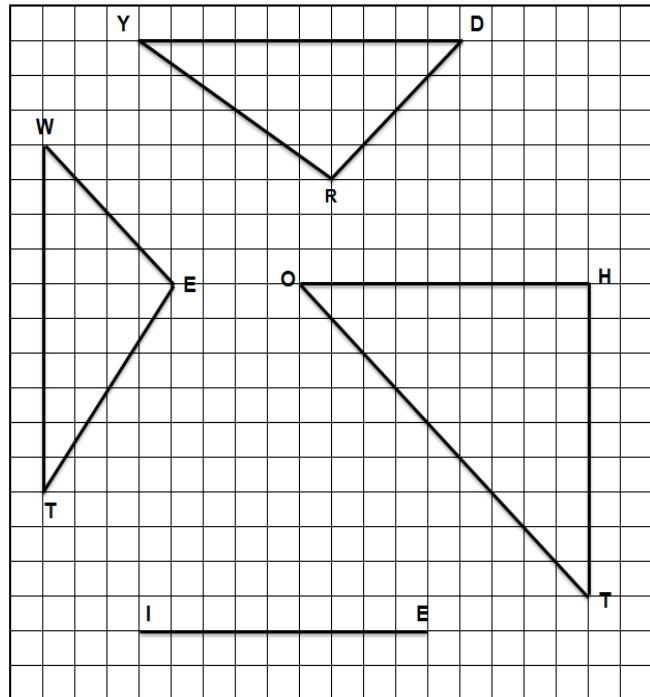
Does garden DRY and WET have the same sizes and shapes? Are they congruent?

If yes, how can we prove that the two triangles are congruent?

How can we apply triangle congruence theorem to prove that the two triangles are congruent?

Let us first review the previous topic by answering the activity in the next page.

Enjoy!





What's In

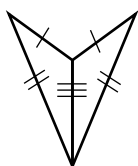
Activity: Match Me!

Directions: Match each figure in Column A with the corresponding triangle congruence postulate in column B. Choose the letter of the correct answer. Use a separate sheet of paper.

Column A

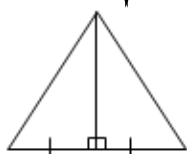
Column B

_____ 1.



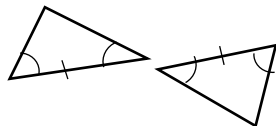
A. Angle-Side-Angle (ASA)

_____ 2.



B. Side-Angle-Side (SAS)

_____ 3.

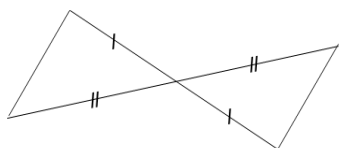


C. Side-Side-Side (SSS)

_____ 4.



_____ 5.



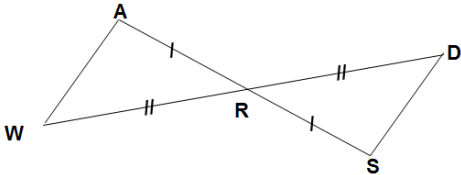
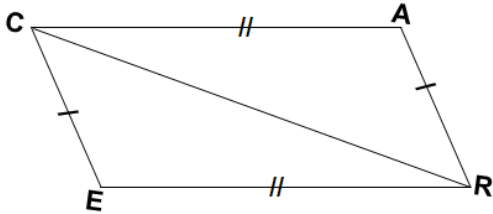
Questions:

1. How did you find the activity? Do you find it difficult to identify what congruence postulate is illustrated in each figure?
2. What is your basis in determining which congruence postulate is illustrated in each given figure?



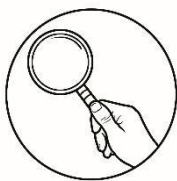
What's New

Directions: List down the corresponding congruent parts of each pair of congruent triangles and answer the questions that follow.

Congruent Triangles	Corresponding Congruent parts
<p>1.</p> 	
<p>2.</p> 	

Questions:

- Were you able to list down the corresponding congruent parts of each pair of congruent triangles? If yes, what is your basis in identifying the congruent parts?
- Are the corresponding congruent parts enough to prove that the two triangles are congruent?
- What congruence postulate can be used to prove the congruence of each pair of triangles?



What is It

In the previous lesson, you learned that triangles are congruent if they have exactly the same shape and size. Specifically, two triangles are congruent if and only if their vertices can be made to coincide so that corresponding sides are congruent and corresponding angles are congruent. However, in certain cases, if the three pairs of corresponding parts are congruent, then it is sufficient to prove that two triangles are congruent.

Let us now recall the conditions under which two triangles are congruent.

1. SAS (Side-Angle-Side) Congruence Postulate

If two sides and an included angle of one triangle are congruent to the corresponding two sides and the included angle of another triangle, then the triangles are congruent.

2. ASA (Angle-Side-Angle) Congruence Postulate

If two angles and the included side of one triangle are congruent to the corresponding two angles and an included side of another triangle, then the triangles are congruent.

3. SSS (Side-Side-Side) Congruence Postulate

If three sides of one triangle are congruent to the corresponding three sides of another triangle, then the triangles are congruent.

From these congruence postulates mentioned above, other theorems can be deduced.

Suppose you are given the measures of two angles and a non-included side, are these information sufficient to prove the congruence of the two triangles?

AAS (Angle-Angle-Side) Congruence Theorem

If two angles and the non-included side of one triangle are congruent respectively to the two angles and the non-included side of another triangle, then the two triangles are congruent.

In Figure 1, given that $\triangle DOG \cong \triangle CAT$, $\angle D \cong \angle C$, $\angle G \cong \angle T$ as marked, while $\overline{DO} \cong \overline{CA}$ tells the congruence of the non-included sides. The proof of this theorem is presented in Example 3.

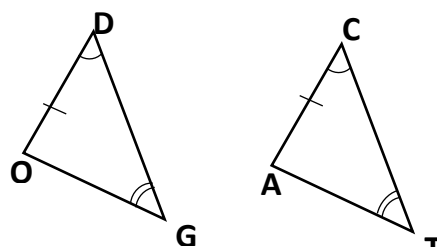


Figure 1

Another theorem that can be deduced from the congruence postulates are the congruence of right triangles. Below are the illustrations and descriptions of each theorem.

HyA (Hypotenuse-Acute angle) Congruence Theorem

If the hypotenuse and an acute angle of one right triangle are congruent to the corresponding hypotenuse and an acute angle of another right triangle, then the triangles are congruent.

This theorem can be applied to right triangles only. Figure 2 at the right shows two right triangles $\triangle RGA$ and $\triangle MSA$. The hypotenuses \overline{AR} and \overline{AM} are congruent as marked, $\angle RGA$ and $\angle MSA$ are congruent because they are right angles, $\angle GAR$ and $\angle SAM$ are acute angles and $\angle GAR \cong \angle SAM$ because they are vertical angles. Using these congruent parts, the two triangles are congruent by **HyA Congruence Theorem**. The proof of this theorem is shown in Example 4.

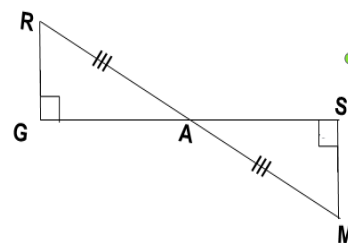


Figure 2

HyL (Hypotenuse-leg) Congruence Theorem

If the hypotenuse and a leg of one right triangle are congruent to the corresponding hypotenuse and a leg of another triangle, then the triangles are congruent.

Again, this theorem can be applied to right triangles only. As shown in Figure 3 at the right, $\triangle USB \cong \triangle ARC$. \overline{US} and \overline{AR} are legs, and \overline{UB} and \overline{AC} are the hypotenuses of the given triangles, $\overline{US} \cong \overline{AR}$ and $\overline{UB} \cong \overline{AC}$ as marked, while $\angle S \cong \angle R$, because both are right angles. The two right triangles are congruent by **HyL Congruence Theorem**. The proof of this is found in Example 5.

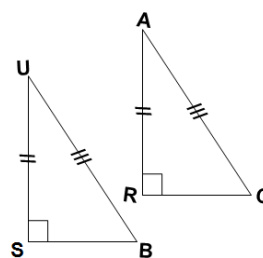


Figure 3

LA (leg-acute angle) Congruence Theorem

If a leg and an acute angle of one right triangle are congruent to a leg and an acute angle of another right triangle, then the triangles are congruent.

In the two right triangles in Figure 4, \overline{GU} and \overline{RA} are the congruent legs as marked, and $\angle M$ and $\angle P$ are congruent acute angles as marked, and $\angle G \cong \angle R$ because both are right angles. Hence, \overline{GU} and $\angle M$ are congruent respectively to \overline{RA} and $\angle P$. The two right triangles are congruent by LA Congruence Theorem. The proof of this theorem is shown in Example 6.

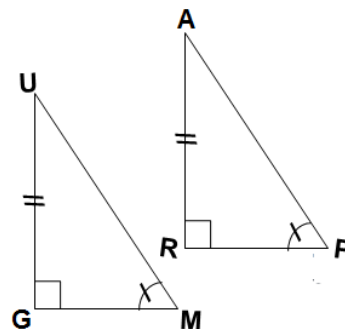


Figure 4

LL (Leg Leg) Congruence Theorem

If the legs of one right triangle are congruent to the legs of another right triangle, then the triangles are congruent.

In the right triangles in Figure 5 at the right, \overline{AB} and \overline{OC} are the longer legs while \overline{BG} and \overline{CP} are the shorter legs of the two triangles. $\overline{AB} \cong \overline{OC}$ and $\overline{BG} \cong \overline{CP}$ as marked. Hence, we can conclude that the two legs of $\triangle BAG$ are congruent to the two legs of $\triangle COP$ so, $\triangle BAG \cong \triangle COP$ by LL Congruence Theorem. The proof of this theorem is shown in Example 7.

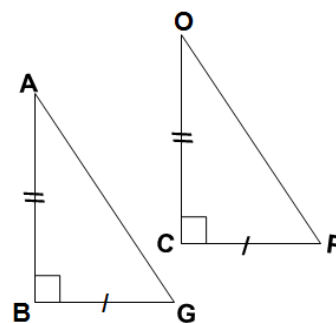


Figure 5

How do we prove that two triangles are congruent? How do we prove the different triangle congruence theorems? Here are the steps in proving that two triangles are congruent.

Steps in Proving Congruence of Two Triangles

Step 1. Identify what the given are, and what is to be proved. Mark the given information on

the diagram.

Step 2. Identify the congruence theorem to be used and the additional information needed

and why.

Now, let us apply the congruence postulates and theorems in proving congruent triangles using the illustrative examples below.

Example 1. Prove that $\triangle RAW \cong \triangle RSD$ in Figure 6 at the right.

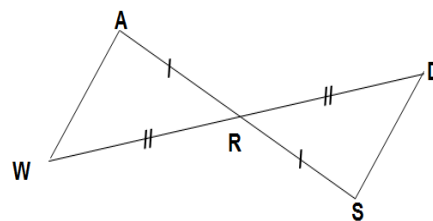


Figure 6

Step 1. Identify what the given are and what is to be proved.

Given: $\overline{AR} \cong \overline{SR}$, $\overline{WR} \cong \overline{DR}$

Prove: $\triangle RAW \cong \triangle RSD$

Step 2. Identify the congruence theorem to be used and the additional information needed and why.

$\angle WRA$ and $\angle DRS$ are vertical angles so they are congruent by Vertical Angle Theorem, hence **SAS congruence postulate can be used to prove $\triangle RAW \cong \triangle RSD$.**

Step 3. Write down the statements and the reasons in a two-column proof. Make sure the last statement contains what should be proved.

Two-column proof:

Statements	Reasons
1). $\overline{AR} \cong \overline{SR}$, $\overline{WR} \cong \overline{DR}$	1). Given
2). $\angle WRA$ and $\angle DRS$ are vertical angles	2). Definition of vertical angles
3). $\angle WRA \cong \angle DRS$	3). Vertical Angle Theorem
4). $\triangle RAW \cong \triangle RSD$	4). SAS Congruence Postulate

We were able to show the congruence of the two triangles using **SAS congruence postulate**.

Example 2. Based on Figure 7 at the right, prove that $\triangle REC \cong \triangle CAR$.

Step 1. Identify what the given are and what is to be proved.

Given: $\overline{RE} \cong \overline{CA}$, $\overline{EC} \cong \overline{AR}$

Prove: $\triangle REC \cong \triangle CAR$

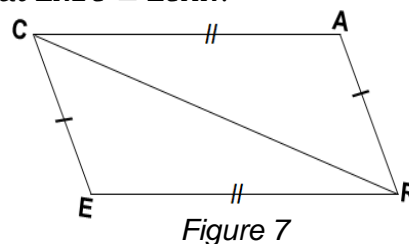


Figure 7

Step 2. Identify the congruence theorem to be used and the additional information needed and why.

\overline{CR} is the common side of $\triangle REC$ and $\triangle CAR$, so by reflexive property. Hence, **SSS postulate can be used to prove** $\triangle REC \cong \triangle CAR$ because each of the three sides of $\triangle REC$ is congruent respectively to the three sides of $\triangle CAR$.

Step 3. Write down the statements and the reasons in a two-column proof. Make sure the last statement contains what should be proved.

Two-column proof:

Statements	Reasons
1). $\overline{RE} \cong \overline{CA}$, $\overline{EC} \cong \overline{AR}$	1). Given
2). $\overline{CR} \cong \overline{CR}$	2). Reflexive Property
3). $\triangle REC \cong \triangle CAR$	3). SSS Congruence Postulate

Example 3. In the Figure 8 at the right, prove that

$$\triangle DOG \cong \triangle CAT$$

Step 1. Identify what the given are and what is to be proved

Given: $\overline{DO} \cong \overline{CA}$, $\angle D \cong \angle C$, $\angle G \cong \angle T$

Prove: $\triangle DOG \cong \triangle CAT$

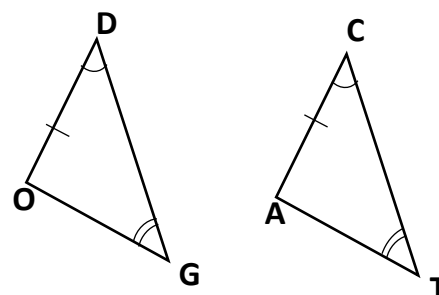


Figure 8

Step 2. Identify the congruence theorem to be used and the additional information needed and why.

Since it is given that $\angle D \cong \angle C$, $\angle G \cong \angle T$, then by Angle Sum Theorem, $\angle O \cong \angle A$.

Angle Sum Theorem states that the sum of the measures of the angles of a triangle is 180° . Because the two corresponding pairs of angles are already congruent, so the third pair of angles are also congruent.

Thus, the two triangles can be proven either by **ASA congruence postulate** or by **AAS congruence theorem**.

Step 3. Write down the statements and the reasons in a two-column proof. Make sure the last statement contains what should be proved.

Two-column proof:

Statements	Reasons
1). $\overline{DO} \cong \overline{CA}$, $\angle D \cong \angle C$, $\angle G \cong \angle T$	1). Given
2). $\triangle DOG \cong \triangle CAT$	2). AAS Congruence Theorem

or

1). $\overline{DO} \cong \overline{CA}$, $\angle D \cong \angle C$, $\angle G \cong \angle T$	1). Given
2). $m\angle D = m\angle C$; $m\angle G = m\angle T$	2). Definition of Congruent Angles
3). $m\angle D + m\angle O + m\angle G = 180^\circ$ $m\angle C + m\angle A + m\angle T = 180^\circ$	3). Angle Sum Theorem In a triangle
4a). $m\angle O = 180^\circ - m\angle D - m\angle G$ 4b). $m\angle A = 180^\circ - m\angle C - m\angle T$	4). Addition Property of Equality
5). $m\angle A = 180^\circ - m\angle D - m\angle G$	5). Substitution Property (2)
6). $m\angle O = m\angle A$	6). Transitive Property of Equality (4a & 5)
7). $\angle O \cong \angle A$	7). Converse of the Definition of Congruent Angles
8). $\triangle DOG \cong \triangle CAT$	8). ASA Congruence Postulate

You have noticed that in this example, we were able to prove the congruence of the given triangles by ASA postulate and at the same time by **AAS theorem**.

Let us try to use AAS congruence theorem in proving triangle congruence in the next example.

Example 4. In the right triangles in Figure 9 at the right, prove that $\triangle ARG \cong \triangle AMS$.

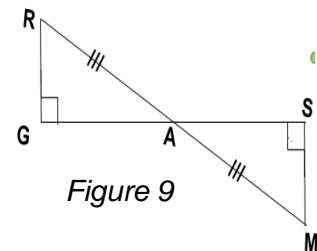
Step 1. Identify what the given are and what is to be proved.

Given: $\overline{RA} \cong \overline{MA}$, $\angle G$ and $\angle S$ are right angles

Prove: $\triangle ARG \cong \triangle AMS$

Step 2. Identify the congruence theorem to be used and the additional information needed and why.

Since \overline{RM} intersects \overline{GS} , vertical angles are formed, and vertical angles are congruent. Thus, AAS congruence theorem can be used to prove $\triangle ARG \cong \triangle AMS$.



Step 3. Write down the statements and the reasons in a two-column proof. Make sure the last statement contains what should be proved.

Two-column proof:

Statements	Reasons
1). $\overline{RA} \cong \overline{MA}$	1). Given
2). $\angle G$ and $\angle S$ are right angles	2). Given
3). $\angle G \cong \angle S$	3). Right Angle Theorem
4). $\angle RAG$ and $\angle MAS$ are vertical angles	4). Definition of Vertical Angles
5). $\angle RAG \cong \angle MAS$	5). Vertical Angles Theorem
6). $\triangle ARG \cong \triangle AMS$	6). AAS Congruence Theorem

Hence, the two triangles are congruent by **AAS theorem**.

Example 5. Figure 10 at the right shows that the hypotenuses and legs of the two right triangles are given to be congruent respectively. Prove that $\triangle BUS \cong \triangle CAR$.

Step 1. Identify what the given are and what is to be proved

Given: $\overline{US} \cong \overline{AR}$, $\overline{BU} \cong \overline{CA}$, $\angle S$ and $\angle R$ are right angles

Prove: $\triangle BUS \cong \triangle CAR$

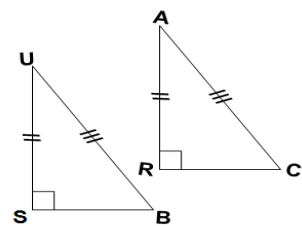


Figure 10

Step 2. Identify the congruence theorem to be used and the additional information needed and why.

The congruence of the $\triangle BUS$ and $\triangle CAR$ can be proven using SSS congruence postulate and HyL congruence theorem.

Case 1): In using the SSS congruence postulate, there is a need to show the congruence of \overline{SB} and \overline{RC} using the Pythagorean equation since the given triangles are right triangles.

Case 2); $\triangle BUS \cong \triangle CAR$ can also be proven using the HyL congruence theorem because the given triangles are right triangles.

Step 3. Write down the statements and the reasons in a two-column proof. Make sure the last statement contains what should be proved.

Two-column proof:**Case 1:**

Statements	Reasons
1). $\overline{US} \cong \overline{AR}$, $\overline{BU} \cong \overline{CA}$	1). Given
2). $ US = AR $, $ BU = CA $	2) Definition of congruent segments
3) $\angle S$ and $\angle R$ are right angles.	3). Given
4). $\triangle BUS$ and $\triangle CAR$ are right triangles.	4). Definition of right triangle.
5a). $ BU ^2 = US ^2 + SB ^2$ 5b). $ CA ^2 = AR ^2 + RC ^2$	5). Pythagorean Theorem
6a). $ BU ^2 - US ^2 = SB ^2$ 6b). $ CA ^2 - AR ^2 = RC ^2$	6). Addition Property of Equality

7). $ BU ^2 - US ^2 = RC ^2$	7). Substitution Property (2)
8a). $ SB = \sqrt{ BU ^2 - US ^2}$	8). Raising Each Side to a Power Property
8b). $ RC = \sqrt{ BU ^2 - US ^2}$	
9). $ SB = RC $	9). Transitive Property
10). $\overline{SB} \cong \overline{RC}$	10). Converse of the Definition of Congruent segments
11). $\triangle BUS \cong \triangle CAR$	11). SSS Congruence Postulate

Case 2:

Statements	Reasons
1). $\overline{US} \cong \overline{AR}$, $\overline{BU} \cong \overline{CA}$	1). Given
2) $\angle S$ and $\angle R$ are right angles.	2). Given
3). $\angle S \cong \angle R$	3). Right Angle Theorem
4). $\triangle BUS$ and $\triangle CAR$ are right triangles.	4). Definition of right triangle.
5). \overline{US} and \overline{AR} are legs of the right triangles	5). Definition of leg of a right triangle. (A side of a right triangle opposite an acute angle is called leg.)
6). \overline{BU} and \overline{CA} are the hypotenuses of the right triangles.	6). Definition of hypotenuse.
7). $\triangle BUS \cong \triangle CAR$	7).) HyL Congruence Theorem

Example 6. In Figure 11 at the right, corresponding congruent parts of the right triangles are similarly marked. Prove that $\triangle GUM \cong \triangle RAP$.

Step 1. Identify what the given are and what is to be proved,

Given: $\overline{GU} \cong \overline{RA}$, $\angle M \cong \angle P$, $\angle G$ and $\angle R$ are right angles

Prove: $\triangle GUM \cong \triangle RAP$

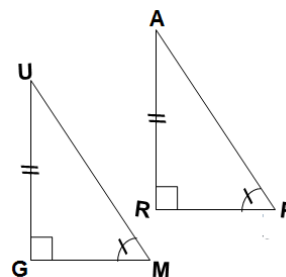


Figure 11

Step 2. Identify the congruence theorem to be used and the additional information needed and why.

By the Right-Angle theorem, $\angle G \cong \angle R$, plus the given above, we can prove that $\triangle GUM \cong \triangle RAP$ by **AAS congruent theorem**. And since the given triangles are right triangles, $\triangle GUM \cong \triangle RAP$ can be proven by **LA congruence theorem**.

Step 3. Write down the statements and the reasons in a two-column proof. Make sure the last statement contains what should be proved.

Two-column proof:

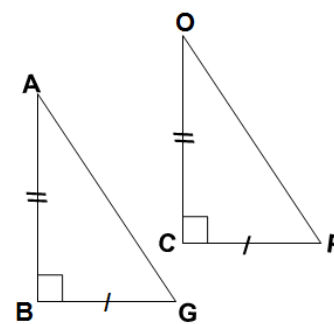
Statements	Reasons
1). $\overline{GU} \cong \overline{RA}$, $\angle M \cong \angle P$	1). Given
2). $\angle G$ and $\angle R$ are right angles	2). Given
3). $\angle G \cong \angle R$	3). Right Angle Theorem
4). \overline{GU} and \overline{RA} are legs of the right triangles.	4). Definition of leg of a right triangle Leg is a side of a right triangle opposite an acute angle.
5). $\angle M$ and $\angle P$ are acute angles in the right triangles	5). Acute angle in a right triangle is opposite to a leg.
6). $\triangle GUM \cong \triangle RAP$	6). AAS Congruence Theorem
7). $\triangle GUM \cong \triangle RAP$	7) LA Congruence Theorem

Example 7. The two legs of $\triangle BAG$ are congruent respectively to the two legs of $\triangle COP$. The two triangles are right. Prove that $\triangle BAG \cong \triangle COP$

Step 1. Identify what the given are and what is to be proved

Given: $\overline{BA} \cong \overline{CO}$, $\overline{BG} \cong \overline{CP}$, $\triangle ABG$ and $\triangle OCP$ are right triangles

Prove: $\triangle BAG \cong \triangle COP$



Step 2. Identify the congruence theorem to be used and the additional information needed and why.

Since the two triangles, $\triangle ABG$ and $\triangle OCP$ are right, hence $\angle B \cong \angle C$. Based on the given congruent parts, $\triangle BAG \cong \triangle COP$ by **SAS congruence postulate**.

The congruence of the corresponding legs of the two triangles will lead us to show that $\overline{AG} \cong \overline{OP}$ through the use of Pythagorean Theorem. So, $\triangle BAG \cong \triangle COP$ can also be proven using SSS Congruence Postulate.

And since the corresponding legs of the right triangles are congruent, it can also be proven by **LL congruence theorem, as shown in step 3 below**

Step 3. Write down the statements and the reasons in a two-column proof. Make sure the last statement contains what should be proved.

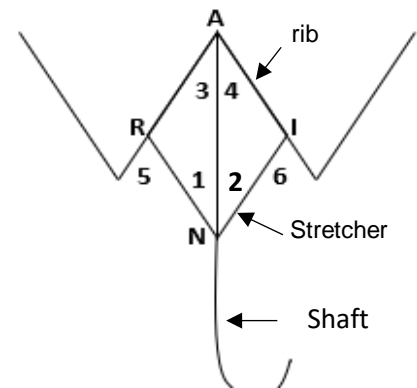
Two-column proof:

Statements	Reasons
1). $\overline{BA} \cong \overline{CO}$, $\overline{BG} \cong \overline{CP}$	1). Given
2). $\triangle ABG$ and $\triangle OCP$ are right triangles	2). Given
3). $\angle B$ and $\angle C$ are right angles	3). Given in the figure. Definition of right triangle.
4). $\angle B \cong \angle C$	4). Right-Angle Theorem

5). \overline{BA} and \overline{CO} , \overline{BG} and \overline{CP} are pairs of corresponding legs of the right triangles	5). Legs are sides of a right triangle opposite the acute angles.
6). $\triangle BAG \cong \triangle COP$	6). LL Congruence Theorem

Now let us try to explore examples of triangle congruence in the real-world and prove that these triangles are congruent.

Example 8. Shapes formed by the ribs, stretchers, and shaft are congruent whether an umbrella is open or closed. Prove that the triangles formed by the shaft and the ribs are congruent.



Step 1. Identify what the given are and what is to be proved

Given: Quadrilateral RAIN is a parallelogram, and \overline{AN} is a diagonal.

Prove: $\triangle ARN \cong \triangle INA$

Step 2. Identify the congruence theorem to be used and the additional information needed and why.

The congruence between $\triangle ARN$ and $\triangle INA$ can be proven by SSS, ASA, and SAS congruence postulates.

Step 3. Write down the statements and the reasons in a two-column proof. Make sure the last statement contains what should be proved.

Proof: Case 1:

Statements	Reasons
1). Quadrilateral RAIN is a parallelogram.	1). Given
2). \overline{AN} is a diagonal.	2). Given
3). $\overline{AR} \cong \overline{NI}$ and $\overline{RN} \cong \overline{IA}$	Opposite sides of a parallelogram are congruent.
4). $\overline{AN} \cong \overline{AN}$	4). Reflexive Property
5). $\triangle ARN \cong \triangle INA$	5). SSS Congruence Postulate

Proof: Case 2:

Statements	Reasons
1). Quadrilateral RAIN is a parallelogram.	1). Given
2). \overline{AN} is a diagonal.	2). Given
3). $\angle R \cong \angle I$	Opposite angles of a parallelogram are congruent.
4). $\overline{AR} \cong \overline{NI}$ and $\overline{RN} \cong \overline{IA}$	Opposite sides of a parallelogram are congruent.
5). $\triangle ARN \cong \triangle NIA$	5). SAS Congruence Postulate

Proof: Case 3:

1). Quadrilateral RAIN is a parallelogram.	1). Given
2). \overline{AN} is a diagonal.	2). Given
3). $\overline{AR} \parallel \overline{NI}$ and $\overline{RN} \parallel \overline{IA}$	Opposite sides of a parallelogram are parallel.
3). $\angle 1 \cong \angle 4$ and $\angle 2 \cong \angle 3$	Alternate interior angles are congruent if parallel lines are cut by a transversal
4). $\overline{AN} \cong \overline{AN}$	4). Reflexive Property
5). $\triangle ARN \cong \triangle NIA$	5). ASA Congruence Postulate

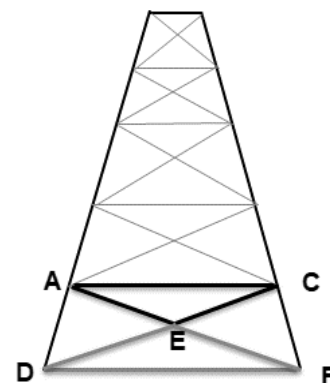
Example 9. Determine if triangles are congruent. Consider the figure at the right. The diagonals of the tower form triangles. Consider $\triangle AEC$ and $\triangle FED$ as illustrated.

Step 1. Identify what the given are and what is to be proved.

Given: Quadrilateral ADFC is an isosceles trapezoid, and $\angle EAC \cong \angle EFD$.

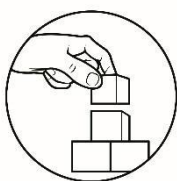
Prove: $\triangle AEC \cong \triangle FED$

Step 2. Identify the congruence theorem to be used and the additional information needed and why.



$\angle EAC \cong \angle EFD$	given congruent angles
$\angle ECA \cong \angle EDF$	congruent because Alternate Interior Angles
$\angle AEC \cong \angle FED$	congruent by Vertical Angle Theorem
\overline{AC} and \overline{DF} are not congruent	Parallel sides of a trapezoid are not congruent.
\overline{AE} and \overline{EF} are not congruent \overline{CE} and \overline{ED} are not congruent	Diagonals of an isosceles trapezoid do not bisect each other.
$\triangle AEC$ and $\triangle DEF$ are not congruent.	Notice that the three pairs corresponding angles of the two triangles are congruent. However, this fact will not be sufficient to prove triangle congruence

The Angle-Angle-Angle (AAA) combination shows that the two triangles may have the same shape but different sizes. In the figure, $\triangle AEC$ and $\triangle FED$ have corresponding angles congruent but their sizes are not the same, hence their corresponding sides are not congruent. Therefore, $\triangle AEC$ and $\triangle FED$ are not congruent.

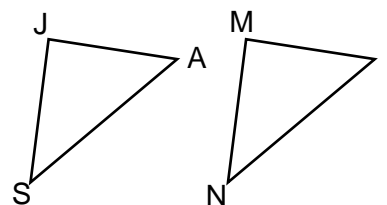


What's More

Activity 1: Give Me!

- I. Using the given congruent parts, state the postulate or theorem that proves $\triangle JAS \cong \triangle MIN$

- $\overline{JS} \cong \overline{MN}$, $\angle J \cong \angle M$, $\overline{JA} \cong \overline{MI}$
- $\angle S \cong \angle N$, $\angle A \cong \angle I$, $\overline{SA} \cong \overline{NI}$
- $\angle J \cong \angle M$, $\angle A \cong \angle I$, $\overline{SA} \cong \overline{NI}$

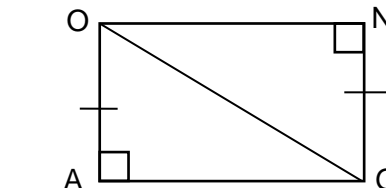
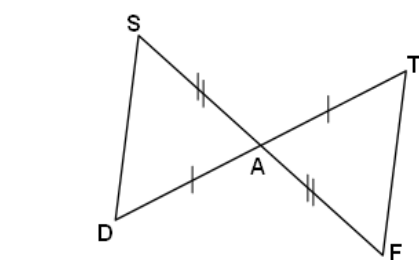
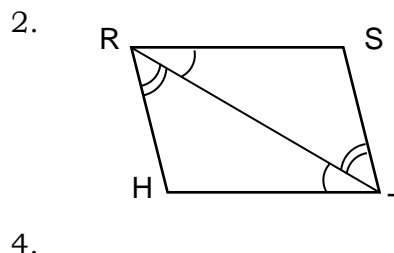
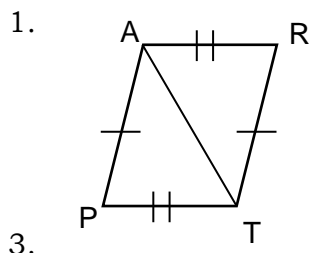


- II. Give additional congruent parts needed to prove that $\triangle CAR \cong \triangle TOY$ by the indicated postulate.

- $\overline{AR} \cong \overline{OY}$, $\angle R \cong \angle Y$; SAS
- $\angle C \cong \angle T$, $\overline{AC} \cong \overline{OT}$; ASA
- $\overline{TO} \cong \overline{CA}$, $\overline{TY} \cong \overline{CR}$; SSS

III.

In each figure, congruent parts are marked. Give additional parts to prove that the triangles are congruent and name the postulate or theorem that justifies the congruence.

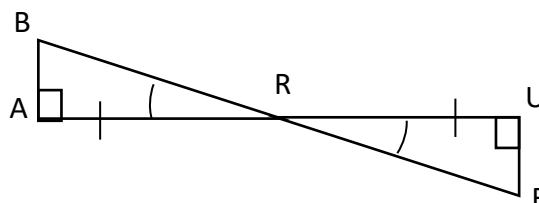


Questions:

1. How did you identify the postulate or theorem in part I?
2. How did you identify the additional part to prove that the two triangles are congruent in Part II?
3. Were you able to give an additional congruent part in Part III to prove that the two triangles are congruent? If yes, how?

Activity 2: Fill Me Up

Given: R is the midpoint of \overline{AU}
 $\angle A$ and $\angle U$ are right angles
 Prove: $\triangle BAR \cong \triangle FUR$



Proof: (Complete the table)

Statements	Reasons
1. R is the midpoint of \overline{AU}	1.
2.	2. Definition of Midpoint
3.	3. Given
4. $\angle A \cong \angle U$	4.
5. $\angle BRA$ and $\angle FRU$ are vertical angles	5. Definition of Vertical Angles
6.	6. Vertical Angle Theorem
7. $\triangle BAR \cong \triangle FUR$	7.

Questions:

- 1). What are the identified congruent corresponding parts?
- 2). What congruence postulate and congruence theorem can be used to prove that

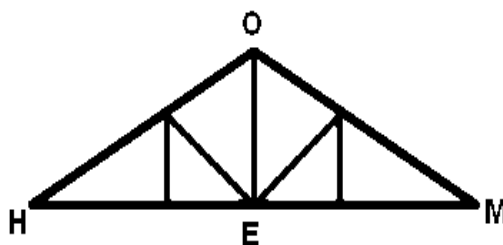
$$\triangle BAR \cong \triangle FUR ?$$

Activity 3. My Home

Trusses in our houses form congruent triangles similar to the picture at the right.

Given: $\overline{HO} \cong \overline{MO}$, $\overline{HE} \cong \overline{ME}$

Prove: $\triangle HOE \cong \triangle MOE$

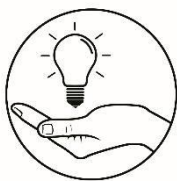


Proof:

Statements	Reasons
1. $\overline{HO} \cong \overline{MO}$, $\overline{HE} \cong \overline{ME}$	1.
2.	2. Reflexive Property
3. $\triangle HOE \cong \triangle MOE$	3.

Questions:

1. How do you find the activity? Did you find it difficult to prove the congruence of two triangles?
2. Why do you think the trusses in our houses form congruent triangles?
3. List down at least five things in your house, buildings or other structures found in your community where congruent triangles are used. Explain the importance of congruent triangles in these structures.



What I Have Learned

PROVING CONGRUENT TRIANGLES

Name and describe the triangle congruence postulates and theorems by filling up the blanks with the correct answer

Fill each blank with correct answer.

Two triangles are congruent if and only if their _____ can be made to coincide such that corresponding _____ are congruent and corresponding _____ are congruent.

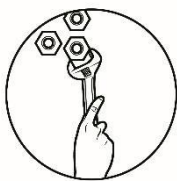
Instead of showing the six corresponding congruent parts, the congruence postulates and theorems reduce these into three. The Congruence Postulates are _____, _____, and _____.

The Congruence Theorems are _____, _____, _____, _____, and _____. The AAS Congruence Triangle Theorem is deduced from _____ Congruence Postulate. _____ and _____ are theorems which can be proved using the AAS Congruence Theorems while _____ and _____ are theorems which can be proved

Illustrate the three triangle congruence postulates

Answer this!

Why do congruent triangles appear in many structures such as buildings and towers?



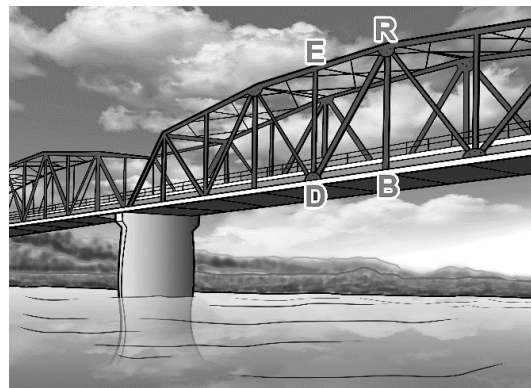
What I Can Do

The **Buntun bridge** spans the Cagayan River. It is 1,369 m long and said to be the longest river **bridge** in the country. It was opened in 1947 and has been known as the gateway to the City of Tuguegarao. The **bridge** has never been flooded by the river but it almost did when typhoons Ondoy and Juan hit the city. One of the reasons why it still stands magnificently until today because of its structures that are made of congruent triangles.

In the figure at the right, it is given that

$$\overline{ED} \cong \overline{RB} \text{ and } \overline{ED} \parallel \overline{RB}.$$

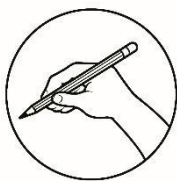
Prove that $\triangle RED \cong \triangle DBR$ by completing the two-column proof below.



Proof:

Buntun Bridge

Statements	Reasons
1. $\overline{ED} \cong \overline{RB}$ $\overline{ED} \parallel \overline{RB}$	1.
2. $\angle BRD$ & $\angle EDR$ are alternate interior angles	2.
3. $\angle BRD \cong \angle EDR$	3.
4. $\overline{RD} \cong \overline{RD}$	4.
5. $\triangle RED \cong \triangle DBR$	5.



Assessment

Directions: Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

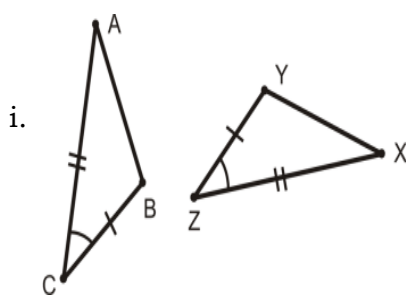
1. Which of the theorems below states that: “If two angles and a non-included side of one triangle are congruent to the corresponding two angles and a non-included side of another triangle, then the triangles are congruent.”?

A. HyA Congruence Theorem C. LL Congruence Theorem
B. HyL Congruence Theorem D. AAS Congruence Theorem

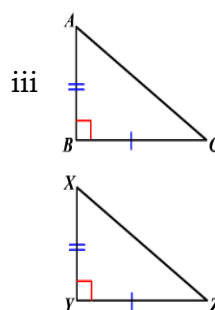
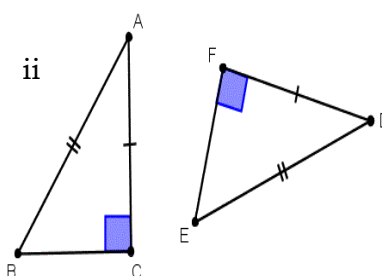
2. Which of the following theorems states that: “If the hypotenuse and an acute angle of one right triangle are congruent to the corresponding hypotenuse and an acute angle of another right triangle, then the triangles are congruent.”?

A. HyA Congruence Theorem C. LA Congruence Theorem
B. HyL Congruence Theorem D. LL Congruence Theorem

3. Which of the following pairs of triangles below are congruent and can be proved by SAS Congruence?



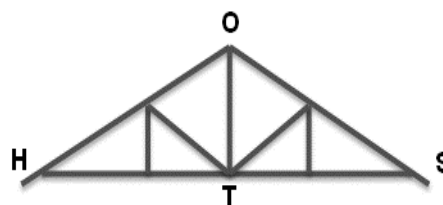
i



A. i and ii
B. ii and iii

C. iii
D. i, ii, and iii

4. In the figure at the right, \overline{OT} is a perpendicular bisector of \overline{HS} at T. What triangle congruence theorem can be used to prove that $\triangle HOT \cong \triangle SOT$?



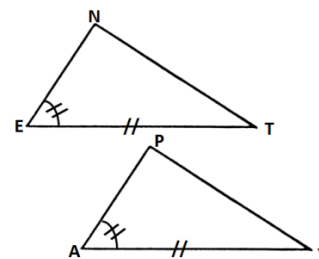
A. LL Congruence Theorem
B. HyA Congruence Theorem

C. HyL Congruence Theorem
D. AAS Congruence Theorem

5. In the figures at the right, $\angle E \cong \angle A$, $\overline{ET} \cong \overline{AY}$. What additional data is needed to prove that $\triangle NET \cong \triangle PAY$ by SAS Congruence?

- A. $\angle N \cong \angle P$
B. $\angle T \cong \angle Y$

- C. $\overline{NE} \cong \overline{PA}$
D. $\overline{NT} \cong \overline{PY}$



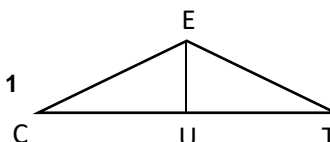
6. Which statement is NOT sufficient to prove the congruence of two triangles?
- A. Two angles and the included side of one triangle are congruent respectively to the two angles and the included side of another triangle.
- B. Two sides of one triangle are congruent respectively to the two sides of another triangle
- C. Two angles and the non-included side of one triangle are congruent respectively to the two angles and the non-included side of another triangle.
- D. Three sides of one triangle are congruent respectively to the three sides of another triangle.

For items 7 to 9, complete the table below. Choose the letter of the correct answer from the choices provided.

- A. $\angle CUE$ and $\angle TUE$ are right angles
B. Given
C. $\overline{EU} \cong \overline{EU}$
D. SAS Congruence Postulate
E. Right Angle Theorem

Given: \overline{EU} is the \perp bisector of \overline{CT}
Prove: $\triangle CUE \cong \triangle TUE$

Figure No. 1



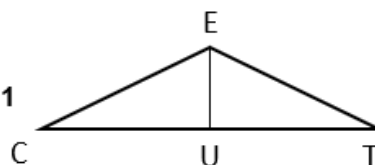
Proof:

Statements	Reasons
1. \overline{EU} is the \perp bisector of \overline{CT}	(7)
2. $\angle CUE$ and $\angle TUE$ are right angles	Definition of Perpendicular Line Segments
3. $\angle CUE \cong \angle TUE$	Right Angle Theorem
4. $\overline{CU} \cong \overline{UT}$	Definition of \perp bisector
5. (8)	Reflexive Property
6. $\triangle CUE \cong \triangle TUE$	(9)

For items 10 to 12, use figure no. 1 and its given data to complete the table below by choosing the letter of the correct answer below.

Given: \overline{EU} is the \perp bisector of \overline{CT}
 Prove: $\triangle CUE \cong \triangle TUE$

Figure No. 1



- A. LA Congruence Theorem
- B. Definition of perpendicular bisector
- C. LL Congruence Theorem
- D. HyL Congruence Theorem
- E. HyA Congruence Theorem

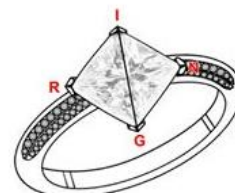
- F. $\triangle CUE \cong \triangle TUE$
- G. $\overline{CU} \cong \overline{UT}$
- H. $\overline{CU} \cong \overline{EU}$
- I. $\overline{EU} \cong \overline{EU}$
- J. $\angle C \cong \angle T$

Statements	Reasons
1. \overline{EU} is the \perp bisector of \overline{CT}	1. Given
2. U is the midpoint of \overline{CT}	2. (10)
3. (11)	3. Definition of Midpoint
4. $\angle CUE$ and $\angle TUE$ are right angles	4. Definition of perpendicular bisector
5. $\angle CUE \cong \angle TUE$	5. Right Angle Theorem
6. $\overline{EU} \cong \overline{EU}$	6. Reflexive Property
7. $\triangle CUE \cong \triangle TUE$	7. (12)

13. You are tasked to make a design of the flooring of your barangay hall using triangles. The available materials are square tiles. How are you going to make the design?

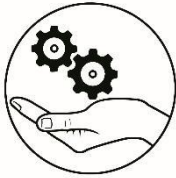
- A. Apply triangle congruence by SSS
- B. Apply triangle congruence by SAS.
- F. Apply triangle congruence by ASA.
- G. Apply triangle congruence by AAS.

14. The figure at the right shows a ring with a rhombus-shaped diamond. \overline{IG} divides the diamond into two congruent triangles. What reason will justify that $\angle IRG \cong \angle ING$?



- A. Adjacent angles congruent
- B. Vertical angles are congruent
- C. Alternate interior angle congruent
- D. Opposite angles of a rhombus are congruent

15. Using the same figure in no. 14, what triangle congruence postulate/theorem may NOT be used to prove that $\triangle RIG \cong \triangle NIG$?
- A. SSS B. SAS C. SAA D. H

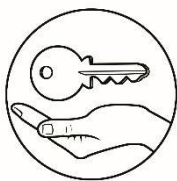


Additional Activities

During a storm, a man supported his hut by using three wires of equal length. The wires are staked to the ground at different points R, S, T which are of the same distance from the base of the hut. Explain and illustrate how you can prove that the angles formed by the wires attached to the ground are all congruent.

Your output will be rated using the following rubrics

10	8	6	4	2
Able to provide correct, logical, explanation and illustration of the hut completely and clearly	Able to provide an explanation and illustration of the hut but not complete and clear	Able to provide explanation and illustration of the hut but the hut being illustrated is not strong enough to survive the storm.	Able to provide a brief explanation without illustration.	Was not able to provide an explanation and illustration.



Answer Key

What I Know

1. B
2. A
3. C
4. B
5. B
6. C
7. C
8. B
9. D
10. A
11. A
12. A
13. B
14. E
15. B

What's In

1. C	2. B	3. A	4. C	5. B
------	------	------	------	------

What's New

1. $\overline{AR} \cong \overline{SR}$	2. $\overline{CE} \cong \overline{RA}$
$\overline{WR} \cong \overline{DR}$	$\overline{ER} \cong \overline{AC}$
$\angle ARW \cong \angle SRD$	$\overline{CR} \cong \overline{CR}$

What's More! Activity 1

- I.
 1. SAS Congruence Postulate
 2. ASA Congruence Postulate
 3. AAS Congruence Theorem
- II.
 1. $\overline{CR} \cong \overline{TY}$
 2. $\angle A \cong \angle O$
 3. $\overline{OY} \cong \overline{AR}$
- III.
 1. $\overline{ATR} \cong \overline{AT}$; SSS Congruence
 2. $\overline{RT} \cong \overline{RT}$; ASA Congruence
 3. $\angle SAD \cong \angle FAT$; SAS Congruence
 4. $\overline{OG} \cong \overline{OG}$; HYL Congruence

What's More

Activity 2

Reasons	1. Given
Statements	2. $\overline{AR} \cong \overline{RU}$
	3. $\angle A$ and $\angle U$ are right angles
	4. Right Angle Theorem
	5.
	6. $\angle BRA \cong \angle FRU$
	7. ASA Congruence Postulate

What's More

Activity 3

Reasons	1. Given
Statements	2. $\overline{OE} \cong \overline{OE}$
	3. SSS Congruence Postulate

Assessment

1. D
2. A
3. D
4. A
5. C
6. B
7. B
8. C
9. D
10. B
11. G
12. C
13. A
14. D
15. D

Additional Activities

(Students' answers vary.)

References

A. Book

1. Emmanuel Abunzo, et. al., *Mathematics 8 Learner's Module* (Pasig City, Philippines: Book Media Press, Inc. and Prinwell, Inc., 2013).
2. A Baccay, *Geometry for Secondary Schools* (Philippines: Phoenix Publishing House), 140-190.
3. Ricardo Crisostomo, Priscila de Sagun, and Alicia Padua, *Our World of Math* (Quezon City, Philippines: Vibal Publishing House, Inc., 2013).
4. Soledad Dilao and Julieta Bernabe, *Geometry* (Quezon City, Philippines: SD Publication, Inc., 2010).
5. Robert Kalin and Mary Kay Corbitt, *Geometry, Prentice Hall* (Jurong, Singapore: Pearson Education, Inc., 1993).
6. Karl Romero and Freidrich Jose, *Geometry in the Real World: Explorations and Applications* (Makati City, Philippines: Salesiana Publishers, Inc., 2003).

B. Electronic Resources

1. www.mathbitnotebook.com/Geometry/BasicTerms/BTauxiliary.html
2. Regional Training for Teachers on Critical Content in Mathematics 8, 2018.
3. https://www.google.com/search?q=free+downloadable+rhombus+shape+ring&tbm=isch&hl=en&chips=q:free+downloadable+rhombus+shape+ring,online_chips:diamond&hl=en&ved=2ahUKEwiuiKerjePqAhVPA6YKHW0YBsUQ4lYoAHoECAEQFg&biw=1349&bih=608#imgrc=pVlCtS2FzkwFJM

For inquiries or feedback, please write or call:

Department of Education - Bureau of Learning Resources (DepEd-BLR)

Ground Floor, Bonifacio Bldg., DepEd Complex
Meralco Avenue, Pasig City, Philippines 1600

Telefax: (632) 8634-1072; 8634-1054; 8631-4985

Email Address: blr.lrqad@deped.gov.ph * blr.lrpd@deped.gov.ph