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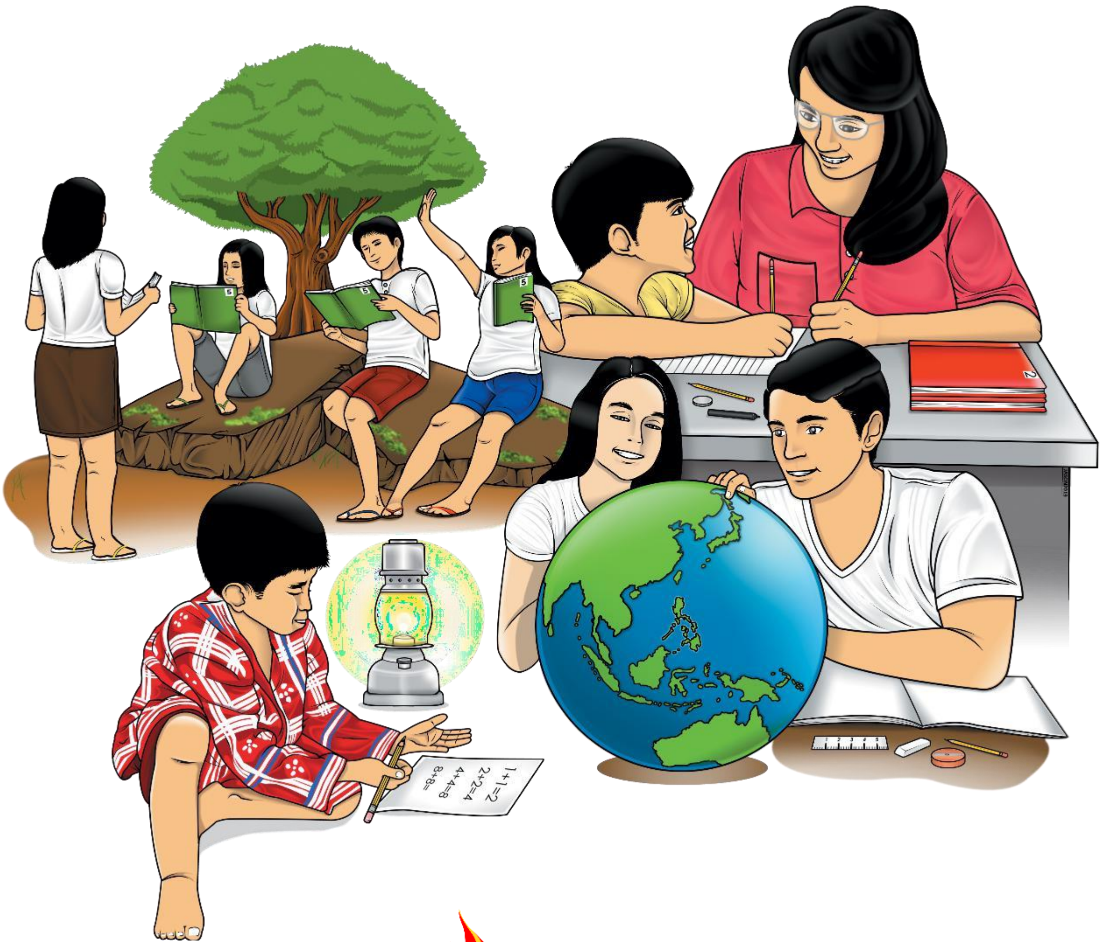
Mathematics

Quarter 1 – Module 7:
Geometric Series



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10

Mathematics

Quarter 1 – Module 7:
Geometric Series

DEPARTMENT OF EDUCATION

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

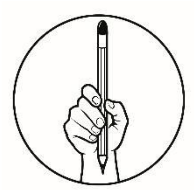
Thank you.

What I Need to Know

This module aims to provide the material necessary to introduce the mathematical concept of geometric sequences to Grade – 10 students. This module will discuss the formula in finding the sum of finite and infinite geometric series. It also includes interesting activities which will help learners understand geometric series well.

After going through this module, the learner should be able to:

a. find the sum of terms of a finite geometric sequence, and
b. find the sum of terms of infinite geometric sequence.



What I Know

A. Multiple Choice. Read and analyze the following items and determine the letter of the correct answer from the given choices. Write your answer on a separate sheet of paper.

1. This refers to the sum of the terms of a geometric sequence.

A. Series
B. Limit
C. Continuity
D. All of these

2. Find the sum of the first six terms of a geometric sequence whose first term is 2 and common ratio of $\frac{2}{3}$.

A. $\frac{1\,234}{4\,567}$
B. $\frac{1\,330}{243}$
C. $\frac{3\,990}{729}$
D. $\frac{760}{4\,551}$

3. Find the sum of the first five terms of the geometric sequence 4, 6, 9, …

A. $\frac{11}{4}$
B. $\frac{211}{4}$
C. $\frac{4}{11}$
D. $\frac{4}{211}$

4. Find the sum of the first five terms of the geometric sequence 2, 8, 32, 128, …

A. 243
B. 682
C. 679
D. 743

5. Find the sum of the terms of the infinite geometric sequence 45, 15, 5, …

A. 68
B. $\frac{137}{2}$
C. $\frac{135}{2}$
D. 69

For items 6 – 10, find specified geometric series of the following geometric sequences:

6.) $3, 12, 48, \ldots$ $\quad S_7$

7.) $2, 6, 18, \ldots$ $\quad S_6$

8.) $125, 25, 5, \ldots$ $\quad S_8$

9.) First term $a_1 = 2$ and common ratio $r = -4$; find S_8

10.) First term $a_1 = \frac{1}{2}$ and common ratio $r = -2$; find S_6

For items 11 – 15, find the geometric series of each infinite geometric sequence.

11.) $12, 6, 3, \ldots$

12.) $125, 25, 5, \ldots$

13.) $1, \frac{1}{5}, \frac{1}{25}, \ldots$

14.) $15, 5, \frac{5}{3}, \ldots$

15.) $1, \frac{2}{3}, \frac{4}{9}, \ldots$

Lesson 1 Finite Geometric Series

What's In

In the previous module, we derived the formula in finding the n[th] term of a geometric sequence. This formula allows you to accurately identify the n[th] term of any geometric sequence.

We shall now proceed to discussing how to find the sum of the terms of geometric sequences.

Consider the geometric sequence 3, 6, 12, 24, … If I let you find the sum of the first five terms of the geometric sequence, maybe, you'll simply generate the five terms then add them. For example, $\mathbf{3 + 6 + 12 + 24 + 48 = 93}$.

Essential Question:
What is the sum of the first 10 terms? First 15 terms? First 20 terms?

How will you be able to answer the above question? Will you do the same of adding the terms one by one to find the sum? How long will it take you answer the question?

This module will discuss to you how to find the sum of the terms of finite and infinite geometric sequences without going through the process of adding the terms one by one.

What's New

PLUS FACTOR

Find the sum of the first five terms of the following geometric sequences.

1.) $7, 14, 28, \ldots$

2.) $3, 12, 48, \ldots$

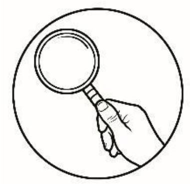
3.) $100, 50, 25, \ldots$

4.) $1, 6, 36, \ldots$

5.) $54, 18, 6, \ldots$

Basically, you will identify first the common ratio of the sequence to generate the next terms of the sequence. For Item number 1, the common ratio (r) is equal to $\frac{14}{7}$= 2. Therefore, the first five terms of the sequence are 7, 14, 28, 56, and 112. Then, add the terms to find the sum.

$$7 + 14 + 28 + 56 + 112 = 217$$



What Is It

Consider item number 1 on your activity, the sum of the first five terms obtained is 217. You were able to find the sum by generating all the terms and then adding them. But, how about the sum of the first 20 terms? Are you still going to generate all the terms?

$$7 + 14 + 28 + 56 + 112 = 217$$

Geometric Series

Geometric series refers to the sum of the terms of a geometric sequence.

Here is the formula in finding the sum of the first n terms of a finite geometric sequence:

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

Where: S_n= the sum of the first n terms
a_1= first term
$\boldsymbol{n}$ = number of terms
$\boldsymbol{r}$ = common ratio

Let's try to use the formula.

Find the sum of the first five terms of the geometric series $7, 14, 28, \ldots$

Solution: a_1 **= 7,** $\boldsymbol{n}$ **= 5,** $\boldsymbol{r}$ **=** $\frac{a_2}{a_1}$ **=** $\frac{14}{7}$**= 2**

Let's substitute the given values in the formula:

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

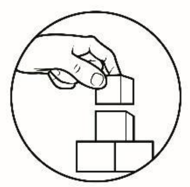
$$S_5 = \frac{7(1 - 2^5)}{1 - 2}$$

$$S_5 = \frac{7(1 - 32)}{-1}$$

$$S_5 = \frac{7(-31)}{-1}$$

$$S_5 = 217$$

Using the formula, it gives the same answer.



What's More

ACTIVITY 1. SUM IT UP

1. Find the sum of the first seven terms of a geometric sequence whose first term is 3 and common ratio is 4.
Solution : $a_1 = 3$, $n = 7$, $r = 4$
Substitute these values in the formula:

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

$$S_7 = \frac{3(1 - 4^7)}{1 - 4}$$

$$S_7 = \frac{3(1 - 16,384)}{-3}$$

$$S_7 = \frac{7(-16,383)}{-3}$$

$$S_7 = \frac{-114,681}{-3}$$

$$S_7 = 38,227$$

2. Find the sum of the first 6 terms of the geometric sequence $\frac{1}{4}, \frac{1}{2}, 1, \ldots$

Solution: $a_1 = \frac{1}{4}$, $\boldsymbol{n} = 6$, $\boldsymbol{r} = \frac{\frac{1}{2}}{\frac{1}{4}} = \frac{4}{2} = \mathbf{2}$

Substitute these values in the formula:

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

$$S_6 = \frac{\frac{1}{4}(1 - 2^6)}{1 - 2}$$

$$S_6 = \frac{\frac{1}{4}(1 - 64)}{-1}$$

$$S_6 = \frac{\frac{1}{4}(-63)}{-1}$$

$$S_6 = \frac{-\frac{63}{4}}{-1}$$

$$S_6 = \frac{63}{4}$$

ASSESSMENT 1. PLUS IT!

1. Find the sum of the first 6 terms of a geometric sequence whose first term is 2 and common ratio is $\frac{2}{3}$.
2. What is the sum of the first 10 terms of the geometric sequence 4, 2, 1?

What I Have Learned

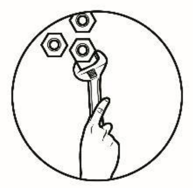
COMPLETE ME!

Fill in the blanks.

1. ________________ is the sum of the terms of a geometric sequence.
2. To find finite geometric series, use the formula _________________.

For numbers $3 - 5$, use the sequence $3, 6, 12, \ldots$ Determine

3. $a_1 =$ ________
4. $r =$ _______
5. $S_5 =$ ________



What I Can Do

Illustrative Example:

Joey saves an amount in his bamboo bank each week. To make it fun, he doubles whatever amount is inside the bank during the next week. On the first week, he saves 1 peso. On the 10th week, how much will be in the bamboo bank in all?

	First Week	Second Week	Third Week	Fourth week
Amount	₱1	₱2	₱4	₱8

The total savings can be computed by

$$1 + 2 + 4 + 8 + \cdots + 512 \longrightarrow S_{10}$$

$$a_1 = 1, r = 2, S_{10} = ?$$

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

$$S_{10} = \frac{1(1 - 2^{10})}{1 - 2}$$

$$S_{10} = 1{,}023$$

Therefore, Joey has a total savings of ₱1,023 after ten weeks.

Problem: Solve completely.

Suppose Rico saves P100.00 in January and each month thereafter he manages to save one-half more than what he saved in the previous month. How much is Rico's savings after 10 months? Round off your final answer to two decimal places.

Lesson 2 Infinite Geometric Series

What's In

In the previous lesson, you have learned about finding the sum of the terms of a finite geometric sequence using a formula. Since the sequence is finite, we can easily find the sum. But, what if the sequence has no last term? Can you possibly find the sum?

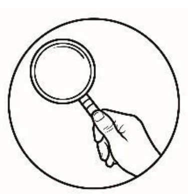
This module will discuss to you how to find the sum of terms of infinite geometric sequences.

What's New

INFINI-TERM

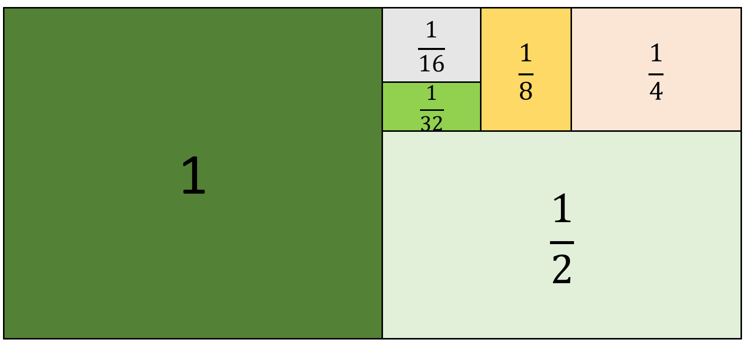
Task: Find the sum of the terms of the geometric sequence $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$

You might be wondering how you can get the sum of the terms of the geometric sequence because its terms are infinite. By the way, is it even possible?
If you want to know the answer to that question, continue learning the next lesson.



What is It

Consider a 1 by 2 rectangular figure. Divide the figure into various portions shown as follows.



The rectangle was divided into various portions such as 1 unit, $\frac{1}{2}$unit, $\frac{1}{4}$ unit, $\frac{1}{8}$ unit, $\frac{1}{16}$unit and so on. The rectangle can still be divided into smaller rectangles up to infinity.

If the areas of all rectangles are added this will give a sum of 2 square units. This scenario indicates that you can still find the sum of the terms of an infinite geometric sequence.

In a mathematical statement, we can write the sum of the areas of the rectangles as follows:

$$1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}\ \boxed{\ldots}\ = \mathbf{2}$$

Sum of terms of an infinite geometric sequence

The three dots or simply called ellipsis indicates infinity.

Based from this example, we can have the formula:

$$S_\infty=\frac{a_1}{1-r}$$

Where: S_∞ =sum to infinity
a_1 = first term
r = common ratio

Let us test if the formula is correct:

$$1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\ \ldots = \mathbf{2}$$

Solution:

$$a_1=1 \qquad\qquad r=\frac{a_2}{a_1}=\frac{\frac{1}{2}}{1}=\frac{1}{2}$$

Let us substitute these values in the formula:

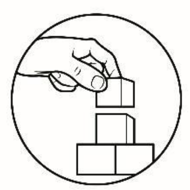
$$S_\infty=\frac{a_1}{1-r}$$

$$S_\infty=\frac{1}{1-\frac{1}{2}}$$

$$S_\infty = \frac{1}{\frac{1}{2}}$$

$$\boxed{S_\infty = 2}$$

We tested it right!



What's More

ACTIVITY 1. SUM UP TO INFINITY

1. Find the sum of the terms of the infinite geometric sequence 3, 1, $\frac{1}{3}$,…

   Solution: $a_1 = 3$, $r = \frac{1}{3}$

   Substitute these values in the formula:

$$S_\infty = \frac{a_1}{1-r}$$

$$S_\infty = \frac{3}{1-\frac{1}{3}}$$

$$S_\infty = \frac{3}{\frac{2}{3}}$$

$$\boxed{S_\infty = \frac{9}{2}}$$

2. Find the sum of an infinite geometric sequence whose first term is 5 and common ratio is $\frac{1}{4}$.

Solution : $a_1 = 5$, $r = \frac{1}{4}$

Substitute these values in the formula:

$$S_\infty = \frac{a_1}{1-r}$$

$$S_\infty = \frac{5}{1-\frac{1}{4}}$$

$$S_\infty = \frac{5}{\frac{3}{4}}$$

$$\boxed{S_\infty = \frac{20}{3}}$$

ASSESSMENT 1. INFINITY

Solve for the specified geometric series. Show your complete solutions.

1.) Find the sum of the terms of an infinite geometric sequence whose first term is 4 and common ratio is $\frac{1}{5}$.

2.) Given the sequence 9, 3, 1,… , find S_∞

What I Have Learned

COMPLETE ME!

Fill in the blanks.

1. _______________ is a set of three dots that indicates infinity.

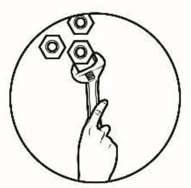
2. To find infinite geometric series, use the formula _____________.

3. In an infinite geometric series, $-1 <$ ___ > 1.

For numbers 4 – 5, use the sequence $2, 1, \frac{1}{2}, \ldots$ Determine

4. $r =$ _______

5. $S_\infty =$ _______



What I Can Do

Illustrative Example:

A ball tossed to a height of 8 meters rebounds to $\frac{1}{2}$ its previous height. Find the distance the ball has travelled when it comes to rest.

Note: 1) The distance travelled going up is the same as the distance travelled going down.
2) When the ball is at rest, the distance is zero.

Direction	Tossing	1st Rebound	2nd Rebound	3rd Rebound
Up	$8m$	$4m$	$2m$	$1m$
Down	$8m$	$4m$	$2m$	$1m$
Total	$2(8)$ *or* $16m$	$2(4)$ *or* $8m$	$2(2)$ *or* $4m$	$2(1)$ *or* $2m$

The total distance can be written as

$2(8) + 2(4) + 2(2) + 2(1) + \cdots$ or $2(8 + 4 + 2 + 1 + \cdots)$ $\quad 2S_\infty$

$$a_1 = 8, r = \frac{1}{2}, S_\infty = ?$$

$$S_\infty = \frac{a_1}{1-r}$$

$$S_\infty = \frac{8}{1-\frac{1}{2}}$$

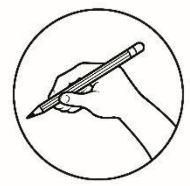
$$S_\infty = 16$$

$$2S_\infty = 32$$

Therefore, the total distance the ball has travelled when it comes to rest is $32m$.

Problem: Solve completely.

A ball tossed to a height of 6 meters rebounds to $\frac{2}{3}$ its previous height. Find the distance the ball has travelled when it comes to rest.



Assessment

A. **Multiple Choice.** Read and analyze the following items and determine the letter of the correct answer from the given choices. Write your answer on a separate sheet of paper. **USE CAPITAL LETTERS ONLY.**

_____ 1. How can you indicate infinite geometric sequences?

A. Use an ellipsis
B. Use an arrowhead
C. Use a bar line above the sequence
D. All of these

_____ 2. Find the sum of the first six terms of a geometric sequence whose first term is 2 and common ratio is $\frac{2}{3}$.

A. $\frac{1330}{243}$
B. $\frac{1234}{4567}$
C. $\frac{760}{4551}$
D. $\frac{3990}{729}$

_____ 3. Find the sum of the first five terms of the geometric sequence 4, 6, 9…

A. $\frac{11}{4}$
B. $\frac{4}{211}$
C. $\frac{4}{11}$
D. $\frac{211}{4}$

_____ 4. Find the sum of the terms of the infinite geometric sequence 45, 15, 5,…

A. 68
B. $\frac{137}{2}$
C. $\frac{135}{2}$
D. 69

_____ 5. Find the sum of the first five terms of the geometric series 2, 8, 32, 128,…

A. 682
B. 243
C. 679
D. 743

For items 6 – 10, find the specified geometric series of each of the following geometric sequences:

____________ 6) $3, 12, 48, \ldots$ S_7

____________ 7) $2, 6, 18, \ldots$ S_6

____________ 8) $125, 25, 5, \ldots$ S_8

____________ 9) First term $a_1 = 2$ and common ratio $r = -4$; find S_8

____________ 10) First term $a_1 = \frac{1}{2}$ and common ratio $r = -2$; find S_6

For items 11 – 15, find the sum of the terms of each of the following infinite geometric sequences:

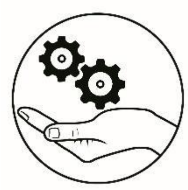
___________ 11) $12, 6, 3, \ldots$

___________ 12) $125, 25, 5, \ldots$

___________ 13) $1, \frac{1}{5}, \frac{1}{25}, \ldots$

___________ 14) $15, 5, \frac{5}{3}, \ldots$

___________ 15) $1, \frac{2}{3}, \frac{4}{9}, \ldots$



Additional Activity

Answer completely.

When a ball is tossed to a height of 4 meters above the ground, it always rebounds to 40% of its previous height until it stops. Find the total distance that the ball has covered when it strikes the ground for the fifth time.

ANSWER KEY

Lesson 1

What I Know

1. A
2. B
3. B
4. B
5. C
6. 16 383
7. 728
8. $\frac{97\,656}{625}$
9. – 26 214
10. $-\frac{21}{2}$
11. 24
12. $\frac{625}{4}$
13. $\frac{5}{4}$
14. $\frac{45}{2}$
15. 3

WHAT'S NEW
Lesson 1

1. 217
2. 1 023
3. $\frac{775}{4}$
4. 1 555
5. $\frac{242}{3}$

What's More.
Lesson 1
Assessment 1

1. $\frac{1\,330}{243}$
2. $\frac{1\,023}{128}$

What I Have Learned
Lesson 1

1. Geometric series
2. $S_n = \frac{a_1(1-r^n)}{1-r}$
3. $a_1 = 3$
4. r = 2
5. $S_5 = 93$

What I Can Do
Lesson 1

₱ 11,333.01

LESSON 2

What's More:
Lesson 2
Assessment 1

1. **5**
2. $\frac{27}{2}$

What I Have Learned

1. Ellipsis
2. $S_\infty = \frac{a_1}{1-r}$
3. r
4. $\frac{1}{2}$
5. 4

What I Can Do

36 m

ASSESSMENT

1. A
2. A
3. D
4. C
5. A
6. 16 383
7. 728
8. $\frac{97\,656}{625}$
9. – 26 214
10. $-\frac{21}{2}$
11. 24
12. $\frac{625}{4}$
13. $\frac{5}{4}$
14. $\frac{45}{2}$
15. 3

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