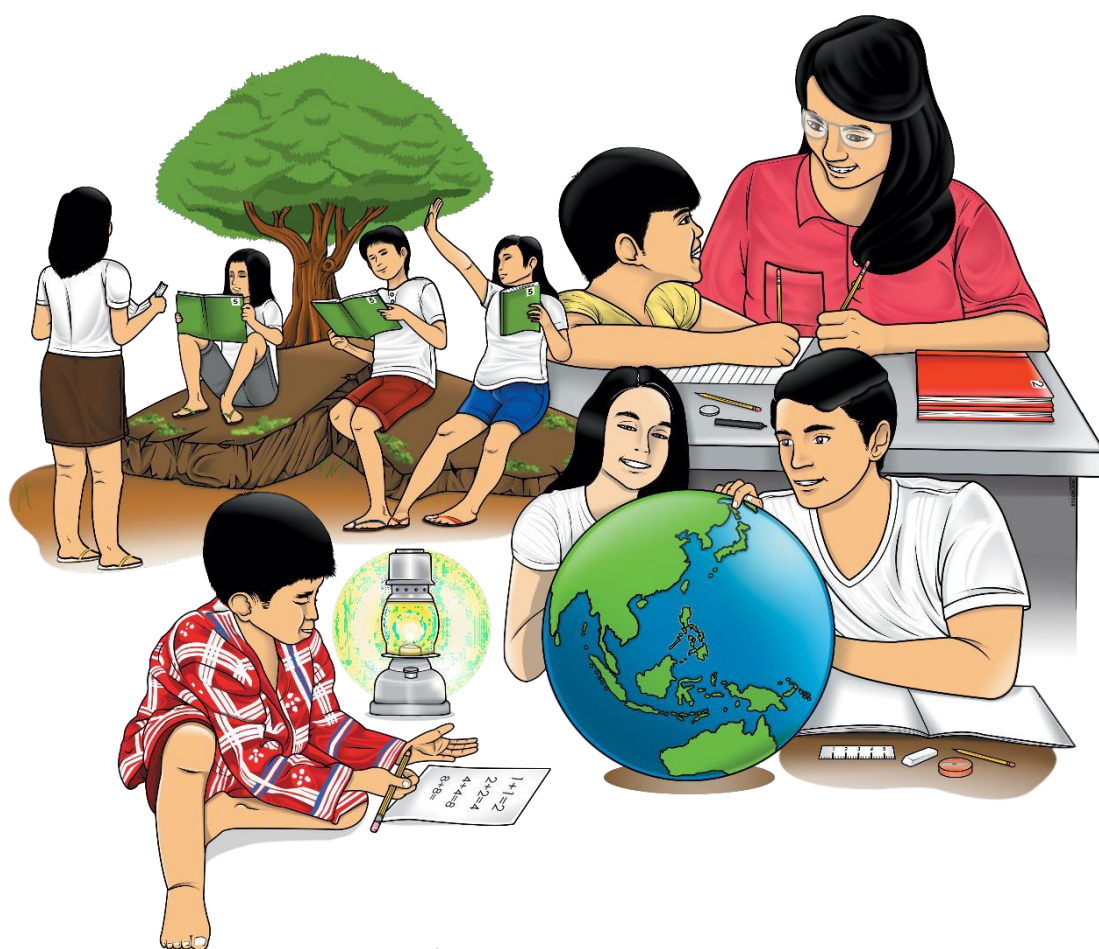


# Mathematics

## Quarter 2 – Module 7: Solving Problems on Circles



**Mathematics – Grade 10**  
**Alternative Delivery Mode**  
**Quarter 2 – Module 7: Solving Problems on Circles (M10GE-If-2)**  
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# Mathematics

## Quarter 2 – Module 7: Solving Problems on Circles

## **Introductory Message**

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

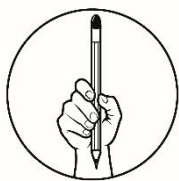


## ***What I Need to Know***

Concepts on circles are basic necessity in many fields such as construction, landscaping and engineering. Aside from these concepts, the properties of lines and its subsets and their relationship to circles are important. These are the lines that either touch or intersect circles.

In this module, we will be applying previously learned concepts on chords, arcs, central angles, inscribed angles, secants, tangents, segments and sectors of a circle to solve problems. Specifically, you will be able to

1. Solve problems involving chords, arcs, central angles, and inscribed angles of circles; and
2. Solve problems involving tangents and secants of circles.

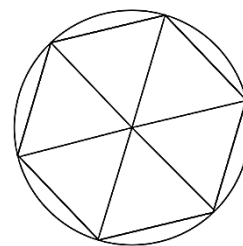


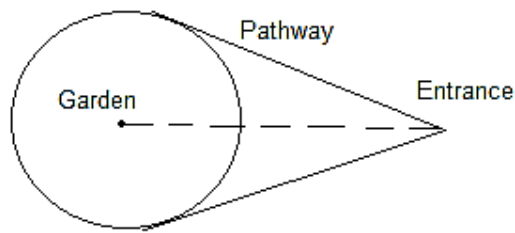
## What I Know

**Description:** This section will test how much you already know about problems involving circles and its chords, arcs, central angles, inscribed angles, tangent lines and secant lines.

**Instruction:** Read each item carefully. Choose the letter of your answer and write it on your answer sheet.

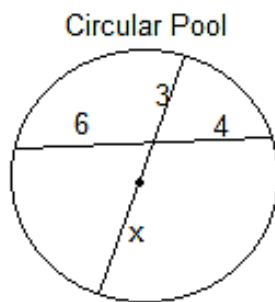
- Which of the following is the measure of the angle determined by the hands of a clock at five o'clock?  
a.  $90^\circ$                       b.  $120^\circ$                       c.  $150^\circ$                       d.  $180^\circ$
- A circle is divided into 6 equal arcs. Which of the following is the degree measure of each arc?  
a.  $30^\circ$                       b.  $60^\circ$                       c.  $90^\circ$                       d.  $120^\circ$
- A wheel has 30 spokes which are evenly spread. Which of the following is the measure of each central angle formed by the spokes?  
a.  $12^\circ$                       b.  $15^\circ$                       c.  $24^\circ$                       d.  $30^\circ$
- In a pie chart showing the expenditures of the Lopez family, the measure of the central angle corresponding to their food expenses is  $120^\circ$ . If their monthly income is Php 35,000.00, how much of it is spent for food?  
a. Php 11,666.67              c. Php 5,666.67  
b. Php 17,500.00              d. Php 12,000.00
- Catherine designed a pendant. It is a regular hexagon set in a circle. Suppose the vertices are connected by line segments and meet at the center of the circle as shown in the figure. Which of the following is the measure of each of the smallest angle formed at the center?  
a.  $22.5^\circ$                       b.  $45^\circ$                       c.  $60^\circ$                       d.  $72^\circ$
- A dart board has a diameter of 40 cm and is divided into 20 congruent sectors. Which of the following is the area of one of the sectors?  
a.  $20\pi \text{ cm}^2$               b.  $40\pi \text{ cm}^2$               c.  $80\pi \text{ cm}^2$               d.  $800\pi \text{ cm}^2$
- Mr. Alex designed a semicircle arch made of bent iron for their gate's entrance. Suppose the diameter of the semicircle is 3 meters, which of the following should be the length of the iron before bending to avoid wastage?  
a.  $1.5\pi \text{ m}$                       b.  $3\pi \text{ m}$                       c.  $6\pi \text{ m}$                       d.  $9\pi \text{ m}$
- A circular garden has pathways which are tangent to the garden whose center is 10 meters away from the entrance as shown in the figure below. If the radius of the circular garden is 5 meters, how long is each pathway?





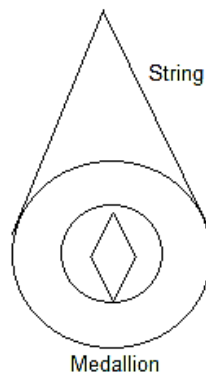
- a.  $3\sqrt{5}$  m      b.  $5\sqrt{3}$  m      c. 25m      d. 75m

9. A family tied a pair of ropes to their circular pool, to serve as a guide for their children who are practicing how to swim. The ropes intersect at the interior forming segment as shown in the figure below. Find the missing segment measure of the rope in meters.



- a. 5m      b. 6m      c. 7m      d. 8m

10. A circular medallion is hung such that the strings are tangent to the circle as shown in the figure. If the major arc measures  $230^\circ$ , find the degree measure of the angle formed by the strings.



- a.  $40^\circ$       b.  $50^\circ$       c.  $130^\circ$       d.  $230^\circ$

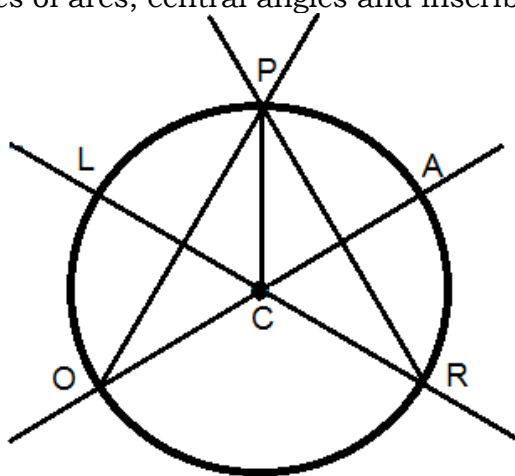


## What's In

Let us have a review of what you have already learned about arcs, central angles, inscribed angles, tangents and secants of circles by doing the following activities.

### Half, equal, or twice as?

In the figure that follows, A, R, O, L and P are points on circle  $\odot C$ . Fill in the blanks with the word **half, equal or twice** to show the relationship of the degree measures of arcs, central angles and inscribed angles formed.

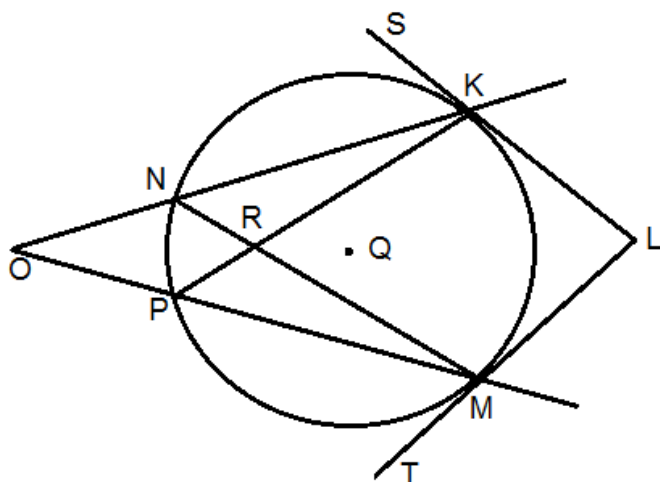


1. The measure of central angle  $\angle OCR$  is \_\_\_\_\_ to the degree measure of  $\widehat{OR}$ .
2. The degree measure of  $\widehat{LO}$  is \_\_\_\_\_ to the measure of central angle  $\angle LCO$ .
3. The measure of inscribed angle  $\angle OPR$  is \_\_\_\_ the measure of central angle  $\angle OCR$ .
4. The measure of central angle  $\angle PCA$  is \_\_\_\_ the measure of inscribed angle  $\angle POA$ .
5. The measure of inscribed angle  $\angle PRL$  is \_\_\_\_ the measure of central angle  $\angle PCL$ .

### Sum or Difference?

In the figure that follows, K, M, P, and N are points on  $\odot Q$ . Fill in the blanks with the word **sum, difference or one-half** to show the relationship of angles formed by intersecting tangents and secants.





1. The measure of  $\angle NOP$  formed by intersecting secants  $\overline{OK}$  and  $\overline{OM}$  outside the circle is equal to one-half the \_\_\_\_\_ of the degree measure of arcs  $\widehat{KM}$  and  $\widehat{NP}$ .
2. The measure of angle  $\angle KLM$  formed by intersecting tangents  $\overline{SL}$  and  $\overline{TL}$  outside the circle is equal to one-half the \_\_\_\_\_ of the degree measure of arcs  $\widehat{KNM}$  and  $\widehat{KM}$ .
3. The measure of angle  $\angle NRK$  formed by intersecting secants  $\overline{NM}$  and  $\overline{PK}$  inside the circle is equal to one-half the \_\_\_\_\_ of the degree measure of arcs  $\widehat{NK}$  and  $\widehat{PM}$ .
4. The measure of angle  $\angle KRM$  formed by intersecting secants  $\overline{NM}$  and  $\overline{PK}$  inside the circle is equal to one-half the \_\_\_\_\_ of the degree measure of arcs  $\widehat{KM}$  and  $\widehat{NP}$ .
5. The measure of angle  $\angle PMT$  formed by intersecting secant  $\overline{OM}$  and tangent  $\overline{LT}$  on the circle is equal to \_\_\_\_\_ the measure of the intercepted arc  $\widehat{PM}$ .

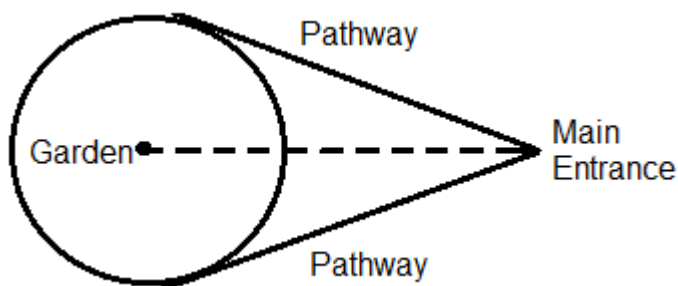


## What's New

In the following tasks, you are going to solve word problems involving segments formed by tangents and secants of a circle.

### How Long!

As illustrated in the figure that follows, there are two pathways from the main entrance where visitors can choose to walk going to the circular garden. The pathways are both tangent to the garden whose center is 40 meters away from the main entrance. If the area of the garden is about  $706.5 \text{ m}^2$ , how long is each pathway?



## Guide Questions

### Step 1: Understand the Problem

- What are you trying to find?
- In the illustration, what are the geometric figures involved? What are the given? What are the unknowns?
- What previously learned concepts on circles can be applied in the situation?

### Step 2: Devising a Plan

- After reviewing the concepts related to circles and the given, what steps should be undertaken to solve for the unknowns?
- Do we need additional markings in the given figure? Do we need to introduce variables to be used in the solution?
- What are the equations to be used to lead to the solution of the problem?
- How will the given values be used or manipulated in order to formulate the equation?

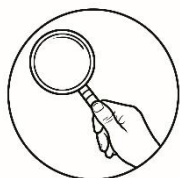
### Step 3: Carrying Out the Plan

- Implement the strategy and perform calculations
- Keep an accurate record of your work

### Step 4: Looking Back

- Check the results in the original problem. Do you think the answer fits in what is being asked? Does the computed value realistic or acceptable in real life situation?
- Don't forget to review your solution.

Good luck!



## What Is It

### Let's Discuss

Were you able to answer the given word problem? If not, do not worry. The procedures that follow will lead us to the detailed solution of the problem.

### Step 1: Understand the Problem

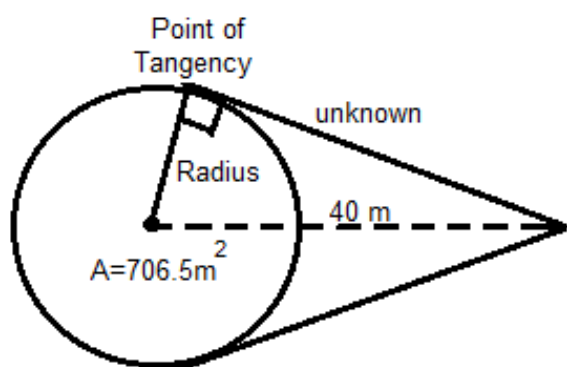
We are asked to solve for the **length of the pathway** which is the tangent segment to the circle. To solve, use the given area of the circle and the distance from

the center of the circle to the main entrance. Let the main entrance be the endpoint of the tangent outside the circle.

Remember that a tangent is a line or a segment that crosses a circle only once at a point called the point of tangency. Also, a theorem on tangent line also states that: "If a line is tangent to a circle, then it is perpendicular to the radius drawn to the point of tangency."

### Step 2: Devising a Plan

After understanding and reviewing the concepts involved in solving our problem, let us put the given markings or available information in the figure then define variables.



Let:

$r$  - radius

$u$  - unknown

$d$  - distance from the center to the main entrance of the garden

$A$  - Area of the circular garden

In the figure, a right triangle is formed;  $r$  and  $u$  are perpendicular since a tangent is perpendicular to the radius that crosses the circle at the point of tangency. Guided with this, the angle between  $r$  and  $u$  is a right angle. Thus, a right triangle is formed. In the right triangle,  $d$  is the hypotenuse, while  $r$  and  $u$  are the legs.

To solve for  $u$ , use the Pythagorean Theorem: "The sum of the squares of the legs of a right triangle is equal to the square of its hypotenuse". Using the defined variables, the Pythagorean theorem can be written as  $r^2 + u^2 = d^2$ .

Reviewing the given, use the area to know the value of  $r$ . To solve, use the formula of the area,  $A = \pi r^2$ .

### Step 3: Carrying Out the Plan

Pursue the plan,

$$A = \pi r^2 \quad \text{area of a Circle}$$

$$706.5 = \pi r^2 \quad \text{Substituting the given}$$

$$\frac{706.5}{\pi} = \frac{\pi r^2}{\pi} \quad \text{Dividing both sides of the equation by } \pi \text{ or } 3.14$$

$$225 = r^2$$

$$\sqrt{225} = \sqrt{r^2} \quad \text{Extracting the square root both sides of the equation}$$

$$15 = r$$

Having the value of  $r$ , we can now solve for the length of the pathway,  $u$  using the Pythagorean Theorem.

$$r^2 + u^2 = d^2$$

Extracting the square root in both sides of the equation

$$\sqrt{d^2 - r^2} = u$$

$$\sqrt{40^2 - 15^2} = u$$

Substitute the known values

$$\sqrt{(40 + 15)(40 - 15)} = u$$

Values inside the radical sign can be factored as sum and difference of two squares

$$\sqrt{(55)(25)} = u$$

Simplify each factor

$$\sqrt{1,375} = u$$

Further simplifying

$$37.08 = u$$

#### Step 4: Looking Back

To check the answer, substitute first the given and the obtained value in  $r^2 + u^2 = d^2$ .

$$r^2 + u^2 = d^2$$

Substitute the obtained value and the given.

$$15^2 + 37.08^2 = 40^2$$

Simplify

$$1600 = 1600$$

Next is to check the given and the computed value in  $A = \pi r^2$ .

$$A = \pi r^2$$

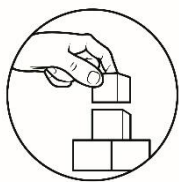
Substitute the computed value and the given.

$$706.5 = (3.14)(15^2)$$

Simplify

$$706.5 = 706.5$$

Therefore, the length of each pathway is about 37.08 meters.



## What's More

### Word Problem 1:

Each central angle formed by consecutive spokes of a Ferris wheel measures  $15^\circ$ . What is the seating capacity if two seats are anchored to each spoke in the Ferris wheel?

#### Step 1: Understand the Problem

In the problem, the angles formed in between consecutive spokes can be considered as central angles of circles. We are asked to determine the seating capacity of the Ferris wheel if a central angle formed by consecutive spokes measures  $15^\circ$ .

To solve, remember that the sum of the central angles of a circle with no common interior points is  $360^\circ$ .

#### Step 2: Devising a Plan

We need to find how many spokes are there in the Ferris Wheel. Since the central angle formed by consecutive spokes measures  $15^\circ$  and a whole revolution is  $360^\circ$ , then get the quotient between  $360^\circ$  and  $15^\circ$ . After which, multiply the quotient by two since there are two seats in each spoke.

#### Step 3: Carrying out the Plan

$$360^\circ \div 15^\circ = 24$$

Getting the quotient of  $360^\circ$  and  $15^\circ$  to find the number of spokes

$$24 \times 2 = 48$$

Multiplying to two to get the number of seating capacity.

#### Step 4: Looking Back

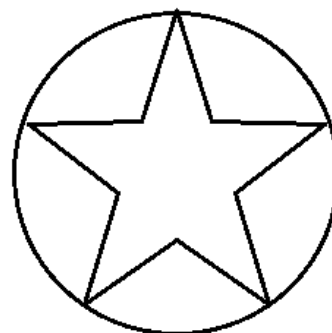
To check whether there are really 48 seating capacity, solve the problem backwards. Verify first if the product of 24 and 2 is really equivalent to 48. Next, verify the product of 24 and  $15^\circ$  if it is really equivalent to  $360^\circ$ . The two steps are easily verified; thus, we can conclude that our answer is correct.

Therefore, there are 48 seating capacity.

### Word Problem 2:

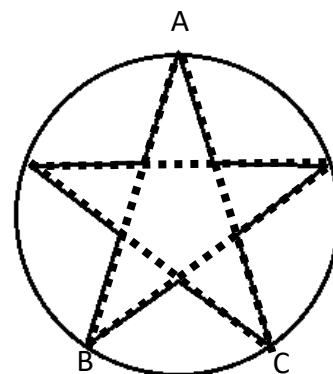
There are circular gardens having paths in the shape of a regular star pentagon like the one shown in the figure.

- Determine the measure of an arc intercepted by an inscribed angle formed when the vertices of the star are connected.
- What is the measure of an inscribed angle in a garden with a five-pointed star?



### Step 1: Understand the Problem

In the garden having paths in the shape of a regular star pentagon, connect each vertex to the other vertices as shown in the figure at the right. The figure shows the intercepted arcs of the inscribed angle formed by two consecutive dotted lines. For example, inscribed angle,  $\angle A$  is intercepted by  $\widehat{CB}$ . In the problem, we need to find the measure of the inscribed angles and intercepted arcs.



### Step 2: Devising a Plan

Remember that from the given, the star pentagon formed is regular and the degree measure of one complete circle is  $360^\circ$ . Thus, to find the measure of each intercepted arc, get the quotient of  $360^\circ$  and the number of intercepted arcs.

To find the measure of an inscribed angle, use the theorem: "If an angle is inscribed in a circle, then the measure of the angle is equal to one-half the measure of its intercepted arc."

### Step 3: Carrying out the Plan

$$360^\circ \div 5 = 72^\circ$$

Getting the quotient of  $360^\circ$  and 5 to find the degree measure of an intercepted arc. Thus, the measure of each intercepted arc of the 5 angles of the regular star pentagon is  $72^\circ$ .

$$72^\circ \div 2 = 36^\circ$$

Getting the half of the degree measure of an intercepted arc since inscribed angle is one-half of its intercepted arc. This results to one inscribed angle equal to  $36^\circ$ .

### Step 4: Looking Back

Check if twice the measure of an inscribed angle is equal to the measure of an intercepted arc and five times the degree measure of an intercepted arc is  $360^\circ$ . In the problem, the measure of an arc intercepted by an inscribed angle formed when the vertices of the star are connected should be  $72^\circ$ . In addition, the sum of the measure of all intercepted arc should be  $360^\circ$ .

### Word Problem 3:

A dart board has a diameter of 40 cm and is divided into 20 congruent sectors. What is the area of one of the sectors?



### Step 1: Understand the Problem

Considering the dart board at the right, the darkest region is a sector of the circular dart board. The area of one sector is what the problem intends to find. To find its area, get the product of the area of the circle and the  $\frac{\text{measure of the corresponding arc of the sector}}{360}$ .



### Step 2: Devising a Plan

From the given, to find the area of a sector of the circle, find first the measure of the arc which corresponds to a sector. This can be done by dividing the degree measure of a whole circle ( $360^\circ$ ) by 20 sectors.

The area (A) of the circle can be computed using  $A = \pi r^2$ . To get the radius, divide the diameter by two.

Finally, to determine the area of each sector of the dart board, get the product of  $\frac{\text{measure of the corresponding arc of the sector}}{360}$  and the area of the circle.

### Step 3: Carrying out the Plan

From the plan, the following steps is followed to find the area of each sector of the dart board

- a. Find the measure of each arc by getting the quotient of  $360^\circ$  and 20.  
 $360^\circ \div 20 = 18^\circ$

- b. Find the radius of the dart board.  
 $40 \div 2 = 20$  cm. is the length of the radius

- c. Find the ratio of the measure of an arc to  $360^\circ$ ,  
Ratio =  $\frac{\text{measure of the arc}}{360}$   
 $= \frac{18}{360}$   
 $= \frac{1}{20}$

- d. Find the area (A) of the circle using the equation,  $A = \pi r^2$   
 $A = \pi(20)^2$                       Substitute the radius from letter b  
 $A = 400 \text{ cm}^2$                       Simplify Exponents

- e. Finally, find the area of each sector by getting the product of the ratio in step c and the area in step d.

$$\begin{aligned}\text{Area of each sector} &= \frac{1}{20} \cdot 400\pi \\ &= 20\pi && \text{Substitute 3.14 to } \pi \\ &= 62.83 \text{ cm}^2\end{aligned}$$

### Step 4: Looking Back

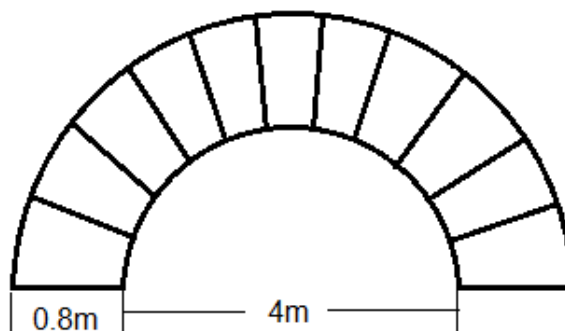
To check the computed area of each sector

- a. Start with  $20\pi$ . Multiply it by the number of sectors which is 20.

- b. Multiply the result by  $\frac{1}{20}$
- c. Divide the answer in b by  $\pi$ . After which, extract the square root.
- d. Multiply the result of c by 2. The product should be 40. If you obtain a different answer, go back to step a.
- e. Finally get the product of 18 and 20. The obtained answer should be equivalent to 360.

#### Word Problem 4:

Mr. Celso designed an arch made of bent iron for the top of a school's main entrance as shown in the figure at the right. Each of the 12 segments between the two concentric semicircles are 0.8 meter long. Suppose the diameter of the inner semicircle is 4 meters, what is the total length of the bent iron used to make this arch?



#### Step 1: Understand the Problem

The problem asked to determine the total length of the bent iron used to make the arch. Each of the 12 segments joining the inner and outer circle measures 0.8 meter and the diameter of the inner circle is 4 meters

#### Step 2: Devising a Plan

To solve the total bent iron used to make the arch, get the sum of the inner arc length and outer arc length. In solving for each arc length, remember that the length of an arc can be determined by using the proportion  $\frac{\text{degree measure of the arc}}{360^\circ} = \frac{\text{arc length}}{2\pi r}$  where  $r$  is the radius of the circle. Multiply  $2\pi r$  both sides of the proportion then simplify. We obtain

$$\text{arc length} = \frac{(\text{degree measure of the arc})(2\pi r)}{360^\circ}$$

In the given, the radius of the inner arc length is 2m and the radius of the outer arc length is the sum of 2 m and 0.8 m. While the inner arc length and outer arc length measures  $180^\circ$  since they are both semicircles.

#### Step 3: Carrying out the Plan

Solve separately the inner and outer arc length then get their sum. Therefore, the solution is as follows:

- a. Finding the Arc length of the inner arc:

$$\text{arc length} = \frac{(\text{degree measure of the arc})(2\pi r)}{360^\circ}$$

$$= \frac{(180^\circ)(2\pi)(2)}{360^\circ}$$

Substituting the arc length of a semicircle and the radius of the inner arc

$$= 2\pi$$

Simplifying



$$= 6.28 \text{ m}$$

Substituting 3.14 for  $\pi$

b. Finding the Arc length of outer arc:

$$\text{arc length} = \frac{(\text{degree measure of the arc})(2\pi r)}{360^\circ}$$

$$= \frac{(180^\circ)(2\pi)(2.8)}{360^\circ}$$

Substituting the arc length of a semicircle and the radius of the outer arc which is the sum of 2 and 0.8

$$= \frac{5.6\pi}{2}$$

Simplifying

$$= 8.79 \text{ m}$$

Substituting 3.14 for  $\pi$  and further simplifying

After obtaining the length of the inner and outer arcs, get the sum to finally obtain the total length of the bent iron used to make the arch.

$$6.28 + 8.79 = 15.07 \text{ m}$$

#### Step 4: Looking Back

Solve backwards to check if the solution is correct. Start with the measure of the length of each arc then solve backwards until the radii is obtained. The solution could be as follows:

a. Radius of the outer arc, starting from  $\frac{5.6\pi}{2}$

Multiply  $\frac{5.6\pi}{2}$  by  $\frac{360^\circ}{(180^\circ)(2\pi)}$ . Obtained value should be 2.8. If not, check what went wrong in our solution.

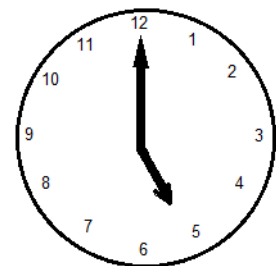
b. Radius of the inner arc starting from  $2\pi$

Multiply  $2\pi$  by  $\frac{360^\circ}{(180^\circ)(2\pi)}$ . Obtained value should be 2. If not, check our solution.

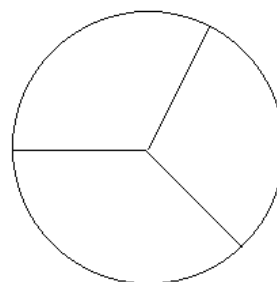
### Assessment 1: It's Your Turn!

Apply the concepts learned in the previous activities to be able to solve the following problems.

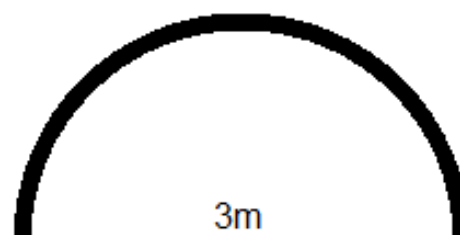
- Rowel is designing a motorcycle wheel. He decided to put 6 spokes which divide the rim into 6 equal parts. What is the degree measure of each angle formed by the spokes at the hub?
- Find the degree measure of the angle formed by the hands of a clock at 5:00 in the afternoon.



3. A fruit pie is divided into 3 equal parts forming central angles. Find the area of each sector formed if the fruit pie has a radius of 6 inches.



4. Mr. Alex designed a semicircle arch made of bent iron for the door of his garage. Suppose the diameter of the semicircle is 3 meters, how long should the iron be before bending to avoid wastage?

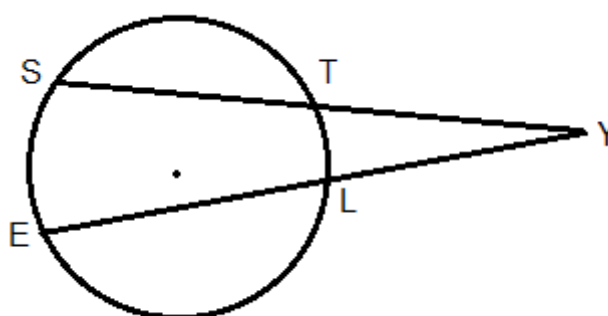


Rubrics: To guide you with your solution, please refer to the following:

Score per item	Descriptors
5	All the steps to answer a word problem are followed and each step is presented accurately.
3	The steps to answer a word problem are followed but the answer provided is incomplete.
1	The solution is erroneous.

### Word Problem 5:

In a circular board, an elastic band is tied to different points of the circle namely points S and E. The band is then stretched to a certain point outside the circle forming secants SY and EY. If SY= 15cm, TY= 6cm, and LY= 7cm, what is the length of secant EY? Refer to the figure at the right.



#### Step 1: Understand the Problem

The word problem deals with secant segments. The problem asked to find the length of secant EY. The given are: length of secant SY, length of external secant TY and the length of external secant LY.

#### Step 2: Devising a Plan

To solve the problem, use the following theorem: If two secant segments are drawn to a circle from an exterior point, then the product of the lengths of one secant segment and its external secant segment is equal to the product of the lengths of the other secant segment and its external secant segment. Applying the theorem in the figure, we have,

$$SY \cdot TY = EY \cdot LY$$

*Step 3: Carrying out the Plan*

- a. Review the given measurements.

$$SY = 15 \text{ cm}$$

$$TY = 6 \text{ cm}$$

$$LY = 7 \text{ cm}$$

- b. To solve the unknown segment EY, substitute the given in the theorem

$$SY \cdot TY = EY \cdot LY$$

$$(15)(6) = (EY)(7)$$

Substituting the given measurements

$$90 = 7(EY)$$

Simplifying

$$\frac{90}{7} = \frac{7(EY)}{7}$$

Dividing both sides of the equation by 7

$$EY = 12.86 \text{ cm.}$$

Simplify to obtain 12.86 cm., the length of EY

*Step 4: Looking Back*

To check if the solution is correct, substitute the given and computed value in the theorem  $SY \cdot TY = EY \cdot LY$  as follows

$$(15)(6) = (12.86)(7)$$

Substituting the values.

$$90 = 90$$

Simplify. Since we obtain 90 on both sides of the equation, thus our solution is correct.

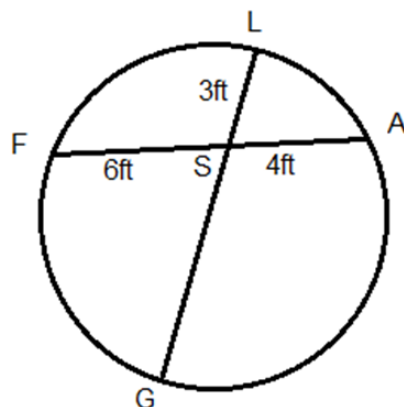
**Word Problem 6:**

An old circular table was repaired by nailing two wooden strips on its back, forming intersecting chords. Find the length of the second piece of wood LG if the dimensions of the chords are as follows:

$$FS = 6 \text{ ft}$$

$$SA = 4 \text{ ft}$$

$$LS = 3 \text{ ft}$$



*Step 1: Understand the Problem*

The given word problem involves intersecting chords of a circle. We are given the length of segment LS, FS and segment SA. Segments FS and SA form chord FA. Using the given and concepts of secant segments, we are asked to find for the length of segment LG.

*Step 2: Devising a Plan*

Since we are dealing with intersecting chords of a circle, use the following theorem: If two chords of a circle intersect, then the product of the measures of the segments of one chord is equal to the product of the measures of the segments of the other chord. In symbols,  $FS \cdot SA = LS \cdot SG$

*Step 3: Carrying out the Plan*

- a. From the problem, the following are the given measurements:

$$FS = 6 \text{ ft}$$

SA=4 ft  
LS=3 ft  
LG is unknown

- b. To solve for LG, start with the theorem.

$$FS \cdot SA = LS \cdot SG \quad \text{Substituting the given values}$$

$$(6)(4) = 3(SG) \quad \text{Getting the product}$$

$$\frac{24}{3} = \frac{3(SG)}{3} \quad \text{Divide both sides by 3}$$

$$SG = 8\text{ft} \quad \text{Simplifying. Therefore, SG is equal to 8 feet.}$$

- c. To finally solve for LG,

$$LG = LS + SG$$

$$LG = 3 + 8 \quad \text{Substituting the given value of LS and the obtained value of SG}$$

$$LG = 11 \quad \text{Simplifying. Therefore, LG is equal to 11 feet}$$

#### Step 4: Looking Back

To check if we had a correct answer, we don't need to start checking the obtained value of LG since we can easily identify that  $3 + 8$  is really equivalent to 11. Thus, to check our answer, we start with the value of SG. We can substitute the obtained value and the given to the theorem  $FS \cdot SA = LS \cdot SG$

$$(6)(4)=(3)(8) \quad \text{Substituting the values}$$

$$24=24 \quad \text{Obtaining equal values on both sides of the equation, thus we obtain a correct value of SG}$$

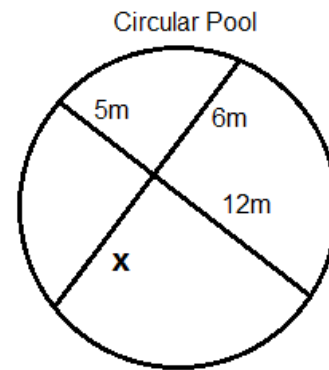
### Assessment 2: Solve my length!

Apply the concepts learned to solve for the given word problems. Use the following guide questions to help you in presenting your solution.

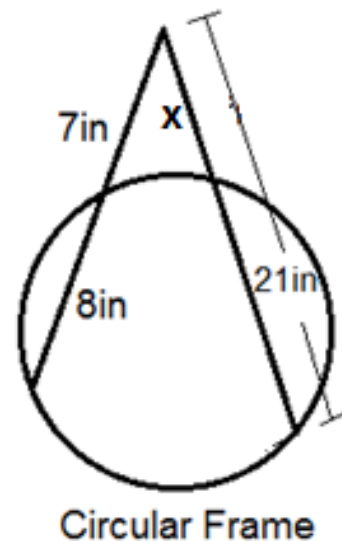
#### Guide Questions:

- ✓ What are the problems all about?
- ✓ What theorems or concepts can you use to solve the problems?
- ✓ How can you formulate equations based from the theorems or concepts?
- ✓ How will you use the equations to solve for the unknowns?
- ✓ Did you check your answer?

1. A family tied a pair of ropes to their circular pool to serve as guide for their children who are practicing how to swim. The ropes intersect at the interior of the pool with three segments formed measuring 5m, 6m, and 12 m as illustrated in the figure at the right. Find the missing segment measure of the rope in meters.

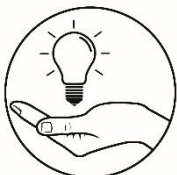


2. A circular frame is supported with sticks on its backside as shown in the figure at the right. Find the unknown measure of the external secant segment  $x$ .



Rubrics: To guide you with your solution, please refer to the following

Score per item	Descriptors
5	All steps to answer word problem are followed and each step is presented accurately.
3	The steps to answer word problem are followed but the answer provided is incomplete.
1	The solution is erroneous.

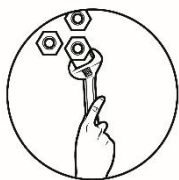


## ***What I Have Learned***

Fill in the blanks with your learnings regarding solving problems on circles

1. If a line is tangent to a circle, then it is \_\_\_\_\_ to the radius drawn to the point of tangency.

2. The sum of the central angles of a circle with no common interior points is \_\_\_\_\_.
3. If an angle is inscribed in a circle, then the measure of the angles is \_\_\_\_\_ the measure of the intercepted arc.
4. The area of a sector of a circle is the product of the area of the circle and the \_\_\_\_\_.
5. If two secant segments are drawn to a circle from an exterior point, then the product of the lengths of one secant segment and its external secant segment is \_\_\_\_\_ to the product of the lengths of the other secant segment and its external secant segment.
6. If two chords of a circle intersect, the product of the measures of the segments of one chord is \_\_\_\_\_ to the product of the measures of the segments of the other chord.



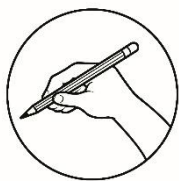
## ***What I Can Do***

### **The Huge Ceiling Ball!**

Ronnie used a golden string to hang a huge circular light ball in the middle of their living room. The string measures 50 centimeters and it reaches until the center of the circular light ball. Suppose the tangent line from the ceiling where the string is hoisted to the circular light ball is 30 cm, what is the radius of the circular light ball?

Rubrics: Your output will be rated as follows

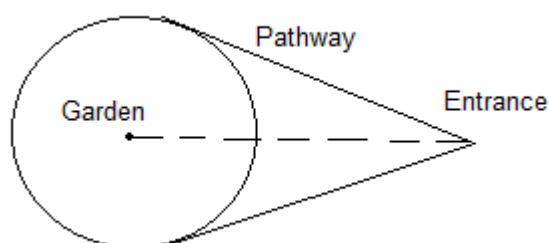
Score	Descriptors
20	The diagram was well illustrated and with correct label. All computations are accurate and the final answer is expressed correctly in a complete sentence.
15	The diagram is illustrated. The computations are correct but the final answer is not clearly expressed.
10	Some parts of the diagram are missing. Some computations are not accurate.
5	A diagram is present and an answer is identified though incorrect.



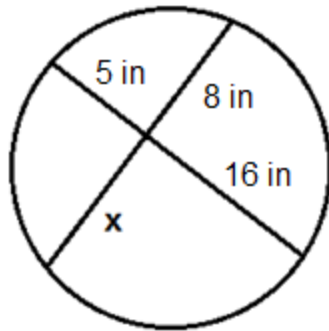
## ***Assessment***

**Instruction:** Read carefully each item. Choose the letter of your answer and write it on your answer sheet.

- Which of the following is the measure of the angle determined by the hands of a clock at four o'clock?  
a.  $90^\circ$       b.  $120^\circ$       c.  $150^\circ$       d.  $180^\circ$
- Which of the following is the degree measure of an arc formed when a circle is divided into 5 equal arcs?  
a.  $30^\circ$       b.  $60^\circ$       c.  $72^\circ$       d.  $144^\circ$
- A wheel has 12 spokes which are evenly spread. Which of the following is the measure of each central angle formed?  
a.  $12^\circ$       b.  $15^\circ$       c.  $24^\circ$       d.  $30^\circ$
- In a pie chart showing the expenditures of the Lopez family, the measure of the central angle corresponding to their food expenses is  $120^\circ$ . If their monthly income is Php 35,000.00, which of the following cost corresponds to their food expenses?  
a. Php 11,666.67      c. Php 5,666.67  
b. Php 17,500.00      d. Php 12,000.00
- Angel wants to draw a pie chart that shows how she spent her 30-day vacation. If she used six days of it practicing her piano recital, which of the following degree measure of a central angle does corresponds to its sector?  
a.  $22.5^\circ$       b.  $45^\circ$       c.  $72^\circ$       d.  $90^\circ$
- A dart board has a diameter of 30 cm. and is divided into 15 congruent sectors. Which of the following is the area of one of the sectors?  
a.  $15\pi \text{ cm}^2$       b.  $20\pi \text{ cm}^2$       c.  $80\pi \text{ cm}^2$       d.  $800\pi \text{ cm}^2$
- Mr. Alex designed a semicircle arc made of bent iron for their gate's entrance. Suppose the diameter of the semicircle is 3 meters, how long should the iron be before bending to avoid wastage? (Round off answer to two decimal places)  
a. 4.71 m      b. 5.71 m      c. 9.42 m      d. 10.55 m
- A circular garden has pathways which are both tangent to the garden whose center is 10 meters away from the entrance as shown in the figure below. If the radius of the circular garden is 5 meters, which of the following is the length of each pathway?



- $3\sqrt{5} \text{ m}$
  - $5\sqrt{3} \text{ m}$
  - 25m
  - 75m
- A circular weaved winnower is supported by a pair of sturdy rattan that intersects at its back as shown in the figure. Which of the following choices is the missing length?



- a. 5 in      b. 10 in      c. 12 in      d. 15 in

10. A circular mirror is hung such that the strings are tangent to the circle as shown in the figure. If the major arc measures  $240^\circ$ , which of the following is the degree measure of the angle formed by the strings.



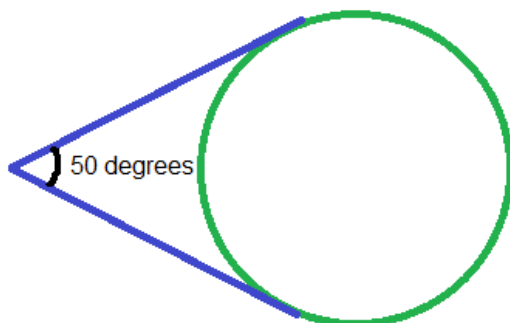
- a.  $60^\circ$       b.  $80^\circ$       c.  $130^\circ$       d.  $230^\circ$



## Additional Activity

### Guess What!

Mario is looking at a circular garden having pathways tangent to the circle from an exterior point as shown in the sketch below. If the angle formed by the tangent pathways in the exterior of the circle is  $50^\circ$ , what are the degree measures of the major arc and the minor arc intercepted by the tangents in the circle? Explain how you arrived at your answer.

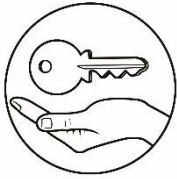




Rubrics:

Score	Descriptors
20	The answer is accurate and explanation is well expressed supported by solution.
15	The answer is accurate but the explanation is not well supported by a solution.
10	The answer is inaccurate though the explanation contains valid points.
5	The explanation is not valid resulting to an inaccurate answer.

You can do it!



## Answer Key

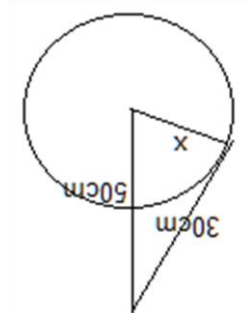
### What I Know

1. c
2. b
3. a
4. a
5. c
6. a
7. a
8. b
9. d
10. b

### What's In

1. equal
2. equal
3. half
4. twice
5. twice
1. difference
2. difference
3. sum
4. sum
5. One-half

### What I Can Do



The radius of the ball is 40 cm.

### What's More :

- Assessment 1
1. 60 degrees
2. 150 degrees
3. 37.68 square inches
4. 4.71 meters
- Assessment 2
1. 10 meters
2. 5 inches

### ADDITIONAL ACTIVITY

The major arc is equal to 230 degrees and the minor arc is 130 degrees.

### ASSESSMENT

1. b
2. c
3. d
4. a
5. c
6. a
7. a
8. b
9. b
10. a

### WHAT I HAVE LEARNED

1. perpendicular
2.  $360^\circ$
3. one-half
4.  $\frac{\text{measure of corresponding arc of the sector}}{360^\circ}$
5. Equal
6. equal

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