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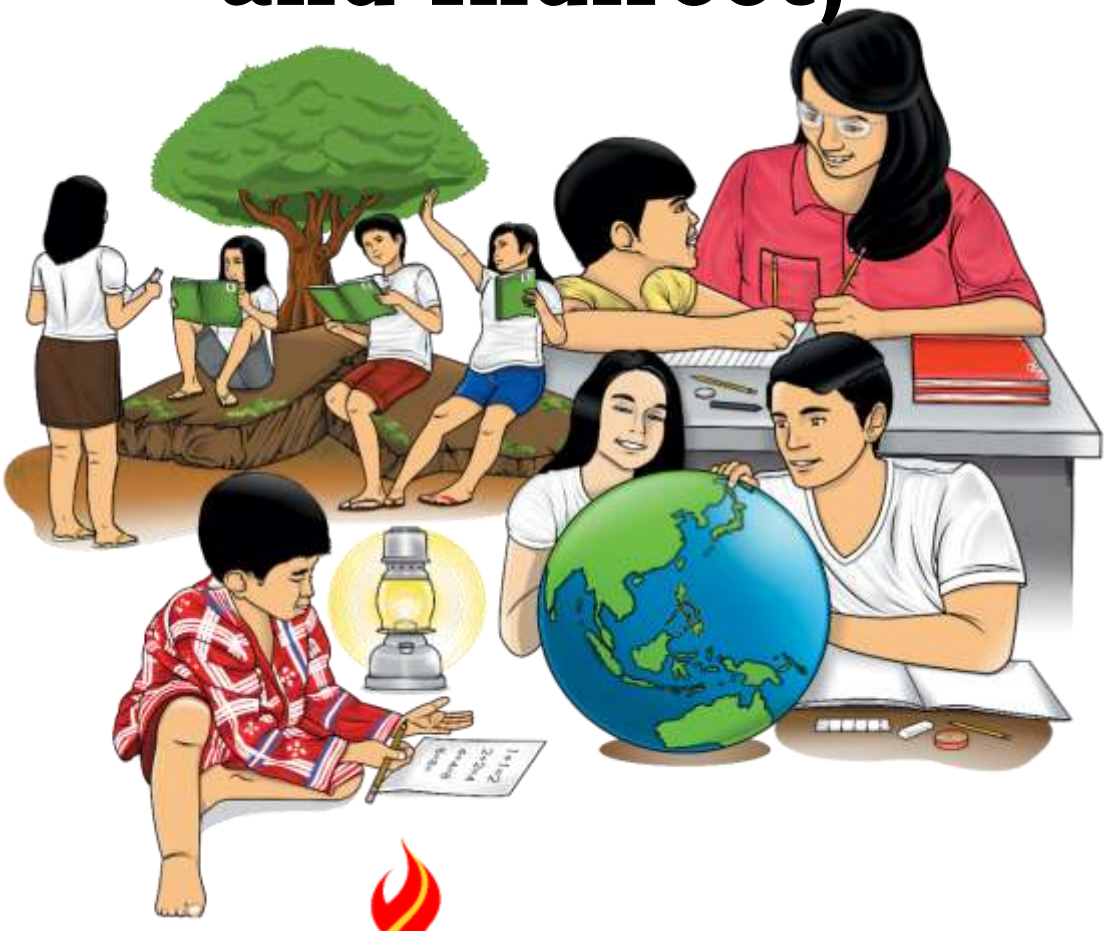
Mathematics
Quarter 2 – Module 14: "Writing Proofs (Direct and Indirect)"



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Secretary: Leonor Magtolis Briones
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Development Team of the Module

Writers:	Rizza Mae S. Bagsican
Language Editor:	Vicente P. Balbuena
Content Editor:	Riza C. Cadavos, Joselito C. Serna, Cecille C. Guimary, Mary Grace R. Prado, Jessel G. Azarcon
Layout Editor:	Jay R. Tinambacan
Reviewers:	Rhodora C. Luga, Charlie May C. Octal, Irelle B. Tesado, Mellanie G. Tuyor, Keziah Grace B. Presto
Illustrator:	Rizza Mae S. Bagsican
Layout Artist:	Rizza Mae S. Bagsican, Leonil Rechie P. Cahanap
Management Team:	Francis Cesar B. Bringas, Isidro M. Biol, Jr., Rhea J. Yparraguirre Maripaz F. Magno, Josephine Chonie M. Obseñares, Josita B. Carmen, Celsa A. Casa, Regina Euann A. Puerto, Bryan L. Arreo, Leopardo P. Cortes, Claire Ann P. Gonzaga, Lieu Gee Keeshia C. Guillen

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Department of Education – Caraga Region

Office Address: Learning Resource Management Section (LRMS)
J.P. Rosales Avenue, Butuan City, Philippines 8600
Telefax No.: (085) 342-8207 / (085) 342-5969
E-mail Address: caraga@deped.gov.ph

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Mathematics

Quarter 2 – Module 14: "Writing Proofs (Direct and Indirect)"

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Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

What I Need to Know

In this module, you will be acquainted with the different ways in writing proofs which will help sharpen your ability to provide logical reasoning about a phenomenon or a situation. The scope of this module enables you to use it in many different learning situations. The lesson is arranged to follow the standard sequence of the course. But the order in which you read them can be changed to correspond with the textbook you are now using.

This module contains:

Lesson 1- Writing Direct Proof and Indirect Proof

After going through this module, you are expected to:

1. define direct proof and indirect proof;
2. differentiate a direct proof from an indirect proof; and
3. write direct proof and indirect proof.

What I Know

Pre-Assessment:

Directions: Choose the letter of the correct answer. Write your answers on a separate sheet of paper.

1. What is a direct proof?
   A. A proof that always involves the multiplication of two values.
   B. A proof that can only use number properties to show that a certain statement is false.
   C. A proof that assumes a statement's hypothesis is true and uses a series of logical deductions to conclude that the statement's conclusion is true.
   D. A proof that assumes that the statement being proven is false and then attempts to find a contradiction to that assumption proving the original statement to be true.

2. What does an indirect proof rely on?
   A. a contradiction
   B. a function
   C. a product
   D. a sum

3. Which of the following is NOT a difference between direct and indirect proofs?
   A. Indirect proofs look for a contradiction to their original assumption, and direct proofs do not.
   B. Direct proofs use other theorems, rules, and definitions in their proofs, and indirect proofs do not.
   C. Indirect proof usually starts with the statement 'assume not' or 'assume the opposite', and direct proofs do not.
   D. Direct proofs involve assuming a hypothesis is true, and indirect proofs involve assuming a conjecture is false.

4. Which of the following statements is true?
   A. Start an indirect proof by assuming that the conclusion is true.
   B. Start an indirect proof by assuming that the conclusion is false.
   C. Both of these statements are true.
   D. Neither of these statements are true.

Consider the statement for numbers 5 and 6.

Prove: "$If\ 2(3m-5)=8, then\ m=3$".

Statement	Reasons
1. $2(3m-5)=8$	Given
2. $2(3m)-2(5)=8$	**(5)**
3. $6m-10=8$	Simplification
4. **(6)**	Addition Property of Equality
5. $6m=18$	Simplification
6. $\frac{6m}{6}=\frac{18}{6}$	Division Property of Equality
7. $m=3$	Simplification

5. What is the reason for the statement $2(3m)-2(5)=8$ in Step 2?
   A. Addition Property of Equality
   B. Subtraction Property of Equality
   C. Distributive Property of Equality
   D. Transitive Property of Equality

6. Which statement is true using Addition Property of Equality in Step 4?
   A. $6m-10-8=8-8$
   B. $6m-10+8=8+8$
   C. $6m-10+10=8+10$
   D. $6m-10-10=8-10$

Consider the statement for numbers 7 and 8.

Prove that if $AB \cong CD$ and $CD \cong EF$, then $AB \cong EF$

Statements	Reasons
1. (**7**)	Given
2. $AB=CD$ and $CD=EF$	Definition of Congruent Segments
3. $AB=EF$	**(8)**
4. $\overline{AB} \cong \overline{EF}$	Definition of Congruent Segments

7. Which statement is correct for Step 1 of the proof?
   A. $\overline{AB} \cong \overline{CD}$
   B. $\overline{CD} \cong \overline{EF}$
   C. $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$
   D. $\overline{AB} \cong \overline{EF}$ and $\overline{AB} \cong \overline{CD}$

8. What is the reason for the statement $AB=EF$ in Step 3 of the proof?
   A. Addition Property of Equality
   B. Distributive Property of Equality
   C. Subtraction Property of Equality
   D. Transitive Property of Equality

9. What assumption would you make from the statement that, *"There can be only one 90° angle in a triangle"* using indirect proof?
   A. All the angles in a triangle add up to 90°.
   B. All the angles in a triangle add up to 180°.
   C. There can be more than one 90° angle in a triangle.
   D. There can be more than one 180° angle in a triangle.

10. What assumption would you make from the statement that *"If* $3x + 7 > 13$*, then* $x > 2$" using indirect proof*?*
   A. $x \leq 2$
   B. $x < 2$
   C. $x \geq 2$
   D. $x = 2$

11. What is the first step of an indirect proof?

   A. Write the given.
   B. Contradict the given.
   C. Assume the negation of the given.
   D. Assume the negation of what you are trying to prove.

12. If you were proving that a given line l, is perpendicular to another line j, using indirect proof, what would your initial assumption be?

   A. Assume that line l is not perpendicular to line j.
   B. Assume that line l is perpendicular to line j.
   C. Assume that line l and line j are parallel.
   D. None of these are correct.

13. Which is a contradiction to the statement $\overline{XY} \cong \overline{RT}$?

   A. $\overline{XY} < \overline{RT}$
   B. $\overline{TR} \cong \overline{YX}$
   C. $\overline{XY} \approx \overline{RT}$
   D. $\overline{XY} \leq \overline{RT}$

14. Given $\angle A$ is an angle. Prove $\angle A \cong \angle A$. What is the reason for the statement $m\angle A \cong m\angle A$ in Step 2 of the proof?

Statements	**Reasons**
1. $\angle A$ is an angle	Given
2. $m\angle A \cong m\angle A$	**(14)**
3. $\angle A = \angle A$	Definition of Congruent Angles

A. Distributive Property of Equality
B. Reflexive Property of Equality
C. Symmetric Property of Equality
D. Transitive Property of Equality

15. Given $\angle A \cong \angle B$ and $\angle B \cong \angle C$. Prove $\angle A \cong \angle C$. What is the reason for the statement $m\angle A = m\angle C$ in Step 3 of the proof?

Statements	Reasons
1. $\angle A \cong \angle B$ and $\angle B \cong \angle C$	Given
2. $m\angle A = m\angle B$ and $m\angle B = m\angle C$	Definition of Congruent Angles
3. $m\angle A = m\angle C$	**(15)**
4. $m\angle A \cong m\angle C$	Definition of Congruent Angles

A. Addition Property of Equality
B. Distributive Property of Equality
C. Subtraction Property of Equality
D. Transitive Property of Equality

Lesson 1 Writing Direct Proof and Indirect Proof

In this lesson, your previous knowledge about properties of equality and inequality, congruence, as well as definitions and postulates in Geometry will be put to test as these are necessary information in making proofs.

What's In

Activity: Equality Express

Directions: To illustrate each of the given property, fill in the blanks with the correct number to complete the following mathematical statements. Write your answers on a separate sheet of paper.

Properties of Equality		**Statements**
1. Addition Property of Equality	→	If $3+2=5$, then $(3+2)+4=5+$ __
2. Subtraction Property of Equality	→	If $5+1=6$, then $(5+1)-2=$ __ -2
3. Multiplication Property of Equality	→	If $(4)(\_)(2)=12(2)$
4. Division Property of Equality	→	If $(8)(2)=16$, then $\frac{(8)(2)}{\_\_}=\frac{16}{\_\_}$
5. Transitive Property of Equality	→	If $7+5=12$ and $12=10+2$, then $7+$ __ $=$ __ $+2$

Questions:

1. Did you easily find the number that will make the equality true? How?
2. How did the Properties of Equality help you in finding the missing numbers to complete the mathematical statements?
3. Do you think that these properties will help you in writing proofs? Explain your answer.

What's New

Knowing the validity of the hypothesis in the previous lesson about conditional statements is an important skill to form statements of proof and make generalization. Thus, you are now ready to study the types of proofs.

Activity: Prove Me Right!

Directions: Study and examine how each of the proof below presents its way of proving the given statement. Then, answer the questions that follow. Write your answers on a separate sheet of paper.

Prove that if $3x+6=18$ then $x=4$.

Given: $3x+6=18$

Prove: $x=4$

Proof A:

Statements	Reasons
1. $3x + 6 = 18$	Given
2. $3x + 6 - 6 = 18 - 6$ $3x = 12$	Subtraction Property of Equality
3. $\frac{3x}{3} = \frac{12}{3}$ $x = 4$	Division Property of Equality

Proof B:

Let $x \neq 4$. If $x \neq 4$, then $x < 4$ or $x > 4$

Situation 1: If $x < 4$

Statements	Reasons
1. $x < 4$	Given
2. $3x < 3(4)$	Multiplication Property of Equality
3. $3x < 12$	Simplification
4. $3x + 6 < 12 + 6$	Addition Property of Equality
5. $3x + 6 < 18$	Simplification

Notice that if $x \neq 4$, then $3x + 6 < 18$. Therefore, this is a contradiction to the given that $3x + 6 = 18$.

Situation 2: If $x > 4$

Statements	Reasons
1. $x > 4$	Given
2. $3x > 3(4)$	Multiplication Property of Equality
3. $3x > 12$	Simplification
4. $3x + 6 > 12 + 6$	Addition Property of Equality
5. $3x + 6 > 18$	Simplification

Notice that if $x \neq 4$, then $3x + 6 > 18$. Therefore, this is a contradiction to the given that $3x + 6 = 18$.

Questions:

1. What can you say about Proof A? Proof B?
2. Is there a difference between Proof A and Proof B? If yes, what is the difference? If no, why do you say so?
3. Why is it important to know how to write proofs given a problem or a situation?

What is It

In the previous activity, two methods of proof were used to prove the given statement – Direct Proof and Indirect Proof. Demonstrating understanding on how to develop these proofs will be easier by considering some of the Properties of Equality, Inequality and Congruence including Postulates as cited below:

Properties of Equality

Let a, b, and c be real numbers.

Properties	Statements
Reflexive Property	$a = a$
Symmetric Property	If $a = b$, then $b = a$.
Transitive Property	If $a = b$ and $b = c$, then $a = c$.
Addition Property	If $a = b$, then $a + c = b + c$.
Subtraction Property	If $a = b$, then $a - c = b - c$.
Multiplication Property	If $a = b$, then $ac = bc$.
Division Property	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.

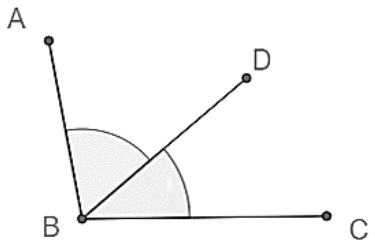
Properties of Inequality

Let a, b, and c be real numbers.

Properties	Statements
Addition Property	If $a < b$, then $a + c < b + c$
Subtraction Property	If $a < b$, then $a - c < b - c$
Multiplication Property	If $a < b$, and $c > 0$, then $ac < bc$. If $a > b$, and $c > 0$, then $ac > bc$. If $a > b$, and $c < 0$, then $ac < bc$. If $a < b$, and $c < 0$, then $ac > bc$.
Division Property	If $a < b$ and $c > 0$, then $\frac{a}{c} < \frac{b}{c}$. If $a < b$ and $c < 0$, then $\frac{a}{c} > \frac{b}{c}$.

Postulates

Postulates	Statements
Segment Addition Postulate	$\overline{AB} + \overline{BC} = \overline{AC}$
Angle Addition Postulate	

	 $\angle ABD + \angle DBC = \angle ABC$

A **proof** is a logical argument in which each statement you make is supported/justified by given information, definitions, axioms, postulates, theorems, and previously proven statements.

Note:

- *An axiom is any mathematical statement that serves as a starting point from which other points are logically derived.*
- *A postulate is a statement that is accepted without proof.*
- *A theorem is a statement accepted after it is proved deductively.*

There are two methods in writing proof. These include:

1. Direct Proof
2. Indirect Proof

Direct proof is a type of logical reasoning that uses accepted facts to reason in a step-by-step manner until the desired statement is obtained.

Steps in writing a direct proof:

1. Assume the statement p is true.
2. Use what you know about p and other facts as necessary to deduce that another statement q is true, that is to show $p \rightarrow q$ (if p then q) is true.

Two-Column Form is one way of organizing a proof. This consists of two columns, one for statements and one for reasons.

Illustrative Example 1.

If $2\,(3x + 4) = 56$, then $x = 8$.
Given: $2\,(3x + 4) = 56$
Prove: $x = 8$

Proof:

Statements	**Reasons**
1. $2(3x + 4) = 56$	Given
2. $2(3x) + 2(4) = 56$	Distributive Property of Equality
3. $6x + 8 = 56$	Simplification
4. $6x + 8 - 8 = 56 - 8$	Subtraction Property of Equality

5. $6x = 48$	Simplification
6. $\frac{6x}{6} = \frac{48}{6}$	Division Property of Equality
7. $x = 8$	Simplification

Illustrative Example 2.

Given: $\overline{AB} = \overline{CD}$
Prove: $\overline{AC} = \overline{BD}$

Proof:

Statements	**Reasons**
1. $\overline{AB} = \overline{CD}$	Given
2. $\overline{BC} = \overline{BC}$	Reflexive Property
3. $\overline{AB} + \overline{BC} = \overline{CD} + \overline{BC}$	Addition Property of Equality
4. $\overline{AC} = \overline{AB} + \overline{BC}$ $\overline{BD} = \overline{BC} + \overline{CD}$	Segment Addition Postulate
5. $\overline{AC} = \overline{BD}$	Substitution Property

Indirect proof is a type of proof in which a statement to be proved is assumed false (by negation) and if the assumption leads to an impossibility, then the statement assumed false has been proved to be true.

Steps in writing an indirect proof:

1. Identify the statement you want to prove. Assume temporarily that this statement is false by assuming that its opposite is true.
2. Reasons logically until you reach a contradiction.
3. Point out that the desired conclusion must be true because the contradiction proves the temporary assumption false.

Illustrative Example 1.

If $x = 2$, then $3x - 4 \neq 8$.
Given: $x = 2$
Prove: $3x - 4 \neq 8$

Proof:

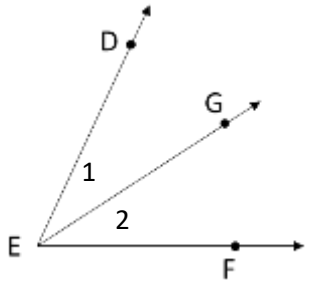
Statements	**Reasons**
1. $3x - 4 = 8$	Negation (Indirect proof assumption)
2. $3x - 4 + 4 = 8 + 4$	Addition Property of Equality
3. $3x = 12$	Simplification
4. $\frac{1}{3}(3x) = \frac{1}{3}(12)$	Multiplication Property of Equality
5. $x = 4$	Simplification

But $x = 4$ contradicts the given statement that $x = 2$. Since our assumption is false therefore, $3x - 4 \neq 8$ is true.

Illustrative Example 2.

Given: $\overrightarrow{ED}$ is not perpendicular to $\overrightarrow{EF}$

Prove: ∠1 and ∠2 are not complementary



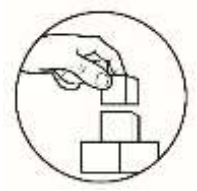
Proof:

Statements	**Reasons**
1. Suppose ∠1 and ∠2are complementary.	Indirect proof assumption
2. $m\angle 1 + m\angle 2 = 90$	The sum of the measures of complementary angles is 90^0.
3. $m\angle 1 + m\angle 2 = m\angle DEF$	Angle Addition Postulate
4. $m\angle DEF = 90$	Substitution
5. $\angle DEF$ is a right angle	Definition of Right Angle
6. $\overrightarrow{ED} \perp \overrightarrow{EF}$	Definition of Perpendicularity

Note that statement 6 is a contradiction to the given hypothesis that $\overrightarrow{ED}$ is not perpendicular to $\overrightarrow{EF}$, hence, the assumption that ∠1 and ∠2are complementary must be false and it follows that ∠1 and ∠2 are not complementary.

Going Back

Let us look back at the Prove Me Right Activity on page 7 where you were asked to determine the difference between the types of proof used given one statement. Considering the discussions presented, Proof A clearly illustrates direct proof since the proof uses accepted facts to reason in a step-by-step manner until the desired statement is obtained. While Proof B shows how a given statement is proved using indirect proof since the proof started with an assumption opposite to the given statement until a contradiction is reached in the process.



What's More

Activity 1: I Need Proof!

Directions: Fill in the blanks by choosing your answers from the box. Write it on a separate sheet of paper.

$2(b+1) = -6$	$2b + 2 = -6$	$\frac{2b}{2} = \frac{-8}{2}$
Given	Prove	by Simplification
Subtraction Property of Equality	Addition Property of Equality	Multiplicative Property of Equality

Prove that if $2(b+1) = -6$, then $b = -4$.

Given:$2(b+1) = -6$

Prove: $b = -4$

Proof:

Statements	Reasons
1. $2(b+1) = -6$	
2.	Distributive Property of Equality
3. $2b + 2 - 2 = -6 - 2$	
4. $2b = -8$	by Simplification
5.	Division Property of Equality
6.$b = -4$	

Activity 2: Prove Me Indirectly!

Directions: Arrange the statements to its logical order using an indirect proof. Use letters A to F. Write your answers on a separate sheet of paper.

Prove that if $y = -3$, then $4(y-3) \neq -20$.

_____1. $4y - 12 = -20$ by distributive property of equality.

_____2. Hence our assumption is false and $4(y-3) \neq -20$ is true

_____3. Using addition property of equality, $4y - 12 + 12 = -20 +$ 12.So $4y = -8$ by simplifying.

_____4. Assume that $4(y-3) = -20$. Take this statement as true and solve for y.

_____5. Thus, division property of equality, $\frac{4y}{4} = \frac{-8}{4}$. So $y = -2$ by simplifying.

_____6. But $y = -2$, contradicts the given statement that $y = -3$.

Activity 3: I Like to Prove It! Prove It!

Directions: Prove the given statement using ONLY ONE of the two methods (direct proof or indirect proof) which you find easier to use. Write your answers on separate sheet of paper.

If $\frac{1}{5}(x+10) = 3$, then $x = 5$

Prove: $\frac{1}{5}(x+10) = 3$

Given: $x = 5$

What I Have Learned

Directions: Given are the steps in writing proofs for both direct and indirect. Classify these steps accordingly to the type of proof.

Steps in writing Direct Proof	**Steps in Writing Indirect Proof**

- Assume the statement p is true.
- Cite the reasons logically until you reach a contradiction.
- Point out that the desired conclusion must be true because the contradiction proves the temporary assumption false.
- Identify the statement you want to prove. Assume temporarily that this statement is false (by negation) by assuming that its opposite is true.
- Use what you know about p and other facts as necessary to deduce that another statement q is true, that is show $p \rightarrow q$ (if p then q) is true.

What I Can Do

Directions: Given the proposition ***"If you will follow the health protocols during COVID-19 pandemic, then you will be safe from getting infected with the virus"***. Do you agree with the statement? Justify your answer by citing your proofs.

Rubric for the Written Proof

Score	Descriptions
(4) Exemplary	A correct and complete proof is given. Some irrelevant information may be included, particularly on timed work where the student is unable to polish up the presentation.
(3) Acceptable	A correct approach to proving the proposition is attempted. Some statements may be unjustified or improperly justified, but errors are minor and could be fixed without substantially changing the proof.
(2) Basic	Statements linked into a reasonable (though perhaps misguided) attempt to prove the theorem. The proof may be left incomplete or may depend upon a major unjustified leap.
(1) Poor	Unconnected, mostly true statements properly deduced from the given. Listing facts without a sense of how to link them to get a correct proof. May just jump to the conclusion without justification.
(0) Unacceptable	Mainly incorrect consequences improperly deduced from the given. Little or no sense of how to prove the result.

Assessment

Post-Assessment

Directions: Choose the letter of the correct answer. Write the chosen letter on a separate sheet of paper.

1. What is an indirect proof?
   A. A proof that always involves the multiplication of two values.
   B. A proof that can only use number properties to show that a certain statement is false.
   C. A proof that assumes a statement's hypothesis is true and uses a series of logical deductions to conclude that the statement's conclusion is true
   D. A proof that assumes that the statement being proven is false and then attempts to find a contradiction to that assumption proving the original statement to be true

2. A proof that $p \rightarrow q$ is true based on the fact that q is true, such proof is known as _____.
   A. Direct Proof
   B. Indirect Proof
   C. Both A and B are correct
   D. Neither A nor B are correct

3. Which of the following occurs with a direct proof?
   A. Statements are supported by known facts and definitions.
   B. A conditional statement is proven.
   C. A series of statements are made.
   D. All statements are correct.

4. Which statement is NOT true?
   A. Start an indirect proof by assuming that the statement is true.
   B. Start an indirect proof by assuming that the statement is false.
   C. Use direct proof through accepted facts to reason in a step-by-step manner until the desired statement is arrived.
   D. Use two-column form to show a way of organizing a proof. This form consists of two columns, one for statements and one for reasons.

5. What assumption would you make from the statement that $\overline{AB}$ is congruent to $\overline{BC}$ using indirect proof?
   A. $A \neq B$
   B. $B \neq C$
   C. $AB \neq BC$
   D. $\overline{BA} \leq \overline{CB}$

Consider the statement for your answers in numbers 6 and 7.

Given: A, B, C, and D are collinear in that order and $AB = CD$. Prove: $AC = BD$

Statements	**Reasons**
1. A, B, C,and D are collinear, in that order.	Given
2. $AB = CD$	Given
3. $BC = BC$	**(6)**
4. $AB + BC = BC + CD$	Addition Property of Equality
5. $AB + BC = AC$ $BC + CD = BD$	**(7)**
6. $AC = BD$	Substitution Property

6. What is the "reason" for the "statement" $BC = BC$ in Step 3 of the proof?
   A. Commutative Property
   B. Reflexive Property
   C. Symmetric Property
   D. Transitive Property

7. What is the "reason" from the "statement" $AB + BC = AC$, $BC + CD = BD$ in Step 5 of the proof?
   A. Angle Addition Postulate
   B. Segment Addition Postulate
   C. Commutative Property of Equality
   D. Distributive Property of Equality

Consider the statement for your answers in numbers 8 and 9.

Prove: If $y + \frac{4}{3} = \frac{10}{3}$, then $y = 2$

Statement	**Reasons**
1. $y + \frac{4}{3} = \frac{10}{3}$	Given
2. $3y + 4 = 10$	Multiplication Property of Equality
3. **(8)**	Subtraction Property of Equality
4. $y = 2$	**(9)**

8. Which statement is true using Subtraction Property of Equality in Step 3?
   A. $3y + 4 - 10 = 10 - 10$
   B. $3y + 4 - 4 = 10 - 4$
   C. $3y + 4 + 10 = 10 + 10$
   D. $3y + 4 + 4 = 10 + 4$

9. What is the reason for the statement $y = 2$ in Step 4 of the proof?
   A. Addition Property of Equality
   B. Multiplication Property of Equality

C. Subtraction Property of Equality
D. Substitution Property

10. What assumption would you make from the statement that $\triangle ABC \cong \triangle DEF$ using indirect proof?
A. $\triangle ABC \neq \triangle DEF$
B. $\triangle ABC \ncong \triangle DEF$
C. $\triangle ABC \leq \triangle DEF$
D. $\triangle ABC \geq \triangle DEF$

Consider the statement for your answers in numbers 11 and 12.
If $x = 2$, then $3x - 5 \neq 10$. Prove this statement by contradiction.

Statements	**Reasons**
1. $3x - 5 = 10$	
2. $3x - 5 + 5 = 10 + 5$	Addition Property of Equality
3. $3x = 15$	by Simplification
4. $x = 5$	Division Property of Equality

11. What is the reason for the statement $3x - 5 = 10$ in Step 1?
A. by Contradiction
B. by Negation
C. by Simplification
D. by Substitution

12. What concluding statement can be drawn from the proof presented?
A. Since $x = 2$ is false, therefore $3x - 5 = 10$.
B. Since $x = 2$ is true, therefore $3x - 5 = 10$.
C. Since $x = 5$ is false, therefore $3x - 5 \neq 10$.
D. Since $x = 5$ is true, therefore $3x - 5 \neq 10$.

For the statement in numbers 13, 14,15.
Consider that points $A, B,$ and C are collinear.
Prove: If $AB = \frac{1}{2}AC$, then B is the midpoint of $\overline{AC}$.

Statements	**Reasons**
1. $AB = \frac{1}{2}AC$	Given
2.**(13)**	Multiplication Property of Equality
3. $AB + AB = AC$	Substitution Property of Equality
4. $AB + BC = AC$	**(14)**
5. $AB + AB = AB + BC$	Transitive Property of Equality
6. $AB = BC$	**(15)**
7. B is the midpoint of $\overline{AC}$	Definition of a Midpoint

13. Which statement is true using Multiplication Property of Equality in Step 2?

A. $\frac{1}{2}AC$
B. $AB = 2AC$
C. $2AB = AC$
D. $2AB = 2AC$

14. What is the reason for the statement $AB + BC = AC$ in Step 4?

A. Line Addition Postulate
B. Segment Addition Postulate
C. Line Segment Addition
D. Congruent Segment Addition

15. What is the reason for the statement $AB = BC$ in Step 6?

A. Addition Property of Equality
B. Subtraction Property of Equality
C. Multiplication Property of Equality
D. Division Property of Equality

Additional Activities

Directions: Write a proof for the situation given below. Write your answers on separate sheet of paper.

Walking down a hallway in the school, you notice that the library is halfway between the canteen and the Science laboratory. The Science laboratory is halfway between the library and the computer laboratory. Prove that the distance between the entrances of the canteen and library is the same as the distance between the entrances of the Science laboratory and computer laboratory.

Answer Key

Pre-Assessment

1. C
2. A
3. B
4. B
5. C
6. C
7. C
8. D
9. C
10. A
11. D
12. A
13. A
14. B
15. D

What's In

1. 4
2. 6
3. 3
4. any number except 0
5. $7 + \mathbf{5} = \mathbf{10} + 2$

What's More

Activity 1: I Need Proof!

1. Given
2. $2b + 2 = -6$
3. Subtraction Property of Equality
5. $\frac{2b}{2} = \frac{-8}{2}$
6. by Simplification

Activity 2: Prove Me Indirectly!

1. B
2. F
3. C
4. A.
5. D.
6. E

Activity 3: I Like to Prove It! Prove It

Direct Proof

1. $\frac{1}{5}(x + 10) = 3$	Given
2. $\frac{1}{5}(x) + \frac{1}{5}(10) = 3$	Distributive Property of Equality
3. $\frac{1}{5}(x) + 2 = 3$	by Simplification
4. $\frac{1}{5}(x) + 2 - 2 = 3 - 2$	Subtraction Property of Equality
5. $\frac{1}{5}(x) = 1$	by Simplification
6. $5\left[\frac{1}{5}(x)\right] = 5[1]$	Multiplication Property of Equality
7. $x = 5$	by Simplification

Indirect Proof
This is to prove that $x \neq 5$
Situation 1: If $x > 5$

1. $x > 5$	Given
2. $x + 10 > 5 + 10$	Addition Property of Inequality
3. $x + 10 > 15$	by Simplification
4. $\frac{1}{5}(x + 10) > \frac{1}{5}(15)$	Multiplication Property of Inequality
5. $\frac{1}{5}(x + 10) > 3$	by Simplification

This is a contradiction that $\frac{1}{5}(x + 10) = 3$. Therefore, $x = 5$

Situation 2: If $x < 5$

1. $x < 5$	Given
2. $x + 10 < 5 + 10$	Addition Property of Inequality
3. $x + 10 < 15$	by Simplification
4. $\frac{1}{5}(x + 10) < \frac{1}{5}(15)$	Multiplication Property of Inequality
5. $\frac{1}{5}(x + 10) < 3$	by Simplification

This is a contradiction that $\frac{1}{5}(x + 10) = 3$.
Therefore, $x = 5$

What I Have Learned

Steps in Writing Direct Proof	**Steps in Writing Indirect Proof**
1. Assume the statement p is true. 2. Use what you know about p and other facts as necessary to deduce that another statement q is true, that is show $p \rightarrow q$ (if p then q) is true.	1.Identify the statement you want to prove. Assume temporarily that this statement is false by assuming that its opposite is true. 2. Reason logically until you reach a contradiction. 3. Point out that the desired conclusion must be true because the contradiction proves the temporary assumption false.

What I Can Do

Answers may vary.

Post-Assessment

1. D
2. A
3. D
4. A
5. C
6. B
7. B
8. B
9. B
10. B
11. B
12. C
13. C
14. B
15. B

Additional Activities

Answers may vary

References

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For inquiries or feedback, please write or call:

Department of Education - Bureau of Learning Resources (DepEd-BLR)

Ground Floor, Bonifacio Bldg., DepEd Complex
Meralco Avenue, Pasig City, Philippines 1600

Telefax: (632) 8634-1072; 8634-1054; 8631-4985

Email Address: blr.lrqad@deped.gov.ph * blr.lrpd@deped.gov.ph