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Mathematics

Quarter 2 – Module 3:
The Relations among Chords, Arcs, Central Angles and Inscribed Angles

ALTERNATIVE DELIVERY MODE
ADM

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Alternative Delivery Mode
Quarter 2 – Module 3: The Relations among Chords, Arcs, Central Angles and Inscribed Angles
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The Relations among Chords, Arcs, Central Angles and Inscribed Angles

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Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

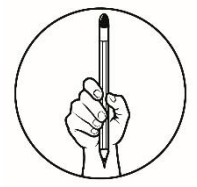
If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

What I Need to Know

This module was designed and written with you in mind. It is here to help you define, identify, name, and illustrate the parts of a circle and derive inductively the relations among chords, arcs, central angles, and inscribed angles. The scope of this module permits it to be used in many different learning situations. The language used recognizes the diverse vocabulary level of students. The lessons are arranged to follow the standard sequence of the course but the order in which you read and answer this module is dependent on your ability.

After going through this module, you are expected to derive inductively the relations among chords, arcs, central angles, and inscribed angles.



What I Know

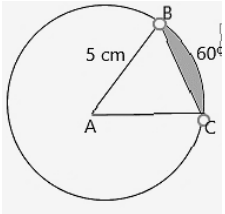
Directions: Read and analyze each item very carefully. Write the letter of the correct answer on your answer sheet.

1. Which angle has its vertex on a circle and has sides that contain chords of the circle?
   A. central angle
   B. circumscribed angle
   C. inscribed angle
   D. intercepted angle

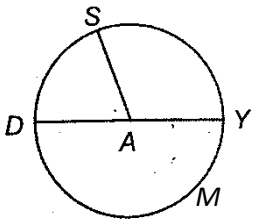
2. A circle with a $5\,cm$-radius has an arc that measures 30°. What is the measure of its arc length? Note: $\pi = 3.14$
   A. $2.62\,cm$ B. $2.3\,cm$ C. $1.86\,cm$ D. $1.5\,cm$

3. What is the sum of the measures of the central angles of a circle with no common interior points?
   A. 120° B. 240° C. 360° D. 480°

4. What do you call the shaded region of circle A?
   A. area
   B. pi
   C. sector
   D. segment



5. Given circle A, what kind of arc is $\overset{\frown}{DMY}$ if $\overline{DY}$ is a diameter?



A. circle
B. major arc
C. minor arc
D. semicircle

6. Given circle A with semicircle $\overset{\frown}{YSD}$ and $m\angle SAD = 70$, what is the measure of $\angle SAY$?
A. 20º
B. 70º
C. 110º
D. 150º

7. Which of the following is NOT a measure of a minor arc?
A. 180º
B. 179º
C. 150º
D. 100º

8. Suppose $\overset{\frown}{RS} = 95°$, what must be added to its measure to make a semicircle?
A. 60º
B. 75º
C. 80º
D. 85^0

9. In the Arc Addition Postulate, which of the following is true about the measure of an arc formed by two adjacent non-overlapping arcs?
A. It is equal to the difference of the measures of these two arcs.
B. It is equal to the sum of the measures of these two arcs.
C. It is equal to the quotient of the measures of these two arcs.
D. It is equal to the product of the measures of these two arcs.

10. Which of the following statements is true about the intercepted arcs of a central angle?
A. The measure of the intercepted are is equal to the measure of the central angle.
B. The measure of the intercepted arc is less than the measure of the central angle.
C. The measure of the intercepted arc is greater than the sum of the measures of its central angles.
D. The measure of the intercepted arc is equal to the sum of the measure of its central angles.

11. Which of the following statements is true about the radii of congruent circles?
A. They are similar.
B. They are congruent.
C. They are equal.
D. They are equivalent.

12. Under what condition can two minor arcs of two congruent circles be congruent?
A. If and only if their corresponding intercepted arcs are congruent.
B. If and only if their corresponding adjacent angles are congruent.
C. If and only if their corresponding central angles are congruent.
D. If and only if their corresponding adjacent arcs are congruent.

13. In what condition can a diameter or a radius bisect a chord and its arc?
    A. When the diameter or radius is equal/congruent to the chord
    B. When the diameter or radius is parallel to the chord
    C. When the diameter or radius is perpendicular to the chord
    D. When the diameter or radius is collinear to the chord

14. Which part is used to name a circle?
    A. center B. chord C. diameter D. tangent

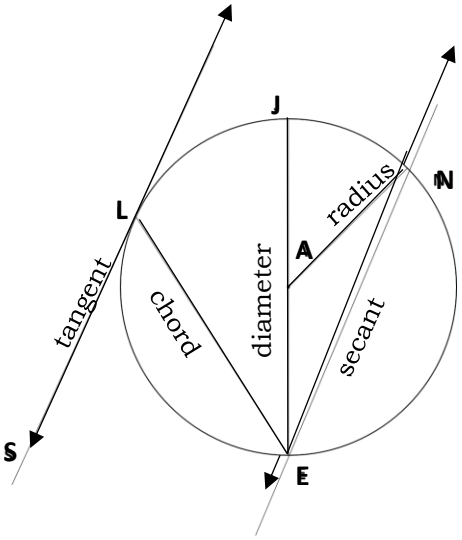
15. Which angle has a vertex on the center of the circle and sides that are radii of that circle? ____ angle.
    A. acute B. central C. inscribed D. intercepted

What's In

Start this module by reviewing some of the terms related to circle for you to be able to go on with the activities. Read the given terms and definitions.

Gallery of Knowledge

A **circle** is a set of all points with the same distance from a fixed point called the center. The center is used to name a circle.
Example: *circle A* or $\odot A$



A **radius** of a circle is a segment whose endpoints are the center and a point on the circle. It is the distance from the center to any point on the circle.
Example: $\overline{AJ}$, $\overline{AN}$, $\overline{AE}$

A **chord** of a circle is a line segment that has its endpoints on the circle.
Example: $\overline{LE}$, $\overline{JE}$, $\overline{EN}$

A **diameter** of a circle is a chord that passes through the center of the circle. It is the longest chord of a circle and its length is twice as long as its radius.
Example: $\overline{JE}$

A **tangent** is a line, a segment, or a ray that intersects a circle at exactly one point, and the point of intersection is called the point of tangency.
Example: *line s* with *point L* as the *point of tangency*

A **secant** is a line, a segment, or a ray that intersects a circle at exactly two points.
Example: $\overleftrightarrow{NE}$ or $\overline{NE}$ or $\overrightarrow{NE}$

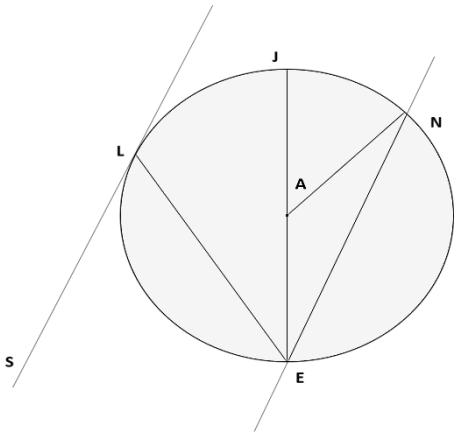
What is It

Lesson 1 Chords, Arcs, and Central Angles

The prior knowledge and skills you have on circle will help you understand it even more. As you undergo with the lesson, be guided by the question: "*How do the relationships among chords, arcs, and central angles of a circle facilitate finding solutions to real-life problems and develop skills in decision making*?" To find the answer, perform each activity. You may refer to the modules you have studied or to other references.

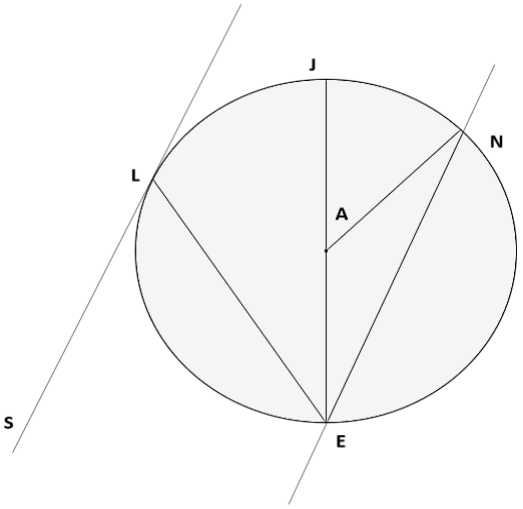
What's New

Identify what parts related to ⊙A are named.



1. $\overline{AJ}$, $\overline{AE}$, $\overline{AN}$
2. $\overline{JE}$
3. $\overline{LE}$,$\overline{JE}$,$\overline{EN}$
4. $\widehat{JNE}$ or $\widehat{JLE}$
5. $\widehat{LJ}$, $\widehat{LN}$, $\widehat{LE}$, $\widehat{EN}$
6. $\widehat{LEN}$, $\widehat{LNJ}$, $\widehat{JLN}$, $\widehat{NLE}$, $\widehat{LNE}$
7. $\angle JAN$, $\angle NAE$
8. A
9. $\widehat{JN}$ in relation to $\angle A$ and $\angle JEN$, $\widehat{LN}$ in relation to $\angle LEN$

An **arc** is a part of a circle between any two points. It is measured in terms of degrees. A circle is in itself an arc that measures 360°. Arcs can be classified as minor, semicircle, and major. The symbol $\overset{\frown}{}$ is used to denote an arc.



A **semicircle** is an arc which is *one-half* of a circle. It measures exactly 180°. A **minor arc** is less than a semicircle and its measure is between 0° and 180° while a **major arc** is greater than a semicircle which measures between 180° and 360°.

A minor arc is named using the two endpoints of the arc. A semicircle is named using three points. The first and the third points are the endpoints of the diameter and the middle point is any point of the arc between the endpoints. In cases where there are only two points given on a circle, the semicircle is named using the two points. Lastly, a major arc is named using three points. The first and the third are the endpoints and the middle point is any point on the arc between the endpoints. Using ⊙A,

$\overset{\frown}{JNE}$ and $\overset{\frown}{JLE}$ are semicircles;

$\overset{\frown}{JN}$ is a minor arc; and

$\overset{\frown}{JLN}$ and $\overset{\frown}{NEL}$ are major arcs.

A **central angle** of a circle is an angle formed by two radii and its vertex is the center of the circle. Using ⊙A,

$\angle JAN, \angle NAE$ and $\angle JAE$ are central angles.

An **intercepted arc** is created when lines or subsets of a line cuts across a section of the circumference of a circle. The intersection of the lines or subsets of it are on the circle, thus forming an inscribed angle. Using ⊙A,

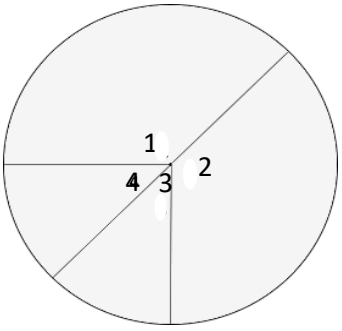
$\overset{\frown}{LN}$ is a minor arc intercepted by inscribed ∠LEN and

$\overset{\frown}{JN}$ is a minor arc intercepted by ∠JEN and ∠JAN.

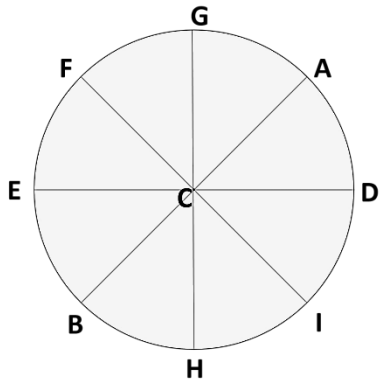
Sum of Central Angles

The sum of the measures of the central angles of a circle with no common interior points is 360°. In the figure, $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$ are examples of central angles without common interior points. Thus,

$m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 360.$



The examples below show the relationship between the central angle intercepting the arc of a circle. Using ⊙C, the following can be shown:



central ∠FCG intercepts $\widehat{FG}$;

central ∠ECI intercepts $\widehat{EI}$;

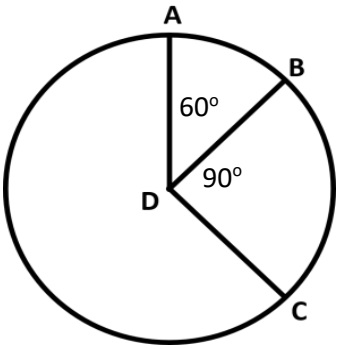
central ∠ACG intercepts $\widehat{GBA}$;

$\widehat{AI}$ is intercepted by central angle ∠ACI; and

$\widehat{FDH}$ is intercepted by central angle ∠FCH.

Arc Addition Postulate

The measure of an arc formed by two adjacent non-overlapping arcs (arcs that share exactly one point) is equal to the sum of the measures of these two arcs. Using ⊙D,

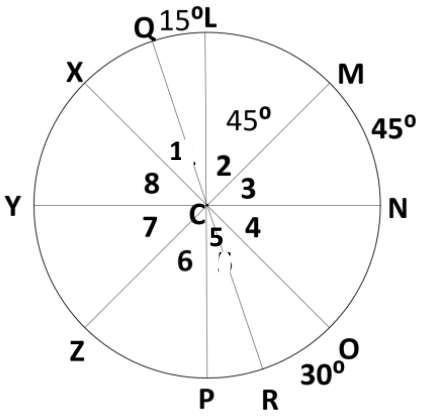


$m\widehat{AC} = m\widehat{AB} + m\widehat{BC}$

$m\widehat{AC} = 60 + 90$

$m\widehat{AC} = 150$

Example: Determine the measures of the given arcs and angles. Refer to the figure that follows. C is the center of the circle while the other points are points on the circle.



1. m∠2 or m∠LCM

Solution: *The measure of ∠2 is* 45°.

2. m$\widehat{LM}$

Solution: *The measure of* $\widehat{LM}$ *is* 45° *because the measure of* ∠LCM *that intercepts* $\widehat{LM}$ *is* 45°.

3. measure of $\widehat{XQ}$

Solution: $\widehat{XQ}$ *is* 30°. ∠XCQ *and* ∠RCO *are vertical angles and a pair of vertical angles are congruent. It follows that their intercepted arcs also are congruent.*

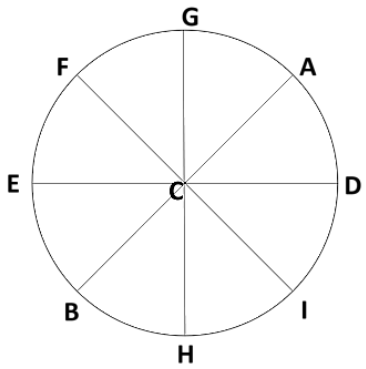
4. measure of $\widehat{LN}$

Solution: *The measure of* $\widehat{LN}$ *is* 90°. *The intercepted arc of* ∠LCM *is* 45°. *Through the Arc Addition Postulate, we can say that the sum of the measures of* $\widehat{LM}$ *and* $\widehat{MN}$ *is equal to the measure of* $\widehat{LN}$.

What's More

Activity 1. Intercepts, Arcs, and Angles

Identify what is being asked in the given items. Refer to $\odot C$.



_____1. The angle intercepting $\widehat{GA}$

_____2. The angle intercepting $\widehat{DH}$

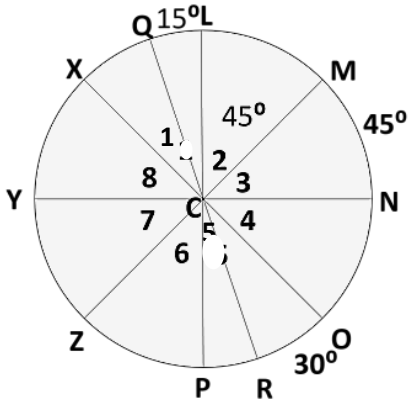
_____3. The arc intercepted by $\angle ACH$

_____4. The angle intercepting $\widehat{FA}$

_____5. The arc intercepted by $\angle ECI$

Activity 2. Arc and Angle Measures

Determine the measures of the given arcs and angles. Refer to $\odot C$ to the right. All the other points are points on the circle.



_____1. Measure of ∠PCR

_____2. Measure of $\widehat{PR}$

_____3. Measure of $\widehat{PO}$

_____4. Measure of ∠4

_____5. Measure of ∠8

What can you say about the activity? Have you discovered the relationship among arcs and central angles?

What I Have Learned

You are now going to wrap up your learning by answering the question:

How are arcs and central angles related to each other?

_____

_____

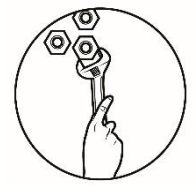
_____

_____

_____.

Your output will be evaluated using the rubric below.

Score	Descriptors
4	The organization of the ideas is clear. There is coherent connection with each idea.
3	The organization of the ideas is clear but there is no coherent connection with each idea.
2	The organization of the ideas is slightly unclear. There is no coherent connection with each idea.
1	The organization of the ideas is unclear and there is no coherent connection with each idea.



What I Can Do

To find out how you can apply in real life the concepts you have just learned, do the task described below.

Capture or cut out pictures that show the concepts of chords, arcs, and central angles in real life. Explain why you believe these concepts were used in those situations. Paste the three pictures in one long bond paper only.

Every picture taken with its respective description will be scored using the rubrics that follow.

Score	Descriptors for the Content
5	The explanation is correct, substantial, specific, and convincing.
4	The explanation is correct, substantial, and specific but not convincing.
3	The explanation is correct and substantial but not specific and convincing.
2	The explanation is correct but not substantial, specific, and convincing.
1	There is explanation but it is not correct, substantial, specific, and convincing.
Score	**Descriptors for the Picture**
5	The picture taken is appropriate to the concept. The concept is clearly presented. The entirety of the picture is seen and is not blurred or cloudy.
4	The picture taken is appropriate to the concept. The concept is clearly presented. The entirety of the picture is seen but is blurred or cloudy.
3	The picture taken is appropriate to the concept. The concept is clearly presented. Some parts of the picture are unseen and are blurred or cloudy.
2	The picture taken is appropriate to the concept. The concept is not clearly presented. Some parts of the picture are unseen and are blurred or cloudy.
1	Pictures are presented but not appropriate to the concept.

Lesson 2 Chords, Arcs, and Inscribed Angle

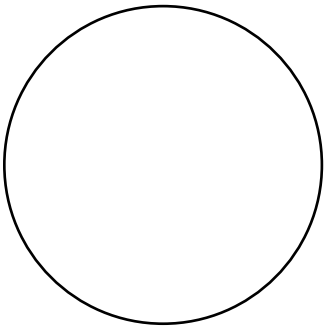
In Lesson 1, we talked about the relationships among chords, arcs, and central angles. We learned that the measure of a central angle is equal to the measure of its intercepted arc. Conversely, the degree measure of an arc is equal to the measure of the central angle that intercepted it.

In Lesson 2, we will be dealing about the relationships among chords, arcs, and inscribed angles. As you go through this lesson, be guided by the question "*How are the relationships among arcs and inscribed angles of a circle used in finding solutions to real-life problems and in making decisions?"* In order to find the answers, perform the succeeding activities. You may refer to the other modules studied earlier or to other references.

What's New

Read and Draw

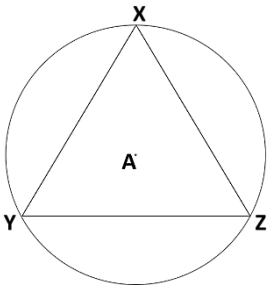
Read the instructions carefully and do what are being asked for.



Name the circle $\odot A$. On the circle, locate a point and name it X. Locate two more points, Y and Z, on the circle such that when you connect points X, Y, and Z they will form an equilateral triangle inside the circle.

What is It

In the activity that you have just done you were able to illustrate examples of inscribed angles.



An **inscribed angle** is an angle whose vertex lies on the circle and the sides contain chords of the circle. Using $\odot A$, $\angle Y$ *intercepts* $\widehat{XZ}$, $\angle X$ *intercepts* $\widehat{YZ}$, *and* $\angle Z$ *intercepts* $\widehat{XY}$.

There are three different ways or cases in which an angle can be inscribed in a circle.

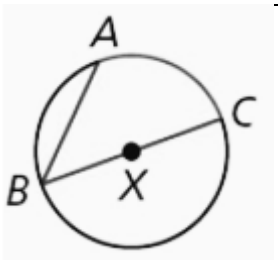
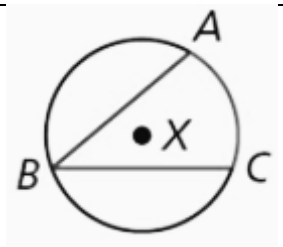
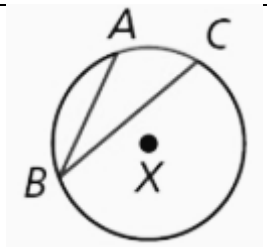
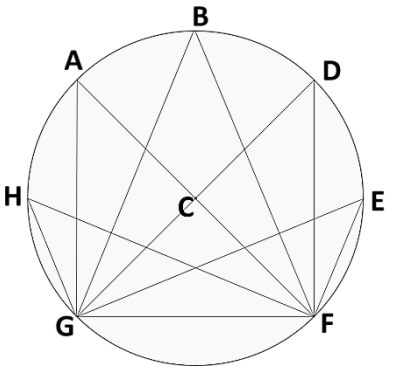
		
The center of the circle is on one side of the inscribed angle.	The center of the circle is in the interior of the inscribed angle.	The center of the circle is on the exterior of the inscribed angle.

Image credits to https://slideplayer.com/slide/7315628/

Arcs and Angles

Study these examples.

Given $\odot C$:



$\widehat{HD}$ is intercepted by $\angle HFD$;

$\widehat{AB}$ is intercepted by $\angle AGB$;

$\widehat{HA}$ is intercepted by $\angle HFA$;

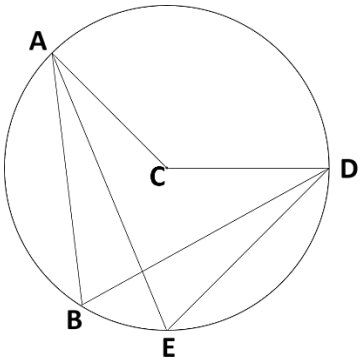
$\angle DGF$ intercepts $\widehat{DF}$; and

$\angle HFG$ intercepts $\widehat{HG}$.

Arc and Angle Measure

> The measure of an inscribed angle is equal to *one-half* the measure of its intercepted arc.

Example: In $\odot C$, m∠ACD = 100, find



a. measure of $\widehat{AD}$;

b. measure of $\angle AED$;

c. measure of $\angle ABD$.

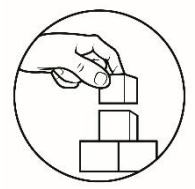
Solution:

a. *The measure of central* $\angle ACD$ *is equal to its intercepted arc* $\widehat{AD}$. *Thus,* $m\widehat{AD} = m\angle ACD = 100$.

b. *The inscribed* $\angle AED$ *intercepts* $\widehat{AD}$.

So, $m\angle AED = \frac{1}{2}m\widehat{AD} = \frac{1}{2}(100) = 50$.

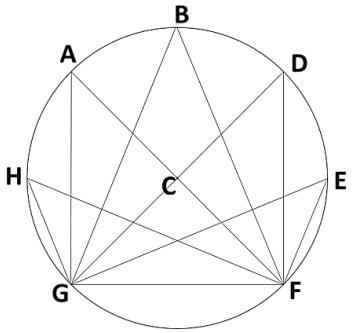
c. *Both* $\angle ABD$ *and* $\angle AED$ *are inscribed angles to intercepted arc* $\widehat{AD}$. *Since* $\angle AED$ *measures* 50°, *then* $\angle ABD$ *also measures* 50°.



What's More

Activity 1. Arcs and Angles

Decide if the statement is correct or not. Write TRUE if it is correct and FALSE if it is incorrect.



_____1. $\angle AGF$ intercepts $\overset{\frown}{AHF}$.

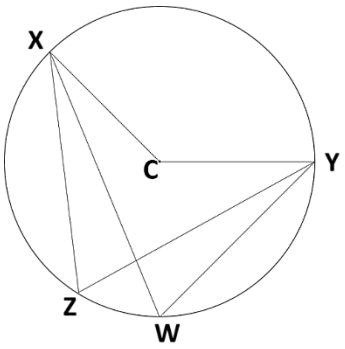
_____2. $\overset{\frown}{AB}$ is intercepted by $\angle AFD$.

_____3. $\angle AFD$ and $\angle AGD$ intercept $\overset{\frown}{AD}$.

_____4. $\overset{\frown}{HAF}$ is intercepted by $\angle HGF$.

_____5. $\angle DGF$ intercepts $\overset{\frown}{DF}$.

Activity 2. Arc and Angle Measure



In $\odot C$, m$\angle XCY = 164$, find the measure of:

_____1. $\overset{\frown}{XY}$

_____2. $\angle XWY$

_____3. $\angle XZY$

What I Have Learned

Doing the series of activities, you are now to wrap up your learning by answering the question that follows:

What is the relationship between arcs and inscribed angles of a circle?

_____

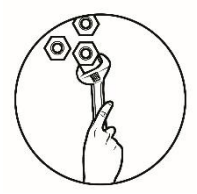
_____

_____

_____

Your output will be evaluated using the rubric that follows.

Score	Descriptors
4	The organization of the ideas is clear. There is coherent connection with each idea.
3	The organization of the ideas is clear but there is no coherent connection with each idea.
2	The organization of the ideas is slightly unclear. There is no coherent connection with each idea.
1	The organization of the ideas is unclear and there is no coherent connection with each idea.



What I Can Do

Read the selection and help Mr. Cruz come up with a logical and correct solution to his problem.

Mr. Cruz would like to place a fountain in his circular garden. He wants the pipe, where the water will pass through, to be located at the center of the garden. Mr. Cruz does not know where it is. Suppose you were asked by Mr. Cruz to find the center of the garden, how would you do it?

The output will be evaluated using the rubrics below.

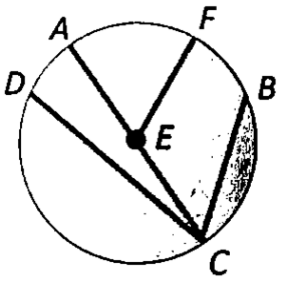
Score	Descriptors
5	Used an appropriate strategy with the correct solution, arrived at a correct answer and with clear presentation
4	Used an appropriate strategy with the correct solution, arrived at a correct answer but with minor unclear presentation
3	Used an appropriate strategy with the correct solution, arrived at a correct answer but with unclear presentation
2	Used an appropriate strategy but the solution and the answer are incorrect and the presentation is not clear
1	Tried to solve the problem but used an inappropriate strategy that led to an incorrect solution and answer

Assessment

Directions: Read and analyze each item very carefully. On your answer sheet, write the letter of the correct answer.

1. Which refers to a set of points that are with the same distance from a fixed point?
   A. center B. circle C. radius D. secant

Refer to ⊙E *for items 2 to 8.*



2. What part of ⊙E is $\overset{\frown}{AFC}$ if $\angle AEC = 180°$?
   A. central angle C. minor arc
   B. major arc D. semicircle

3. What part of ⊙E is $\angle AEF$?
   A. central angle C. intercepted arc
   B. inscribed angle D. right angle

4. In ⊙E, $\overset{\frown}{AD}$ is a/an ____ of $\angle DCA$.
   A. central angle C. intercepted arc
   B. inscribed angle D. right angle

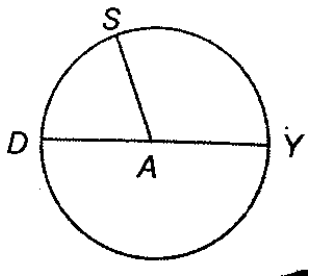
5. What is the measure of $\overset{\frown}{ADB}$?
   A. between 0° and 180° C. between 180° and 360°
   B. exactly 180° D. exactly 360°

6. What is the measure of $\angle FEC$ if $\overset{\frown}{FC} = 115°$?
   A. 230° B. 115° C. 57.5° D. 28.75°

7. What is the measure of $\overset{\frown}{DAB}$ if $\angle BCD = 73°$?
   A. 36.5° B. 73° C. 146° D. 292°

8. What is the measure of $\overset{\frown}{DF}$ if $\angle DCA = 10°$ and $\angle AEF = 40°$
   A. 60° B. 40° C. 20° D. 10°

Refer to ⊙A *for items 9 to 12.*



9. What is the measure of $\angle DAS$ if $\overset{\frown}{DSY}$ is a semicircle and $\angle YAS = 110°$?
   A. 20° C. 110°
   B. 70° D. 150°

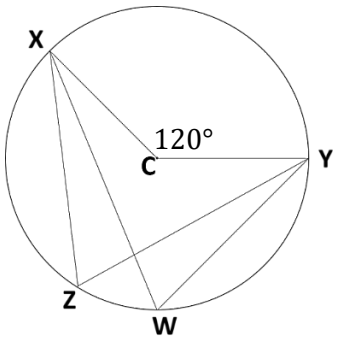
10. Suppose the measures of all the arcs in ⊙A are given. Which of the following concepts should be used to get the measure of $\overset{\frown}{DYS}$ in ⊙A?
   A. Arc Addition Postulate C. Sum of Central Arcs
   B. Intercepted Angle Postulate D. Sum of Inscribed Angles

11. What is the measure of the biggest arc in ⊙A?
   A. between 0° and 180° C. between 180° and 360°
   B. exactly 180° D. exactly 360°

12. Which of the following is true about the measures of ∠SAY and $\overset{\frown}{SY}$?
   A. The measure of ∠SAY is equal to the measure of $\overset{\frown}{SY}$.
   B. The measure of ∠SAY is less than the measure of $\overset{\frown}{SY}$.
   C. The measure of ∠SAY is greater than the measure of $\overset{\frown}{SY}$.
   D. The sum of their measures is equal to the circumference of the circle.

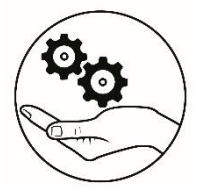
13. What is the relationship between the measure of an inscribed angle to the measure of its intercepted arc?
   A. The measure of the inscribed angle is *one-half* of the measure of its intercepted arc.
   B. The measure of the inscribed angle is *one-third* of the measure of its intercepted arc.
   C. The measure of the inscribed angle is *one-fourth* of the measure of its intercepted arc.
   D. The measure of the inscribed angle is *one-fifth* of the measure of its intercepted arc.

Refer to ⊙C *for items 14 and 15.*



14. What is the measure of $\overset{\frown}{XWY}$?
   A. 60° C. 120°
   B. 80° D. 240°

15. What is the measure of $\angle XZY$?
   A. 60° C. 120°
   B. 80° D. 240°



Additional Activity

More on Circles Please

1. **Arc Length** – The length of an arc can be determined by using the proportion $\frac{A}{360} = \frac{l}{2\pi r}$, where A is the degree measure of the arc, l is the arc length, and r is the radius of the circle.

Example: What is the length of a 60° arc of $\odot A$ whose radius is 5 cm long?

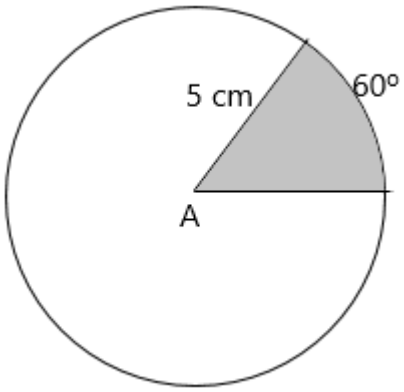
$$\frac{A}{360} = \frac{l}{2\pi r}$$

$$\frac{60}{360} = \frac{l}{2 \times 3.14 \times 5}$$

$$\frac{1}{6} = \frac{l}{31.4}$$

$$l = \frac{31.4 \times 1}{6}$$

$$l = 5.23\ cm$$



2. **Sector of a Circle** – A sector of a circle is the region bounded by an arc of the circle and the two radii or radiuses to the endpoints of the arc. In finding the area of a sector, we multiply the $\frac{measure\ of\ the\ arc}{360}$ and the area of the circle.

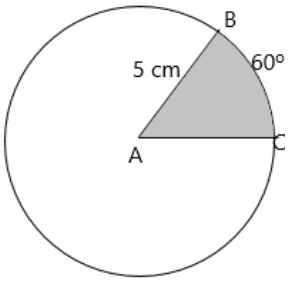
Example: The area of $\odot$A is 78.5 $square\ cm$. What is the area of sector BAC, the shaded region of $\odot$A?

$$Area\ of\ Sector\ BAC = \frac{measure\ of\ the\ arc}{360} \times Area_{circle}$$

$$Area\ of\ Sector\ BAC = \frac{60}{360} \times 78.5$$

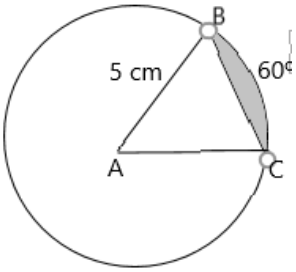
$$Area\ of\ Sector\ BAC = \frac{1}{6} \times 78.5$$

$$Area\ of\ Sector\ BAC = 13.08\ square\ centimeters$$



Segment of a Circle – A segment of a circle is the region bounded by an arc and the segment/chord joining its endpoints. To find the area of a sector, we subtract the area of the triangle formed by the radii and the chord from the area of the whole sector where the segment of the circle is found.

Example: What is the area of the segment/ shaded region in ⊙A? $\overline{BC} = 5\ cm$ and the area of $\triangle BAC = 5.41\ sq.cm.$



$$A_{segment} = A_{sector\ BAC} - A_{\triangle BAC}$$
$$A_{segment} = 13.08\ sq.cm - 5.41\ sq.cm$$
$$A_{segment} = 7.67\ square\ centimeters$$

3. All radii of the same circle are congruent.
4. In a circle or in congruent circles, two minor arcs are congruent if and only if their corresponding central angles are also congruent.
5. In a circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are also congruent.
6. In a circle, a diameter bisects a chord and an arc sharing the same endpoints if and only if the diameter is perpendicular to the chord.

It's your turn.

Answer the following questions. You will be rated using the rubric in *What I Can Do.*

1. Five points on a circle separate the circle into five congruent arcs.

   a. What is the degree measure of each arc?

   b. If the radius of the circle is $3\ cm$, what is the length of each arc?

   c. Suppose the points are connected consecutively with line segments. How do you describe the figure formed?

2. Do you agree that if two lines intersect at the center of the circle, then the lines intercept two pairs of congruent arcs? Explain your answer.

3. The length of an arc of a circle is $6.28\ cm$. If the circumference of the circle is $36.68\ cm$, what is the degree measure of the arc? Explain how you arrived at your answer.

Answer Key

What I Know

1. C
2. A
3. C
4. C
5. D
6. C
7. A
8. D
9. B
10. A
11. B
12. C
13. C
14. A
15. B

Lesson 1.

What's New

1. radiuses/radii
2. a diameter
3. chords
4. semicircles
5. minor arcs
6. major arcs
7. central angles
8. the center
9. intercepted arcs

Lesson 1.

What's More

<u>Assessment 1</u>

1. angle GCA
2. angle DCH
3. arc AH
4. angle FCA
5. arc EI

<u>Assessment 2</u>

1. 15 degrees
2. 15 degrees
3. 45 degrees
4. 45 degrees
5. 45 degrees

Lesson 1.

What I Have Learned

The measure of a central angel and the arc it intercepts are equal. If the measure of the central angle is between zero and 180 degrees, then the intercepted arc is a minor arc. Furthermore, if the measure of the central angle is 180 degrees, then the arc it intercepts is a semicircle and it measures exactly 180 degrees. Moreover, if the measure of the central angle is between 180 and 360 degrees, then the arc it intercepts is a major arc.

Lesson 1.

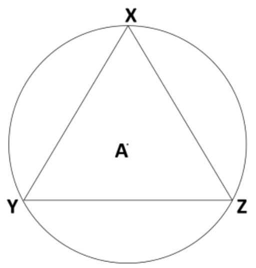
What I can Do

The outputs will be scored using the rubrics presented.

Lesson 2.

What's New

The illustration is similar to this.



Lesson 2.

What's More

Assessment 1

1. FALSE
2. FALSE
3. TRUE
4. TRUE
5. TRUE

Assessment 2

1. 164 degrees
2. 82 degrees
3. 82 degrees

Lesson 2.

What I Have Learned

The measure of an inscribed angle is one-half the measure of the arc it intercepts.

Lesson 2.

What I can Do

Draw two chords on the garden. The center of the perpendicular bisector to the chords is the center of the circular garden.

Assessment

1. B
2. D
3. A
4. C
5. C
6. B
7. C
8. A
9. B
10. A
11. D
12. A
13. A
14. A
15. C

Additional Activities

1. a. 72 degrees
   b. 3.768 centimeters
   c. regular pentagon
2. Yes. There are two pairs of congruent central angles/vertical angles formed and they intercept congruent arcs.
3. 61.64 degrees. (Evaluate students' explanations. They are expected to use the proportion $\frac{A}{360} = \frac{l}{2\pi r}$ to support their explanations.)

References

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