8

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Mathematics

Quarter 2 – Module 6:
"Determining the Dependent and Independent Variables and Finding the Domain and Range of a Function"

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ADM

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Mathematics

Quarter 2 – Module 6: "Determining the Dependent and Independent Variables and Finding the Domain and Range of a Function"

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Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

What I Need to Know

In this module, you will learn how to identify the independent and dependent variables by examining real world scenarios. You will be able to relate these variables to domain and range of a function. And finally, you will determine the domain and range given the sets, mappings, graphs, and equations. The scope of this module enables you to use it in many different learning situations. The lesson is arranged to follow the standard sequence of the course. But the order in which you read them can be changed to correspond with the textbook you are now using.

This module contains lesson on:

> Determining the Dependent and Independent Variables and Finding the Domain and Range of a Function.

After going through this module, you are expected to:

1. identify the independent and dependent variables;
2. identify the domain and range of a function expressed in words, function rule, symbol, table of values, ordered pairs and graph; and
3. relate the concept of domain and range to real-life situation.

What I Know

Directions: Choose the letter of the correct answer. Write the chosen letter on a separate sheet of paper.

1. Which of the following refers to the value of output in a function?
   A. independent variable
   B. dependent variable
   C. constant
   D. abscissa

2. Marites wants to know the effect of the new fertilizer on the growth of her plants. In the situation, what is the independent variable?
   A. the new fertilizer
   B. Marites' experiment
   C. the growth of plants
   D. the new fertilizer and plant growth

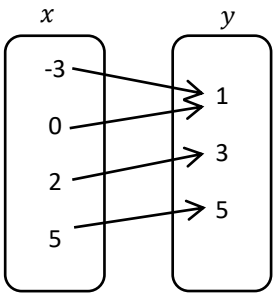
3. In a set of ordered pairs (x_i, y_i), what do you call the set of the first coordinates?
   A. domain
   B. range
   C. x −axis
   D. y −axis

4. In a set of ordered pairs (x_i, y_i), what do you call the set of the second coordinates?
   A. domain
   B. range
   C. x −axis
   D. y −axis

5. What is the domain of the set of ordered pairs {(1,2), (2,4), (3,6), (4,8)}?
   A. {1, 2, 3, 4}
   B. {2, 4, 6, 8}
   C. {1, 2, 6, 8}
   D. {1, 2, 3, 4, 6, 8}

6. What is the range of the set of ordered pairs {(-2,4), (-1,1), (0,0), (1,1), (2,4)}?
   A. {0,1,2,4}
   B. {-2, -1, 0, 1, 2}
   C. {4, 1, 0}
   D. {4, 1, 0, 1, 4}

7. What is the domain of the diagram shown at the right?
   A. {1, 3, 5}
   B. {-3, 0, 3, 5}
   C. {-3, 0, 2, 5}
   D. {1, 2, 3, 5}



8. Using the diagram in #7, what is the range?
   A. {1, 3, 5}
   B. {2, 3, 5}
   C. {-3, 0, 2, 5}
   D. {1, 1, 3, 5}

9. Mary Ann helped her mother in selling banana cue. Each banana cue costs Php5. The amount of money she makes is a function of the number of banana cue. What is the domain of the function if she has 10 banana cues to sell?

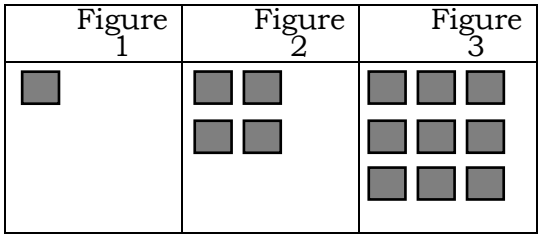
A. Domain: {0, 5, 10, …, 45, 50}
B. Domain: {0, 1, 2, …, 9, 10}
C. Domain: {5, 10, 15, …, 45, 50}
D. Domain: {1, 2, 3, …, 9, 10}

10. Consider the situation in # 9, what is the range of the function?
A. Range: {0, 5, 10, …, 45, 50}
B. Range: {0, 1, 2, …, 9, 10}
C. Range: {5, 10, 15, …, 45, 50}
D. Range: {1, 2, 3, …, 9, 10}

11. The table at the right describes y as a function of x. Which of the following correctly describes the value of -3?
A. It is an input.
B. It is part of the range.
C. It is part of the domain.
D. It is an independent value.

x	y
-1	-7
2	-3
6	7
9	13

12. Observe the figure at the right. The number of squares that make up the figures is a function of the indicated figure number. Which of the following statements is true?
A. The domain of the function is the number of squares.
B. The range of the function is the figure's number.
C. The domain is the set {1, 2, 3} while range is the set {1, 4, 9}.
D. The domain is the set {1, 2, 3, …} while the range is the set {1, 4, 9, …}.



13. In figure 1 at the right, the domain is $\{x \in \mathbb{R} \mid x \geq -3\}$ because the graph extends horizontally to _____.
A. the left
B. the right
C. both the left and the right
D. neither the left nor the right

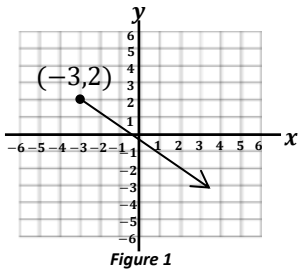
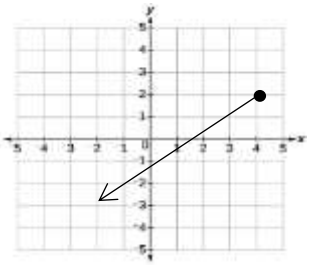


Figure 1

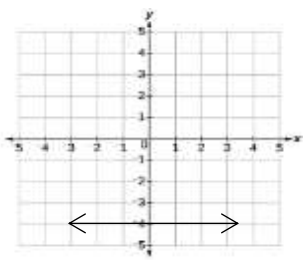
14. Using figure 1 in item 13, the range of the function is $\{y \in \mathbb{R} \mid y \leq 2\}$ because the graph discontinued at 2 and extends vertically _____.
A. upward
B. downward
C. both upward and downward
D. neither the left nor the right

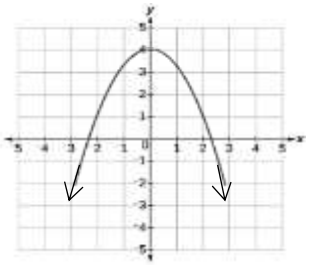
15. Which graph below describes the domain D = $\{x \in \mathbb{R} \mid x \geq -4\}$?

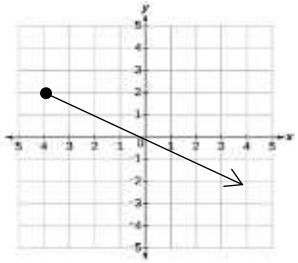
A.



C.



B. 

D. 

Lesson 1 Determining the Dependent and Independent Variables and Finding the Domain and Range of a Function

In our day-to-day life, we are dealing with quantities that change in relation to one another. You learned from your previous lessons that these quantities are referred to as variables. In mathematics, these variables can be represented by symbols or by any letter in the alphabet. Let us begin this module by determining the variables in real-life situations.

What's In

Activity 1: The Variables

Directions: Determine the quantities that change (or the variables) in each of the following situation. Item number 1 is done as an example.

1. Rosario is making souvenirs for a friend's wedding. The number of invited people to attend the wedding determines the number of souvenirs she must make.

The variables are:

the number of invited people

the number of souvenirs

2. George is selling brownies online. The more brownies he sells, the more money he earns.

The variables are:

_____

_____

3. A bakery in Cabadbaran City sells different types of bread. The more bread the customers buy, the more sacks of flour they will use.

   The variables are:

_____

_____

4. During the school's Mathematics month celebration, the mathematics teachers initiated different contests categories and bought medals for the winners. The more contests categories there were in the celebration, the more medals the teachers need to buy.

The variables are:

_____

_____

5. Warren is going to take 2nd Quarterly Exam. The number of hours he spends studying determines the score he will get in the exam.

The variables are:

_____

_____

Reminder: Brace yourself with the knowledge of how to determine the different variables as you will be identifying which one can change by its own and which one would be affected by the change of the other.

What's New

Consider the scenario below and perform what is indicated.

Scenario: *In order to earn some money, Robert cleans his neighbor's backyard for several hours during weekends. How much money did he earn?*

1. Determine the two variables in the scenario.
2. Which of the variables could be changed independently?
3. Which of the variables could be affected by the change of the other?
4. Fill in the blank with the appropriate statement in the parenthesis:
   The _____ (number of hours cleaning/amount of money earned) is the independent variable because it _____ (controls/depends on) the _____ (number of hours cleaning/amount of money earned).
5. Fill in the blank with the appropriate statement in the parenthesis:
   The _____ (number of hours cleaning/amount of money earned) is the dependent variable because it _____ (controls/depends on) the _____ (number of hours cleaning/amount of money earned).
6. If for every hour he gets paid Php40, how much will he earn after working 5 hours?
7. Would it be possible for Robert to earn Php800.00 in one day? Why or why not?

What is It

In your previous activity, the two variables in the scenario are "the number of hours Robert works" and "the amount of money Robert earns".

How do we know which variable is independent and which variable is dependent?

DEFINITION

Independent variable is a variable that will change by taking on different values. It changes independently. In a relation, independent variable controls the dependent variable. It is generally represented by the x –variable or what is referred to as the x –value.

Dependent variable is a variable that is affected by the independent variable, and it changes in response to the independent variable. In a relation, dependent variable depends on the independent variable. It is generally represented by the y –variable or what is referred to as the y –value.

In the scenario, which must happen first before the other occurs? Which variable controls the other, and which variable depends on the other?

If you say Robert must work first before he earns money, then you are correct.

Independent variable	Dependent variable
The number of hours Robert works	*The amount of money Robert earns*

If you can see, the amount of money Robert earns is actually a function of the number of hours he works. For instance, if he earns Php40 for every hour he works, then he earns Php200 for working five hours. The relation can be presented using the equation below:

$$f(x) = 40(x)$$
$$f(5) = (Php40)(5)$$
$$f(5) = Php200 \text{ } \textit{the amount he earns if he works for 5 hours}$$

Is it possible to know how much to pay him if we do not know how many hours he worked? The answer is **No**. In order for him to get paid, he needs to work for a certain number of hours first. So, the amount of money he earns is a function of the number of hours he works.

In this scenario, the number of hours Robert works is the ***domain*** of the function while the amount of money he earns is the ***range*** of the function.

DEFINITION

The domain of a function is the set of all acceptable inputs. Generally, domain is the set of x –values or the ***abscissa*** of the ordered pairs of a function.

The range of a function is the set of resulting outputs. Generally, range is the set of y –values or the ***ordinate*** of the ordered pairs of a function.

A common misconception is we can use any numbers for the inputs (or domain) of a function. That is *very wrong*. In fact, *we must inspect the function and determine any limitations on the domain.*

Like in the previous scenario, it is not possible for Robert to earn money if he works zero hour. He has to work first, at least one hour, because the earning he would make is computed per hour. Besides nobody would pay somebody for not working, right? Therefore, we cannot just change the domain (or the inputs) to any

value that we desire. The domain can take only acceptable values, that is, depending on the given situation.

Domain and Range of a Function Given Sets of Ordered Pairs

Example 1. Determine the domain and range of the function
$\{(0,-3),(2,5),(-1,-1)(-3,15)(1,-1)\}$

Solution:

In an ordered pair, (x,y), the first elements (x-values or abscissa) are the elements of domain, and the second elements (y-values or ordinate) are the range. We know that domain is the acceptable inputs, hence, in this case, the domain that is arranged in ascending order, is the set of the numbers $-3,-1,0,1,2$.

$\{(\mathbf{0},-3),(\mathbf{2},5),(\mathbf{-1},-1),(\mathbf{-3},15),(\mathbf{1},-1)\}$

Domain: $\{-3,-1,0,1,2\}$

Note:
You can also write the domain as how it appears in the set of ordered pairs such as: $\{0,2,-1,-3,1\}$. But list down the repeating number once.

The range of the function is the resulting outputs (y-values or ordinate) highlighted below.

$\{(0,\mathbf{-3}),(2,\mathbf{5}),(-1,\mathbf{-1})(-3,\mathbf{15})(1,\mathbf{-1})\}$

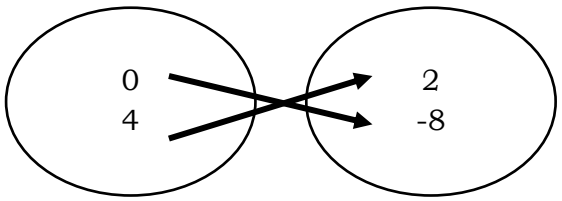
Range: $\{-3,-1,5,15\}$

Note:
You can also write the range as how it appears in the set of ordered pairs such as: $\{-3,5,-1,15\}$. But list down the repeating number once.

Noticed that we *did not write* -1 *twice.*

Domain and Range of a Function using the Mappings/Diagrams

Example 2. Determine the domain and range described by the mapping below.



Solution:

The domain and range are easily recognized in the mapping. The domain is the first set of numbers in the diagram, and the range is the second set of numbers in the diagram.

Domain: {0, 4}
Range: {2, -8} or {-8, 2}

Domain and Range of a Function Given the Notation (or Equation)

Example 3. Given this function $f(x) = \frac{3}{x}$, zero cannot be substituted for x, since division by zero is undefined.

$f(x) = \frac{3}{x}$
$f(0) = \frac{3}{0}$ ⇨ Undefined

Hence, the domain is the set of all numbers except zero. In set notation,

Domain: $\{x \in \mathbb{R} \,/\, x \neq 0\}$

Since you cannot take zero as your input, then zero is also not a possible output. Hence the range is the set of all real numbers except zero. In set notation,

Range: $\{y \in \mathbb{R} \,/\, y \neq 0\}$

Example 4. Determine the domain and range of the function $f(x) = \frac{2x+1}{x+2}$.

Solution:

The denominator $(\boldsymbol{x+2})$cannot be equal to zero since division by zero is *undefined*. Determining the value of the restriction, we get:

$x + 2$	$\neq$	0	*Denominator cannot be zero*
$x + 2 - 2$	$\neq$	$0 - 2$	*Addition property of Equality*
x	$\neq$	-2	*Additive Inverse Property*

Hence, the domain is the set of all real numbers except $-\mathbf{2}$ because -2 will make the whole denominator becomes zero. In set notation,

Domain: $\{x \in \mathbb{R} \,/\, x \neq -2\}$.

To find the range, we need to get the *inverse of the function* by following the procedures below:

$f(x) = \frac{2x+1}{x+2}$	*Given*
$y = \frac{2x+1}{x+2}$	*Replace* $\boldsymbol{f(x)}$ *by* $\boldsymbol{y}$
$x = \frac{\mathbf{2y+1}}{\mathbf{y+2}}$	*Interchange the values of* x *and* y
$x(y+2) = \left(\frac{2y+1}{y+2}\right)(y+2)$	*Multiply the equation by the LCM*
$x(y+2) = 2y+1$	*Multiplicative Inverse Property*
$xy + 2x = 2y + 1$	*Distributive Property*
$xy - 2y = -2x + 1$	*Combining like terms by Addition Property and Additive Inverse Property*
$y(x-2) = -2x + 1$	*Factoring out* y
$[y(x-2)]\left(\frac{1}{x-2}\right) = (-2x+1)\left(\frac{1}{x-2}\right)$	*Multiplication Property*
$y = \frac{-2x+1}{x-2}$	*Multiplicative Inverse Property; Simplify*

The inverse of the given function is $f^{-1}(f(x)) = \frac{-2x+1}{x-2}$. By inspection, the inverse has limitations, too. The denominator $(x-2)$ must not be equal to zero. This restriction applies to the possible outputs (or range) of the original function.

$x + 2 \neq 0$	*Denominator of the Inverse of the function should not be equal to zero*
$x + 2 - 2 \neq 0 - 2$	*Addition Property*
$x \neq -2$	*Additive Inverse Property, Simplify*

Hence, the range (of the original function) is the set of all numbers *except 2*. In set notation,

Range: $\{y \in \mathbb{R} \,/\, y \neq 2\}$

Example 5. Determine the domain and range of the function $f(x) = 2\sqrt{3x-5}$.

Solution:

Remember that the domain is the set of acceptable inputs. It seems that any value can be used. But when we look closely, note that the square root sign (radical symbol) requires some limitations. We know that we cannot take the square root of a negative number. So, whatever under the square root must be greater than or equal to zero. In this case, $(3x-5)$ must be greater than or equal to zero.

$3x - 5 \geq 0$	*Radicand must be greater than or equal to zero*
$3x - 5 + 5 \geq 0 + 5$	*Addition Property*
$3x \geq 5$	*From step 2, Additive Inverse Property*
$x \geq \frac{5}{3}$	*From step 3, Multiplicative Inverse Property*

Therefore, the domain are all values of real numbers greater than or equal to $\frac{5}{3}$. In set notation,

Domain: $\left\{x \in \mathbb{R} \,/\, x \geq \frac{5}{3}\right\}$

So, how do we find the range or acceptable outputs?

Again, consider the square root. We know that this value $(3x - 5)$ must be positive number or zero. Using the given function,

$f(x) = 2(positive\ number\ or\ zero)$
$f(x) = 0\ or\ greater$

The range of the function, $f(x) = 2\sqrt{3x - 5}$ is all real numbers greater than or equal to zero. In set notation,

Range: $\{y \in \mathbb{R} \,/\, y \geq 0\}$

Example 6: Determine the domain and range of the function $f(x) = 2x - 1$.

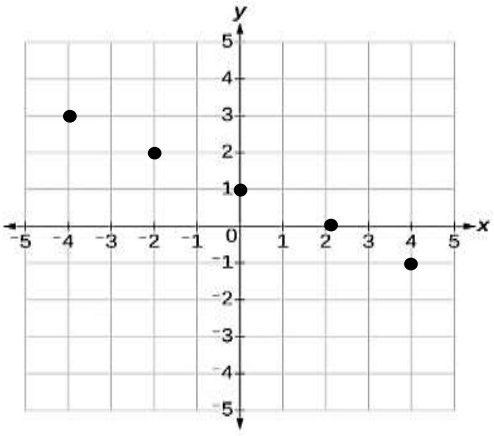
Solution:

Noticed that there are no limitations in the domain of the given function. We can use any values for this function. Also, there is no limitation in the range. Hence, the domain and the range are the set of all real numbers. In set notations,

Domain: $\{x \in \mathbb{R}\}$
Range: $\{y \in \mathbb{R}\}$

Domain and Range of a Function Described by Graphs

Example 7: Determine the domain and range of the given set of points plotted in the Cartesian plane below.



Solution:

To determine the domain and range of the set of points plotted in the Cartesian plane, do the following:

- Enumerate the set of ordered pairs.

  The set of ordered pairs of the points on the Cartesian plane above is$\{(-4,3),(-2,2),(0,1),(2,0),(4,-1)\}$.

- Determine the first elements (x-values or abscissas) of the ordered pairs. These are the set of acceptable inputs (or domain).

  $\{(\mathbf{-4},3),(\mathbf{-2},2),(\mathbf{0},1),(\mathbf{2},0),(\mathbf{4},-1)\}$

  Hence, the domain is:

  D: {-4, -2, 0, 2, 4}

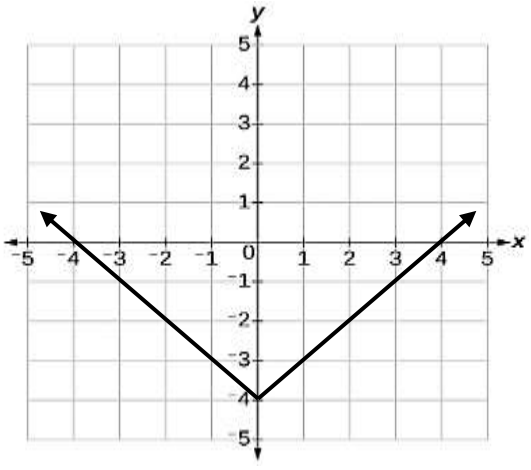
- Determine the second elements (y –values or ordinates) of the ordered pairs. These are the set of acceptable outputs (or range).

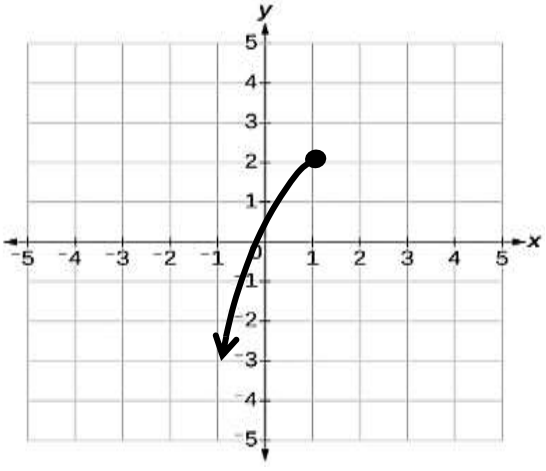
  $\{(-4,\mathbf{3}),(-2,\mathbf{2}),(0,\mathbf{1}),(2,\mathbf{0}),(4,\mathbf{-1})\}$

  Hence, the range is:

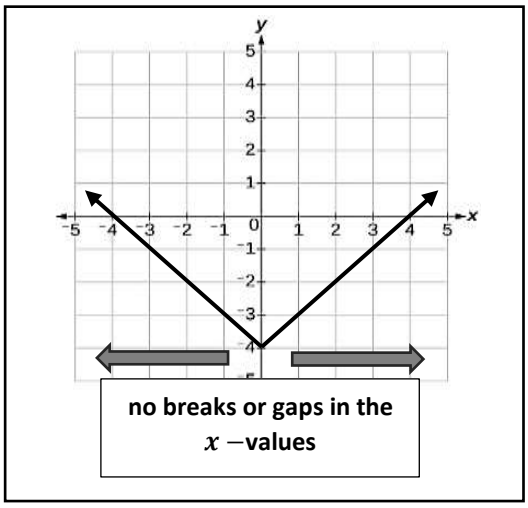
  R: {3, 2, 1, 0, -1}

Example 8: Determine the domain and range of a function described by the graph at the right.





Solution:

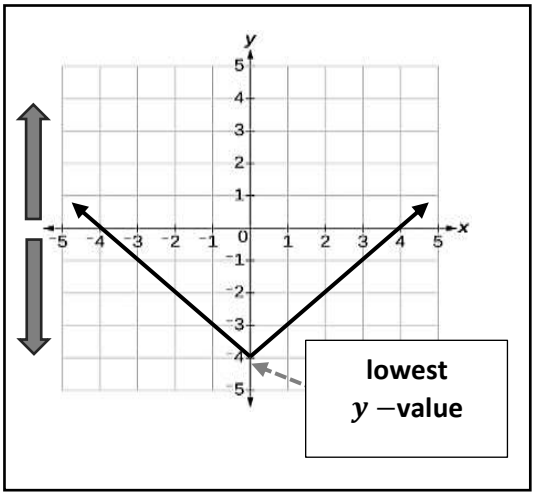


To determine the domain visually, consider the x −values or inputs taken by the graph:

- Looking in the x −values (horizontally) from left to right,
- Looking for breaks (or gaps) in the graph, where it begins and where it ends.

Since this graph goes infinitely left and right without any gaps, the domain is the set of all real numbers.

Domain: $\{x \in \mathbb{R}\}$



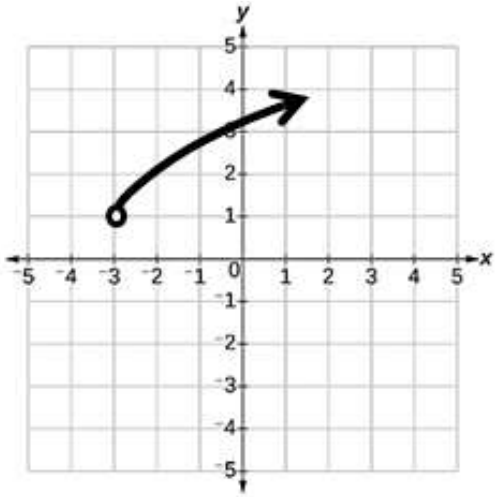
For the range, look at the y-values or outputs taken by the graph:

- Looking vertically, to see if there are any restrictions.
- Looking for breaks (or gaps) in the graph, where it begins and where it ends.

Since the lowest y −value is -4, and the graph extends infinitely upward, then, we can see that the range is all real numbers greater than or equal to -4.

Range: $\{y \in \mathbb{R} \,/\, y \geq -4\}$

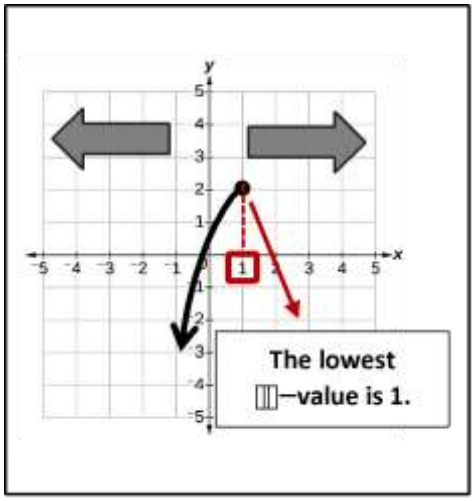
Example 9: Find the domain and range of a function described by the graph at the right.



Solution:

To determine the domain visually, consider the x –values or inputs taken by the graph:

- Looking in the x –values (horizontally) from left to right.
- Looking for breaks (or gaps) in the graph, where it begins and where it ends.

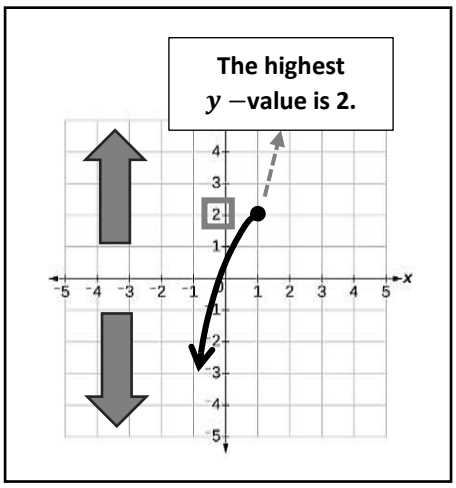


The graph breaks at 1 (x-value), this means that the highest x –value is 1. The graph goes infinitely to the left starting from 1. Therefore, the domain is the set of all real numbers less than or equal to 1. In set notation,

Domain: $\{x \in \mathbb{R} \,/\, x \leq 1\}$

For the range, look at the y –values or outputs taken by the graph:

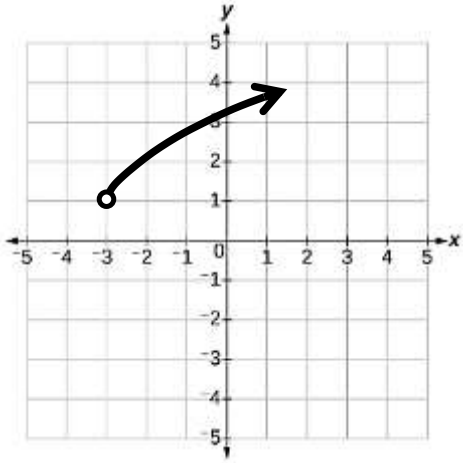
- Looking vertically to see if there are any restrictions.
- Looking for breaks (or gaps) in the graph, where it begins and where it ends.



Since the highest$y$ –value is 2, and the graph goes infinitely down starting from 2, then the range is all real numbers less than or equal to 2. In set notation,

Range: $\{y \in \mathbb{R} \,/\, y \leq 2\}$

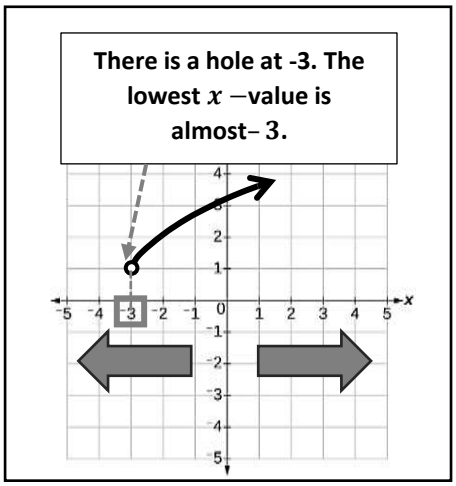
Example 10: Find the domain and range of a function described by the graph at the right.



Solution:

To determine the domain visually, consider the acceptable x −values or inputs taken by the graph:

- Looking in the x −values (horizontally) from left to right.
- Looking for breaks (or gaps) in the graph, where it begins and where it ends.

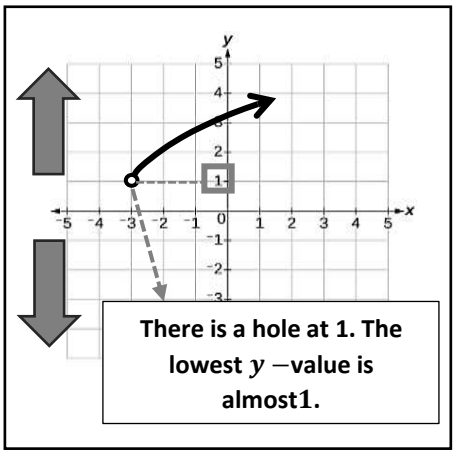


The graph breaks at −3but take note of the hole at point (-3,1). This means that the lowest x −value is almost at −3 but excluding -3. The graph goes infinitely to the right starting from −3, but since there is a hole at point at (−3,1), it means that −3 is not included in the domain of the function described by the graph. Therefore, the domain is the set of all real numbers greater than −3. In notation,

Domain: $\{x \in \mathbb{R} \,/\, x > -3\}$

For the range, look at the y −values or outputs taken by the graph:

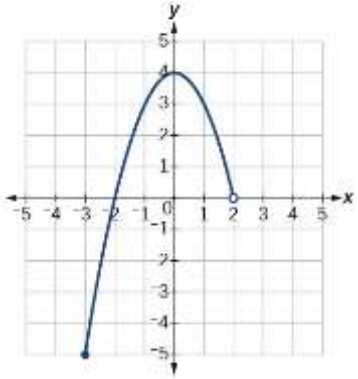
- Looking vertically to see if there are any restrictions.
- Looking for breaks (or gaps) in the graph, where it begins and where it ends.



The graph has a hole at 1. This means that the lowest$\boldsymbol{y}$ −value is almost1.The graph goes infinitely upward starting from 1. Hence, the range is all real numbers greater than 1. In notation,

Range: $\{y \in \mathbb{R} \,/ y > 1\}$

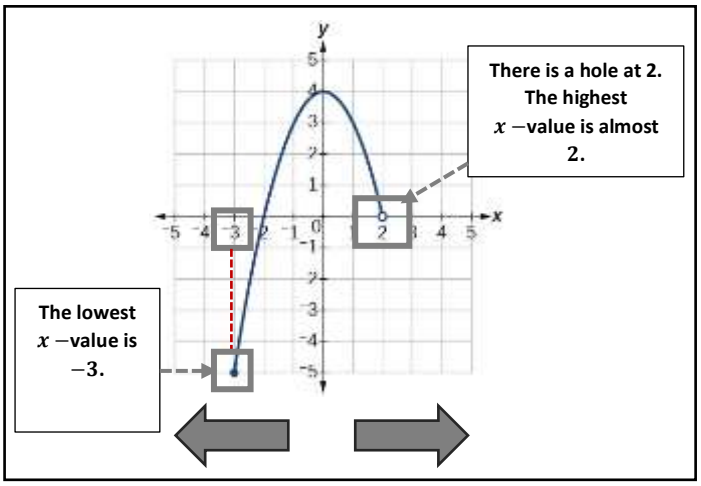
Example 11: Find the domain and range of a function described by the graph at the right.



Solution:

To determine the domain visually, consider the$x$ −values or inputs taken by the graph:

- Looking in the x −values (horizontally) from left to right.
- Looking for breaks (or gaps) in the graph, where it begins and where it ends.

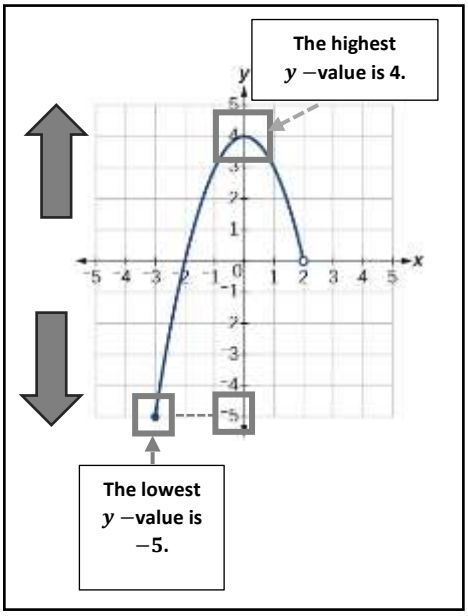


The graph breaks at -3 and almost at 2. Take note of the solid point at (-3, -5) and hole at point (2, 0). The lowest x −value is -3 and the highest x −value is almost 2.Hence, the domain of this function described by the graph is all real numbers greater than or equal to -3 but less than 2. In set notation,

$$\text{Domain: } \{x \in \mathbb{R} \,/ -3 \leq x < 2\}$$

For the range, look at the y −values or outputs taken by the graph:

- Looking vertically to see if there are any restrictions.
- Looking for breaks (or gaps) in the graph, where it begins and where it ends.

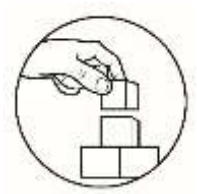


Since the lowest $\boldsymbol{y}$ −value is -5 and the highest $\boldsymbol{y}$ −value is 4, then the range of this function is all real numbers greater than or equal to -5 but less than or equal to 4. In set notation,

$$\text{Range: } \{y \in \mathbb{R} \,/ -5 \leq y \leq 4\}$$

Remember:

To find the domain and range of a function described by its graph, look in the x −direction (horizontally) from left to right for the domain, and y −direction (vertically) upward and downward for the range, then, look for any breaks or gaps in the graph, where it begins and where it ends. Use these data to list your domain and range.



What's More

Activity 1. To whom I depend on?

Directions: Given each situation below, determine each variable (in italicized form) whether independent or dependent, and determine if the said variable controls or depends on the other. Item number 1 was done as an example.

1. Doris was looking for a job. She noticed that the higher paying job requires higher education.

   Higher education is the **independent** variable because it **controls** the variable "higher paying job".

2. Roberto buys 2 kilos of dressed chicken at the market. How much will he pay if the chicken costs Php155 per kilo?

   The cost of the chicken is the _____ variable because it _____ the number of kilograms of chicken.

3. Paul planned to tile his floor. He measures the length and the width of his floor to calculate the area.

   The area of the floor is the _____ variable because it _____ the length and the width of the floor.

4. Jana auditioned in the "Pinoy Got Talent" competition. Her performances got a vote of 4 "Yeses" from the judges.

   Jana's performances are the _____ variable because it _____ the judges' vote.

5. Erick's math grade in the first quarter was 98%. He had completed all 10 requirements.

   The number of requirements completed is the _____ variable because it _____ the math grade.

Activity 2. Limited or Limitless?

Directions: Determine the domain and range of the following items.

1. $\{(-3,-3),(0,3),(-5,-7),(2,7),(1,5)\}$

Domain: _____
Range: _____

2.

x	-2	-1	0	1	3
y	7	4	3	4	12

Domain: _____
Range: _____

3. $f(x) = \frac{x}{x+4}$

Domain: _____
Range: _____

4. $f(x) = \sqrt{4x-8}$

Domain: _____
Range: _____

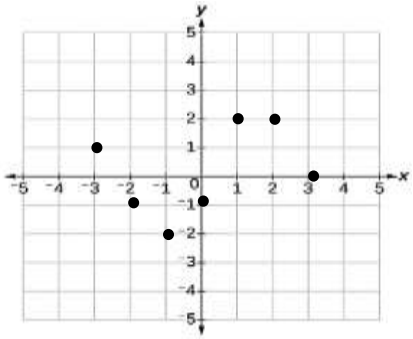
5. $f(x) = x - 6$

Domain: _____
Range: _____

6. $f(x) = x^2 + 1$

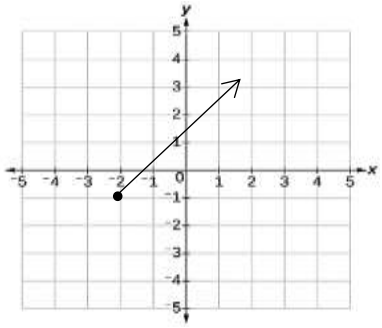
Domain: _____
Range: _____

7.



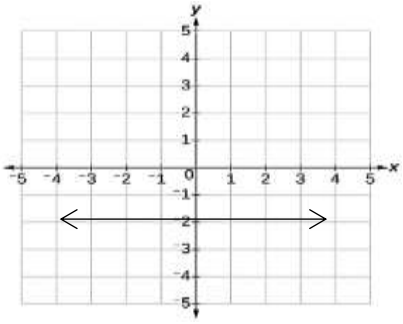
Domain: _____
Range: _____

8.



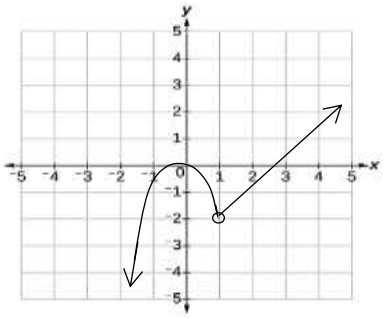
Domain: _____
Range: _____

9.



Domain: _____
Range: _____

10.



Domain: _____
Range: _____

What I Have Learned

After going through with this module, it's now time to check what you have learned from the activities. Read carefully and answer the items that follow.

Directions: Tell whether the following statements are true or false.

_____ 1. The dependent variable changes only in response to the independent variable.

_____ 2. Salary is an independent variable because it controls the number of hours that one works.

_____ 3. The height of a plant is dependent on the number of years it grown.

_____ 4. In a given function, the domain is the set of acceptable inputs while the range is the set of acceptable outputs.

_____ 5. In a set of ordered pairs, the abscissas are the range and the ordinates are the domain.

_____ 6. In a rational function, $f(x) = \frac{1}{2x}$, the domain is the set of real numbers not equal to the number that would make the denominator zero.

_____ 7. In a rational function, $f(x) = \frac{1}{2x}$, the range is the set of real numbers except zero.

_____ 8. In a radical function, $f(x) = 2\sqrt{x+1}$, the domain is the set of real numbers greater than or equal to 0.

_____ 9. To find the domain of a function described by the graph, we look horizontally from left to right, look for breaks or gaps on the graph, where it begins and where it ends.

_____ 10. To find the domain of a function described by the graph, we look vertically up and down, look for breaks or gaps on the graph, where it begins and where it ends.

What I Can Do

It is now time to apply the concept you have learned in finding the domain and range of a function. Read carefully and analyze the problem below.

Noli has to fill up his motorcycle with gasoline to travel from Barobo to Tandag City. The maximum capacity of the fuel tank of his motorcycle is 7 liters and the gasoline price is Php46 per liter. The amount paid (y) is a function of the number of liters of gasoline (x).

a. In the given situation, the number of liters of gasoline is what type of variable? Why?
b. In the given situation, the amount paid is what type of variable? Why?
c. What is the domain and range?
d. Can 5.5 liters of gasoline be the domain in the situation? Explain.
e. Can Php350 an acceptable value of range in the situation? Explain.
f. What could be the domain and range if the tank is quarter full before he had to fill it with gasoline?
g. Is it possible to fill the tank with 7.2 liters of gasoline? Why? Why not?
h. Noli is sitting on his motorcycle while allowing the gasoline station attendant to fill the tank. Is it correct? Why? Why not?
i. If you are in the situation of Noli, what safety precautions would you observe while refuelling your motorcycle?

Assessment

Directions: Choose the letter of the correct answer. Write your answer on a separate sheet of paper.

1. Given the variables: the cost to rent an apartment and the length of time it is rented, which is the independent variable?
   A. the length of time it is rented
   B. the cost of rental and length of time
   C. the cost of rental after a month
   D. the cost to rent an apartment

2. The price of powdered milk is a function of its net weight. In the function, the price of powdered milk is called _____.
   A. Domain
   B. Elements
   C. Inputs
   D. Outputs

3. What is the domain of the given set of ordered pairs$\{(3,1),(2,5),(0,5),(-2,6),(-4,7)\}$?
   A. {3, 2, 0, −4}
   B. {1, 5, 6, 7}
   C. {3, 2, 0, −2, −4}
   D. {1, 5, 5, 6, 7}

4. The teacher asked John to determine the range of the given set of ordered pairs $\{(1,1),(2,4),(3,4),(-2,1),(-4,5)\}$. His answer is$\{1,1,4,4,5\}$. Is he correct?
   A. Yes, since the numbers are increasing.
   B. No, because range is the set of first coordinates.
   C. Yes, numbers 1, 1, 4, 4, and 5 are the second coordinates of the ordered pairs.
   D. No, because the elements should be written once and the answer is $\{1,4,5\}$.

5. The diagram in figure 1 shows y as a function of x. Which of the following is TRUE about the diagram?
   A. 9 is an input
   B. −1 is an output
   C. −3 is part of the range
   D. −1 is part of the domain

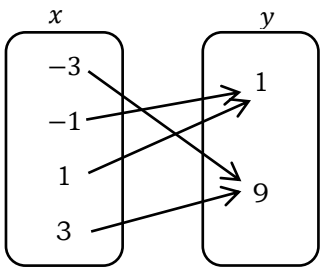


Figure 1

6. Using the diagram in #5, what is the range?
   A. {1, 9}
   B. {1, 3, 9}
   C. {−3, −1, 1, 3}
   D. {1, 1, 9, 9}

7. In a park, Donny wants to rent a bicycle for his son. The rental of bicycle is ₱20/hour. The amount of rental is a function of the number of hours his son will use the bicycle. What is the domain of the function if they are to stay in the park for 4 hours?
   A. Domain: $\{0, 20, 40, 60, 80\}$
   B. Domain: $\{0, 1, 2, 3, 4\}$
   C. Domain: $\{1, 2, 3, 4\}$
   D. Domain: $\{20, 40, 60, 80\}$

8. Consider the situation in # 7, what is the range of the function?
   A. Range: $\{0, 20, 40, 60, 80\}$
   B. Range: $\{0, 1, 2, 3, 4\}$
   C. Range: $\{1, 2, 3, 4\}$
   D. Range: $\{20, 40, 60, 80\}$

9. Given a folded piece of paper, the number of rectangles formed depends on the number of folds it has (as shown in the table below). Which of the following statements is true?

# of folds	1	2	3	4
# of rectangles formed	2	4	8	16

   A. The number of rectangles is the domain.
   B. The number of folds is the range.
   C. The domain is the set {1, 2, 3, 4, …}.
   D. The range is the set {2, 4, 8, 16}.

10. What is the domain of the graph shown in figure 2?
   A. {-4, -1, 0, 1, 4}
   B. {-4, -2, 0, 3, 4}
   C. {(-4, 1), (6, 0)}
   D. {6, 0}

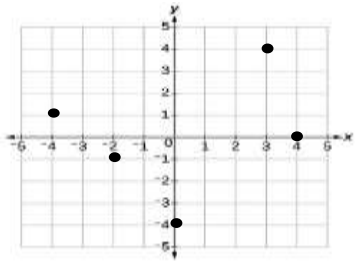


Figure 2

11. In figure 3 at the right, the domain is $\{x \in \mathbb{R}\}$ because the graph extends horizontally to _____.
   A. the right
   B. the left
   C. both left and right
   D. neither the left nor the right

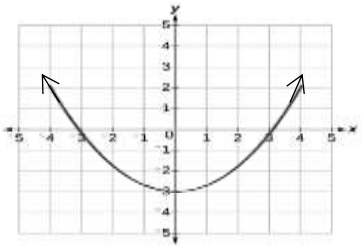
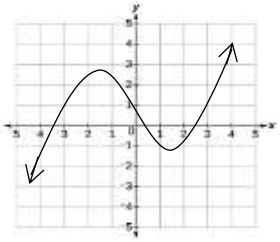
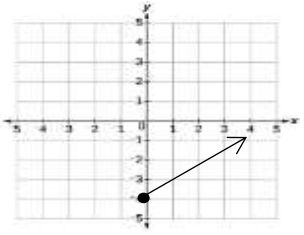


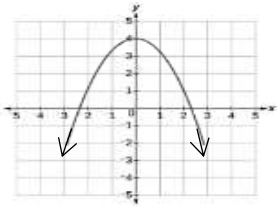
Figure 3

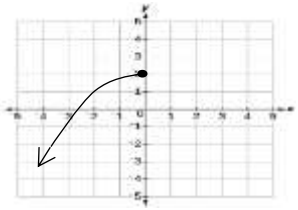
12. What is the range of the graph shown in figure 3?
   A. R = $\{y \in \mathbb{R}\}$
   B. R = $\{y \in \mathbb{R} \mid y \leq -3\}$
   C. R = $\{y \in \mathbb{R} \mid y \geq -3\}$
   D. Neither the left nor the right

13. Which graph below describes the range R = $\{y \epsilon \mathbb{R} \mid y \leq 4\}$?

A. 

C. 

B. 

D. 

14. What is the domain of the function $f(x) = \sqrt{2x-2}$?

A. Domain = $\{x \epsilon \mathbb{R} \mid x \geq 1\}$
B. Domain = $\{x \epsilon \mathbb{R} \mid x \leq 1\}$
C. Domain = $\{x \epsilon \mathbb{R} \mid x \neq 1\}$
D. Domain = $\{x \epsilon \mathbb{R}\}$

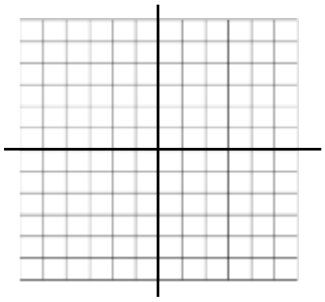
15. What is the range of the function $f(x) = \frac{x+1}{2x}$?

A. Range = $\{y \epsilon \mathbb{R} \mid y \geq \frac{1}{2}\}$
B. Range = $\{y \epsilon \mathbb{R} \mid y \leq -1\}$
C. Range = $\{y \epsilon \mathbb{R} \mid y \neq \frac{1}{2}\}$
D. Range = $\{y \epsilon \mathbb{R} \mid y \geq -1\}$

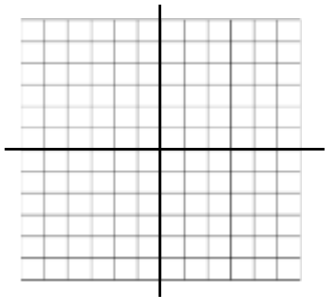
Additional Activities

Directions: Sketch a graph that satisfies the given domain and range in a graphing paper.

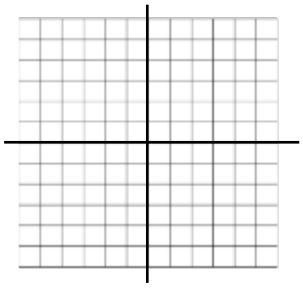
1. D = { x | x ϵ ℝ }
   R = { y | y ϵ ℝ }



2. D = { x | x ϵ ℝ }
   R = { y | y ≥ -3 }



3. D = { x | x ϵ ℝ }
   R = { y | y = 3 }



Questions:

1. What are the things you considered in sketching the graph of the given domain and range?
2. What are your ways in checking whether your graph is correct?

Answer Key

What I know

1. B	9. D
2. A	10. C
3. A	11. B
4. B	12. C
5. A	13. B
6. C	14. B
7. C	15. D
8. A	

What's In

1. Serve as guide
2. the more brownies; the more money
3. the more bread; the more sack of flour
4. the more contest categories; the more medals
5. the number of hours spend studying; the score on the exam

What I Have Learned

1. True	6. True
2. False	7. True
3. True	8. False
4. True	9. True
5. False	10. False

What's In

1. Serve as guide
2. the more brownies; the more money
3. the more bread; the more sack of flour
4. the more contest categories; the more medals
5. the number of hours spend in studying; the score on the exam

What's New

1. number of hours cleaning; amount of money earned
2. the number of hours cleaning
3. the amount of money earned
4. number of hours cleaning; controls; amount of money earned
5. amount of money earned; depends on; number of hours cleaning
6. Php200
7. No. In reality, Robert can only work up to 8 hours. This is the maximum number of working hours in a day, unless he decides to have 2 hours overtime making it 10-hour worked in a day. Also, no one in the neighborhood will allow him to clean their backyard during the night when the owners are already sleeping.

What I Can Do

a. independent variable because it controls the amount paid
b. dependent variable because it depends on the number of liters of gasoline
c. D: $\{x \in \mathbb{R} | 0 \leq x \leq 7\}$
   R: $\{y \in \mathbb{R} | 0 \leq y \leq 322\}$
d. Yes, because it is between zero and 7 liters of gasoline
e. No, because the amount paid for the maximum liters of gasoline that the motorcycle tank can hold is Php322.
f. D: $\{x \in \mathbb{R} | 1.75 < x \leq 7\}$
   R: $\{y \in \mathbb{R} | 80.50 \leq y \leq 322\}$
g. No, because the capacity of the tank is up to 7 liters.
h. No. It is not safe for him.
i. Turn engine off, and get off the motorcycle when refueling.

What's more

Activity 1.

To Whom I Depend On?

1. Serve as guide
2. dependent; depends on
3. dependent; depends on
4. independent; controls
5. independent; control

Activity 2. Limited or Limitless?

1. D: $\{-3,0,-5,2,1\}$
   R: $\{-3,3,-7,7,-5\}$
2. D: $\{-2,-1,0,1,3\}$
   R: $\{7,4,3,12\}$
3. D: $\{x \in \mathbb{R} | x \neq -4\}$
   R: $\{y \in \mathbb{R} | y \neq 1\}$
4. D: $\{x \in \mathbb{R} | x \geq 2\}$
   R: $\{y \in \mathbb{R} | y \geq 0\}$
5. D:$\{x \in \mathbb{R}\}$
   R: $\{y \in \mathbb{R}\}$
6. D: $\{x \in \mathbb{R}\}$
   R: $\{y \in \mathbb{R} | y \geq 1\}$
7. D: $\{-3,-2,-1,0,1,2,3\}$
   R: $\{-2,-1,0,1,2\}$
8. D: $\{x \in \mathbb{R} | x \geq -2\}$
   R: $\{y \in \mathbb{R} | y \geq -1\}$
9. D: $\{x \in \mathbb{R}\}$
   R: $\{-2\}$
10. D: $\{x \in \mathbb{R} | x \neq 1\}$
   R: $\{y \in \mathbb{R} | y \neq -2\}$

Assessment

1. A	6. A	11. C
2. D	7. C	12. C
3. C	8. D	13. B
4. D	9. D	14. A
5. D	10. B	15. C

Additional Activities

1. Answers vary (Any graph of an odd function)
2. Answers vary (Graph of a quadratic function)
3. Graph of a constant function passing through the point (0,3)

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