10

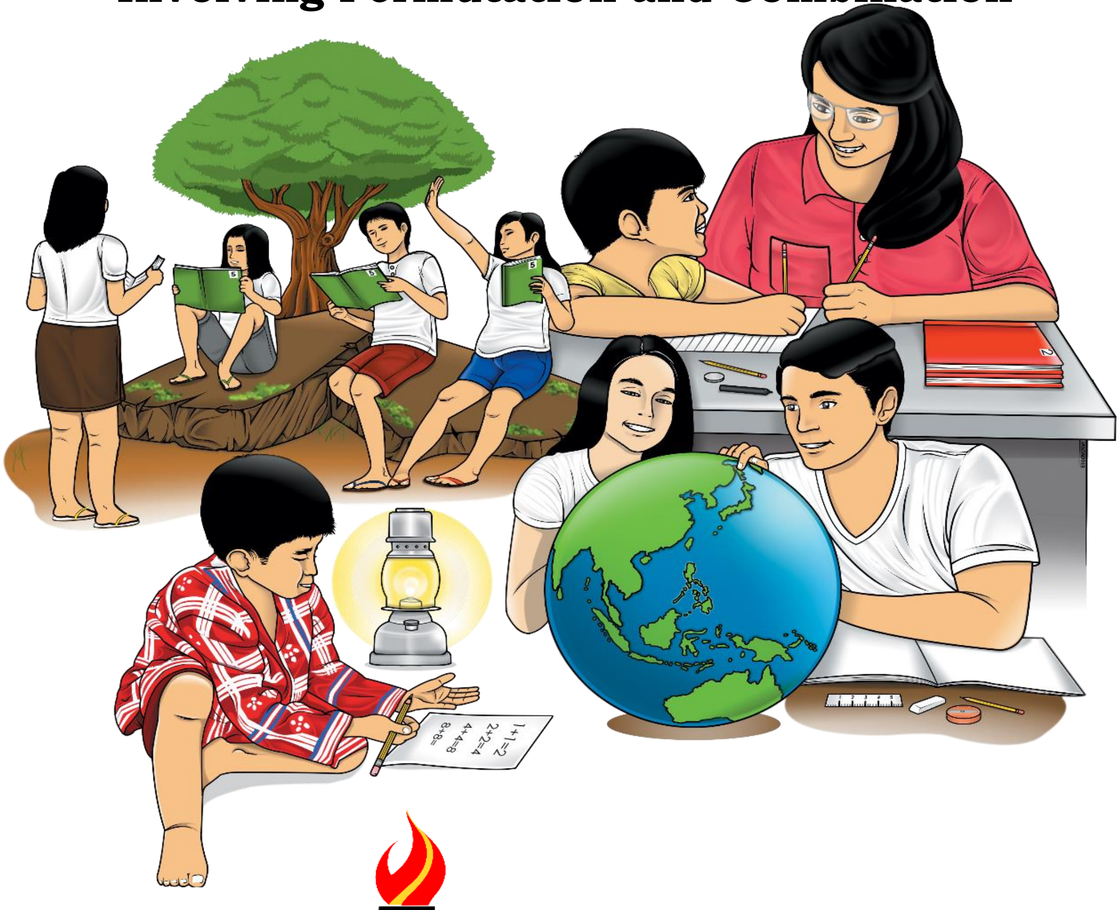
DepED
DEPARTMENT OF EDUCATION

Mathematics

Quarter 3 – Module 28

Combination

Illustrating Combination of Objects, Differentiating Permutation From Combination of *N* Objects Taken *r* at a Time, Solving Problems Involving Permutation and Combination



ALTERNATIVE DELIVERY MODE
ADM

Mathematics – Grade 10
Alternative Delivery Mode
Quarter 3 – Module 28 :Combination
First Edition, 2020

Published by the Department of Education
Secretary: Leonor Magtolis Briones
Undersecretary: Diosdado M. San Antonio

Development Team of the Module

Writer: Selalyn B. Maguilao
Editors: Melchor B. Ticag
Reviewer: Bryan A. Hidalgo
Illustrator:
Layout Artist: Reymark L. Miraples , Jhunness Bhaby A. Villalobos , Rosel P. Patangan
Management Team:
May B. Eclar
Marie Carolyn B. Verano
Carmel F. Meris
Ethielyn E. Taqued
Edgar H. Madlaing
Soraya T. Faculo
Francisco C. Copsiyan,

Printed in the Philippines by ________________________

Department of Education – Cordillera Administrative Region

Office Address : Wangal, La Trinidad, Benguet
Telephone : (074) 422-4074
E-mail Address : car@deped.gov.ph

10

Mathematics

Quarter 3 – Module 28

Combination

Illustrating Combination of Objects, Differentiating Permutation From Combination of *N* Objects Taken *r* at a Time, Solving Problems Involving Permutation and Combination

DEPARTMENT OF EDUCATION

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

What I Need to Know

This module was designed and written with you in mind. It is here to help you understand the concept of combination.

The scope of this module permits it to be used in many different learning situations. The lessons are arranged to follow the standard sequence of the course but the pacing in which you read and comprehend the contents and answer the exercises in this module will depend on your ability.

After going through this module, you are expected to demonstrate understanding of key concepts on combination. Specifically, you should be able to:

1) define combination of n objects taken r at a time,
2) differentiate permutation from combination,
3) solve problems involving permutation and combination.

What I Know

You are task to answer the following questions before you proceed to the lesson. Do not worry, I only want to know how knowledgeable are you with the topics that we will be discussing in this module.

DIRECTION: Read and analyze each item carefully. Write the letter of the correct answer on the blank provided before each number.

_____ 1. Which of the following determines the number of possible groups of a collection of items where the order of the elements does not matter?

A. FCP B. Permutation C. Combination D. Probability

_____ 2. Which of the following requires combination?
A. Entering the PIN of your ATM card.
B. Arranging three people to pose for a picture.
C. Choosing 3 of your friends to attend to your birthday party.
D. Forming 3-digit numbers from the digits 1, 2, 3, 4, 5, 6, ad 7.

_____ 3. Selecting two representatives from 8 candidates requires ______.

A. FCP B. Permutation C. Combination D. Probability

_____ 4. The number of combinations of n objects taken r at a time is defined by the expression _____.

A. $\frac{n!}{r!}$ B. $\frac{n!}{(n-r)!r!}$ C. $\frac{n!}{(n-r)r!}$ D. $n!$

_____ 5. Evaluate: ${}_{12}C_4$.

A. 220 B. 495 C. 11,880 D. 49,500

_____ 6. How many straight lines can be drawn using the 7 points X, Y, Z, M, N, O, and P, such that no three points are collinear?

A. 0 B. 7 C. 21 D. 35

_____ 7. In a classroom, there are 10 male students, 12 female students, and 7 teachers. In how many ways can 3 male students, 7 female students, and 5 teachers be chosen?

A. 933,942 B. 1,995,840 C. 3,994,920 D. 77,558,760

_____ 8. How many diagonals does a pentagon have?

A. 2 B. 3 C. 5 D. 10

For items 9 – 12. A group of 6 women and 9 men must select a six-person committee. How many committees are possible if each committee must consist of the following:

_____ 9. equal number of men and women?

A. 104 B. 540 C. 1,680 D. 1,890

_____ 10. at most 4 women?

A. 2,220 B. 4,110 C. 4,866 D. 4,950

_____ 11. no women?

A. 84 B. 1,890 C. 5,005 D. 60,480

_____ 12. minimum of 4 men?

A. 1,890 B. 2,646 C. 2,703 D. 2,730

_____ 13. Nine dots are randomly placed on a circle, how many triangles can be formed using the dots as the vertices?

A. 9 B. 36 C. 56 D. 84

_____ 14. There are 8 baseball teams during the 2019 CARAA. How many games must be played in order for each team to play every other team exactly once?

A. 16 B. 28 C. 56 D. 70

_____ 15. Thirteen participants for a conference were to shake hands and introduce themselves with each other. How many handshakes have taken place?

A. 13! B. 66 C. 78 D. 286

Lesson 1	Combination of *n* Objects Taken *r* at a Time

What's In

Let's review our previous lesson on permutation by answering the following activity:

Activity 1

A) Fill in the blanks with words or expressions that will best complete the statements below.

The number of possible arrangements of objects/elements in a set when order matters is called __________. It is denoted by the expression __________ and is read as "____."

B) Match each case of permutations of 'n' objects from a set of 'n' distinct objects on the left to its corresponding permutation notation on the right. Write the correct answer on the space provided before each number.

Cases	**Permutation Notation**
____1. Permutation of n objects taken r at a time	A) P = (n–1)!
____2. Permutation of n objects taken n at a time	B) $_nP_r = \frac{n!}{(n-r)!}$
____3. Permutation with identical objects	C) $P = \frac{(n-1)!}{2}$
____4. Permutation of n distinct objects arranged in a circle	D) $_nP_n = n!$
____5. Permutation of n different objects around a key ring	E) $P = \frac{n!}{n_1!n_2!\ldots n_k!}$
	F) P = n!

What's New

Since you know already how to list down possible arrangement of objects,
I think you are ready for the activity below.

Activity 2
Consider the following problems:

1. Using the digits 1, 2, and 3, how many two-digit numbers can be formed if repetition is not allowed? List down all the possible numbers below.	2. Three Players were numbered as 1, 2, and 3. How many teams of two players can be formed? List down all possible teams of two players below.

What is It

What can you say about the given situations on Activity 2? Are they the same? Why? or Why not?

1) 2) {-3, -2, 2}

Let us process the problems given in Activity 2.

3) $(x - 1)^2 (x - 2) (x - 3) = 0$ 4) {-2}

Problem 1. Using the digits 1, 2, and 3, how many two-digit numbers can be formed if repetition is not allowed?

Solution: Since we are tasked to list all two-digit numbers that can be formed using the digits 1, 2, and 3 if repetition is not allowed, then the number of two-digit numbers formed is given by:

$${}_3P_2 = \frac{3!}{(3-2)!} = \mathbf{6} \quad \text{applying permutation}$$

Note: 12 is different from 21 – order is important

Answer: There are 6 two-digit numbers that can be formed if repetition is not allowed, which are 12, 13, 21, 23, 31, 32.

Problem 2. Three Players were numbered as 1, 2, and 3. How many teams of two players can be formed?

Solution: The arrangement of team members does not affect the team composition. To find for the number of teams of two members formed, we can list down possible arrangements.

Note: Players 1 and 2 forming a team is the same as the team formed by players 2 and 1. Thus, order is NOT important. Listing possible arrangements, they are:

12, 21 12 is the same with 21 - 1 combination
13, 31 13 is the same with 31 - 1 combination
23, 32 23 is the same with 32 - 1 combination

Answer: There are 3 teams of two players formed.

Problem 1 is an example of permutation problem because order is important. Problem 2, on the other hand is an example of a combination problem because order is not important. Now, how are we going to answer problem 2 without listing the possibilities? Since order is not important, we will be using the concept of combination to answer problems similar to given problem 2 above.

Notice that the number of arrangements of choosing 2 players from 3 players numbered as 1, 2, and 3 is equal to 2! or 2. The 2 arrangements represent only 1 team or 1 combination. The ratio is 2! :1 or r! : 1.

In the permutation of n objects taken r at a time, the elements of a set r objects taken from the n objects can be arranged in r! different ways. Since in combination order does not matter, the number of arrangements of the elements of the same set of r objects represent only 1 group or 1 combination. The ratio is r! : 1, Hence the following proportion:

$$r! : 1 = {}_nP_r : {}_nC_r$$

Solving for nCr:

$$r! : 1 = {}_nP_r : {}_nC_r$$

$${}_nP_r = r!\,{}_nC_r$$

$$\frac{\frac{n!}{(n-r)!}}{r!} = \frac{r!nCr}{r!}$$

Thus,

$$\frac{n!}{r!(n-r)!} = {}_nC_r$$

Combination is a selection made from a group of ítems without regard to their order.

It is denoted by $_nC_r$, or C(n,r), or C_r^n and read as "the combination of *n* objects taken *r* at a time" or "n choose r" and is given using the formula

$$_nC_r = \frac{n!}{r!(n-r)!}$$

where: ***C*** refers to the number of combinations.
n refers to the total number of objects in a set.
r refers to the number of objects selected from the set.

Let us consider the following problems involving combinations.

Example 1. Three Players were numbered as 1, 2, and 3, how many teams of two players can be formed?

Solution: Since this problem (problem 2) was solved previously by listing method. Now, let us solve it using the combination formula. We are to form teams of 2 players from 3 players. Thus, as explained above, we can use now the formula for combination where n=3 and r = 2.

$$_nC_r = \frac{n!}{r!(n-r)!}$$

$$_3C_2 = \frac{3!}{2!(3-2)!}$$

$$_3C_2 = 3$$

thus, giving us the same answer, which is 3.

Example 2. **Playing Lotto**

Lotto is a game of chance which is played by choosing six different numbers from 1 to 42. How many different bets are possible?

Solution: The problem is an example of combination because the order on how the 6 numbers are chosen is not important.

$_nC_r = \frac{n!}{r!(n-r)!}$ using the formula

$_{42}C_6 = \frac{42!}{6!(42-6)!}$ let n = 42 and r = 6

$_{42}C_6 = 5,245,786$ simplify

Thus, there are 5,245,786 bets possible.

Example 3. **Creating Committees**

How many different committees of 4 people can be formed from a pool of 7 people?

Solution: It's a combination problem because there is no order of the members required in the committee. Since we are to form a group of 4 from 7 people then, we will use combination with n =7 and r= 4.

$${}_7C_4 = \frac{7!}{4!(7-4)!}$$

$${}_7C_4 = 35$$

There are 35 different committees.

Example 4. **Creating Committees**

How many different committees consisting of 8 people can be formed from 12 men and 9 women if the number of men and the number of women as members are equal?

Solution: Since the committee needs equal numbers of men and women to form a committee with 8 members, then, the number of men and number of women are both 4. These 4 men and 4 women will be chosen using the concept of combination since the order of being chosen is not important.

${}_{12}C_4$ - the number of ways that 4 men will be chosen from 12 men.

${}_9C_4$ – the number of ways that 4 women will be chosen from 9 women.

After which, apply FCP by multiplying the two combination results in order to get desired result.

$${}_{12}C_4 \cdot {}_9C_4 = (495)(126)$$

$$= 62{,}370$$

There are 62,370 different committees.

Example 5. **Creating Committees**

A committee of 3 members is to be formed from 6 women and 5 men. The committee must include at least 2 women. In how many ways can this be done?

Solution: Since the committee needs at least 2 women to form a committee of 3 members, then the number of women needed in the committee could either be 2 or 3. So, if there are 2 women in the group, then it needs 1 man to complete the number of members in the committee. If there are 3 women in the group, then, the committee is already complete.

$_6C_2 \cdot {}_5C_1$ - apply FCP to get the number of ways to form a committee with 2 women and 1 man

$_6C_3 \cdot {}_5C_0$ - apply FCP to get the number of ways to form a committee with 3 women and 0 man

Afterwhich, add the combination results to get the desired result.

$$({}_6C_2 \cdot {}_5C_1) + ({}_6C_3 \cdot {}_5C_0) = (15 \cdot 5) + (20 \cdot 1)$$
$$= 75 + 20$$
$$= 95$$

There are 95 different committees.

What's More

Now, your turn.

Activity 3:
Show combination notation in solving each of the following problems.

1) How many combinations of three letters can be made from the letters B, E, A, U, T, and Y?

2) There are 8 persons inside a room. If each person is paired with another person to dance cha- cha, how many pairs of dancers are there in all?

3) From the letters of the word LINEAR, in how many ways can one consonant and two vowels be chosen?

4) I have 5 coins in my purse; 1 five-centavo coin, 1 ten-centavo coin, 1 one-peso coin, 1 five-peso coin and 1 ten-peso coin. If I pull out 3 coins, how many different amounts of the three pulled coins are possible?

5. A box contain 6 distinct red balls and 4 distinct blue balls. In how many ways can 3 balls be drawn randomly from the box if:
   a. the color is not considered?
   b. 2 balls is red and one is blue?
   c. all three balls are red?

What I Have Learned

Summing up, list down what you have learned in our discussion.

Activity 4.
Summarize what you have learned by accomplishing the following activity:

At least **2** concepts/definitions learned about combination:

1. ____

   ____

2. ____

   ____

3. ____

   ____

1 important skill that I should have to understand easily the lesson.

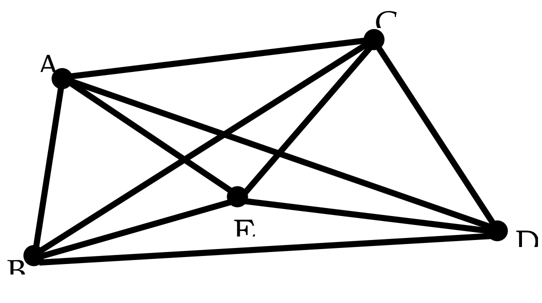
____

What I Can Do

At this part of the module, you will be applying the concepts of combination in some areas like Geometry. Consider the problem below.

Problem: How many triangles can be formed from five points on a plane, no three of which are collinear?

Solution A: Normally, instinct will tell us to draw the points on the plane in order to solve the problem, but mind you, it is possible. We can illustrate it using the figure below.



Now using A, B, C, D, and E to name the points, we can identify the different triangles formed. They are:

ΔABE ΔAED ΔCED ΔBEC ΔABD
ΔBCD ΔABC ΔACD ΔBED ΔAEC

Therefore, there are 10 different triangles formed.

Solution B: Sometimes, it is not practical to draw the figure, especially if you are given a larger number of points. We can apply the concept of combination in order to answer the problem easily.

Let us analyze the problem. How many points are needed in order to form a triangle? How many points do we have? Since we are given five points, no three of which are collinear, any three points chosen can form a triangle. In other words, we are selecting three points out of five given points. Thus, we can solve it using combination where n=5 and r =3.

$${}_nC_r = \frac{n!}{r!(n-r)!}$$

$${}_5C_3 = \frac{5!}{3!(5-3)!}$$

$${}_5C_3 = 10$$

Giving us the same answer, which is 10.

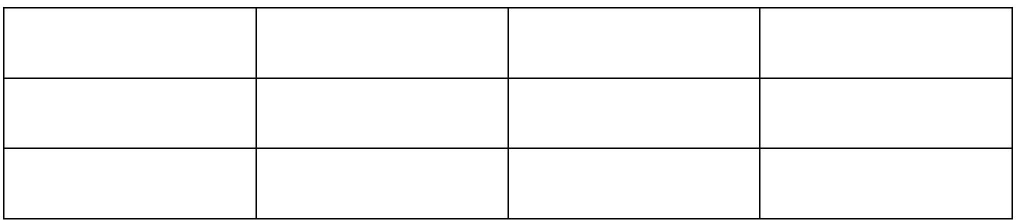
Congratulations, I know that you are ready to apply what you had learned in this module.

Activity 5

Solve the following problems.

1) How many chords can be drawn given 6 points on a circle?

2) The figure below shows 5 vertical lines and 4 horizontal lines which intersect forming right angles. As a result, rectangles are formed.
How many rectangles in all are formed?



Assessment

DIRECTION: Let us determine how much you have learned from this module. Read and answer each item carefully. Write the letter of the correct answer on the blank provided before the number.

_____1. Which of the following situation does not require combination?
A. Forming a committee of councilors.
B. Assigning rooms to conference participants.
C. Selecting the top 3 winners in a singing contest.
D. Choosing two literature books to buy from a variety of choices.

_____2. Which of the following requires combination?
A. Issuing plate numbers.
B. Lining up in paying bills.
C. Arranging 6 people in a round table.
D. Choosing five badminton players from 12 athletes.

_____3. Electing an auditor and a treasurer from 8 candidates is an example of ____.
A. Permutation B. Combination C. Sets D. cannot be determined

_____4. The number of combinations of n objects taken r at a time is denoted by ________.
A. $\frac{n!}{r!}$ B. $\frac{n!}{(n-r)!r!}$ C. $\frac{n!}{(n-r)r!}$ D. $n!$

_____5. Evaluate: ${}_{12}C_3$.
A. 2,200 B. 1,320 C. 220 D. 132

_____6. How many straight lines can be drawn using five points (A, B, C, D, and E), in which no three points are collinear?
A. 0 B. 5 C. 6 D. 10

_____7. In a box, there are 5 black pens, 3 white pens and 4 red pens. In how many ways can 2 black pens, 2 white pens and 2 red pens be chosen?
A. 19 B. 30 C. 38 D. 180

_____8. How many diagonals can be drawn in a 9-sided polygon?
A. 72 B. 36 C. 27 D. 12

For items 9 – 12. A group of 6 women and 9 men will form four-person committees. How many committees are possible if it must consist of the following:

_____9. 2 men and 2 women?
A. 51 B. 102 C. 540 D. 1,365

_____10. at most 3 men?
A. 180 B. 504 C. 1,224 D. 1,239

_____11. a majority of women?
A. 15 B. 180 C. 195 D. 2,700

_____12. minimum of 2 women?
A. 126 B. 735 C. 1,044 D. 1,365

_____13. There are 7 dots randomly placed on a circle, how many triangles can be formed using the dots as vertices?
A. 7 B. 20 C. 35 D. 84

_____14. There are 13 volleyball teams during the 2019 CARAA. How many games must be played in order for each team to play every other team exactly once?
A. 13 B. 26 C. 78 D. 286

_____15. Fifteen participants for the MTAP competition were asked to shake hands and introduce themselves with each other. How many handshakes took place?
A. 15 B. 30 C. 45 D. 105

Additional Activities

Activity 6.
Answer the following problems:

1) A dance instructor asks each student to do 4 out of the 10 dance routines. Of the 10 dance routines, 2 are easy, 5 are moderately difficult and 3 are difficult. In how many ways can a student select each of the following for the 4 dance routines?
a. 4 moderately difficult routines
b. 2 moderately difficult and 2 difficult routines
c. 1 easy and 3 difficult routines

2) From a deck of 52 playing cards, 5 cards are to be drawn. How many sets of 5 cards are possible if:
a. there is no restriction?
b. there are 2 aces and 3 jacks?
c. all cards are black?
d. all cards are diamonds?
e. there are 3 face-cards, 1 king, and 1 spade?

Answer Key

What I Know	Activity 2	Activity 5
1) C 2) C 3) C 4) C 5) B 6) C 7) B 8) C 9) C 10) C 11) A 12) D 13) D 14) B 15) C	1. 12, 13, 21, 23, 31, 32 2. 12, 13, 23 **Activity 3** 1. 20 2. 28 3. 9 4. 10 5. a. 120 b. 60 c. 20	1. 15 2. 60 **Activity 6** 1. a. 5 b. 35 c. 30 d. 2 2. a. 2 598 960 b. 24 c. 65 780 d. 1 287 e. 11 440
Activity 1 1) B 2) D 3) E 4) A 5) C	**Activity 4** 1. NOTATIONS $_nC_r$, or C(n,r), or C^n_r, "the combination of *n* objects taken *r* at a time" or "n choose r" 2. $_nC_r = \frac{n!}{r!(n-r)!}$; C refers to the number of combinations, n refers to the total number of objects in a set, r refers to the number of objects selected from the set	**Assessment** 1) C 2) D 3) A 4) B 5) C 6) D 7) D 8) C 9) C 10) C 11) C 12) B 13) C 14) C 15) D

References:

C.G. Lapinid, Grade 10 Mathematics Pattern and Practicalities, (Don Bosco Press, 2015)

S.M.Esparrago, Next Century Mathematics,(Phoenix Publishing House, INC.,2015)

For inquiries or feedback, please write or call:

Department of Education - Bureau of Learning Resources (DepEd-BLR)

Ground Floor, Bonifacio Bldg., DepEd Complex
Meralco Avenue, Pasig City, Philippines 1600

Telefax: (632) 8634-1072; 8634-1054; 8631-4985

Email Address: blr.lrqad@deped.gov.ph * blr.lrpd@deped.gov.ph