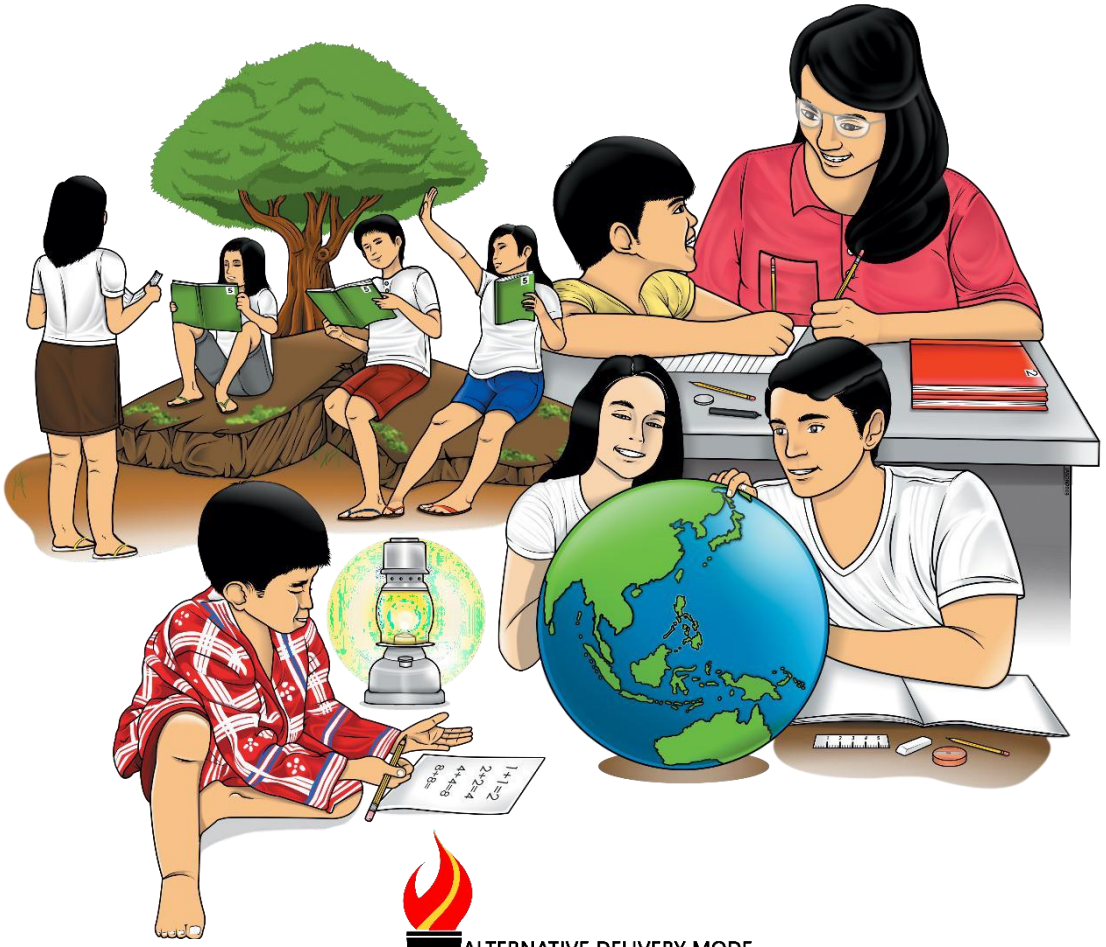
10

DepED
DEPARTMENT OF EDUCATION

Mathematics

Quarter 1 - Module 2: Illustrating an Arithmetic Sequence



ALTERNATIVE DELIVERY MODE
ADM

Mathematics – Grade 10
Alternative Delivery Mode
Quarter I – Module 2: Illustrating an Arithmetic Sequence
First Edition, 2019

Published by the Department of Education
Secretary: Leonor Magtolis Briones
Undersecretary: Diosdado M. San Antonio

Development Team of the Module

Writer's Name:	Evelyn N. Ballatong
Editor's Name:	Heather G. Banagui
Reviewers' Name:	Bryan A. Hidalgo, Laila B. Kiw-isen, Jim D. Alberto, and Selalyn B. Maguilao
Management Team:	May B. Eclar
	Benedicta B. Gamatero
	Carmel F. Meris
	Marciana M. Aydinan
	Ethielyn E. Taqued
	Edgar H. Madlaing
	Lydia I. Belingon

Printed in the Philippines by:

Department of Education – Cordillera Administrative Region

Office Address: Wangal, La Trinidad, Benguet
Telefax: (074) 422-4074
E-mail Address: car@deped.gov.ph

10

Mathematics

Quarter I - Module 2: Illustrating an Arithmetic Sequence

DepED
DEPARTMENT OF EDUCATION

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

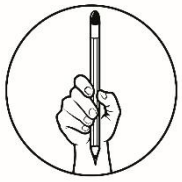
Thank you.

What I Need to Know

This module was designed and written with you in mind. It is here to help you define and illustrate an arithmetic sequence. The scope of this module permits it to be used in many different learning situations. The language used recognizes the diverse vocabulary level of students. The lessons are arranged to follow the standard sequence of the course but the pacing in which you read and answer this module is dependent on your ability.

After going through this module, you are expected to:

1. define an arithmetic sequence,
2. illustrate an arithmetic sequence and
3. solves problems involving arithmetic sequence.



What I Know

Choose the letter of the best answer. Write the letter of your choice on a separate sheet of paper.

1. Which of the following is a correct description of an arithmetic sequence?
   a. A term is obtained by multiplying a constant number from the preceding term.
   b. A term is obtained by squaring the preceding term.
   c. A term is obtained by adding a constant number to the preceding term.
   d. A term is obtained by extracting the square root of the preceding term.

2. Which of the following is the common difference of the arithmetic sequence 4, 7, 10, 13, ...?
   a. 0 b. 1 c. 2 d. 3

3. What is the common difference in the sequence: -4, 3, 10, 17, ...?
   a. 1 b. -7 c. 7 d. -1

4. Which of the following is the missing term of the arithmetic sequence -1, -7, _____, -19, -25, ...?
   a. -10 b. -11 c. -12 d. -13

5. Which of the following of the common difference of the arithmetic sequence -1, -8, -15, -22, ...?
   a. -5, b. -6 c. -7 d. -8

6. Which of the following is the common difference in the arithmetic sequence $\frac{1}{2}$, 1, 1 $\frac{1}{2}$, 2, ...?
   a. $\frac{1}{4}$ b. $\frac{3}{4}$ c. $\frac{5}{2}$ d. $\frac{1}{2}$

7. Which of the following is an arithmetic sequence?
   a. -7, 4, 15,26,. . . c. 21, 15, 9, 2, . . .
   b. -8, -6, -4, 0, ... d. 4, -1, -7, -13, . . .

8. Which of the following is the common difference of an arithmetic sequence whose a_2 = 1, a_5 = 7?
   a. 1 b. 2 c. 3 d. 4

9. Which of the following is the common difference of the arithmetic sequence 3a -1, 3a, 3a + 1, ... ?
   a. -a b. a c. -1 d. 1

10. Which of the following is the first positive term of the arithmetic sequence -8, -6, -4, ...?
    a. 0 b.2 c.4 d. 6

11. Which of the following is the value of **a** to make 3**a** +1, 4**a**, 6**a** + 1, … an arithmetic sequence?

a. -2 b. -1 c. 0 d. 1

12. Which of the following is not an arithmetic sequence?

a. 2, 4, 6, 8, … c. 3, 6, 12, 24,…

b. 4.1, 11.1, 18.1,… d. 5, 10, 15, 20, …

13. Which of the following is not an arithmetic sequence?

a. 4, 7, 10, 13, … c. 100, 98, 96, 94, …

b. -2, -6, -10, -14, … d. -4,-3, 10, 17, …

14. Which of the following is not true about the arithmetic sequence
5, 3, 1, -1, -3, …

a. a_2 = -3
b. the common difference is -2
c. a_6 = -5
d. one way to obtain a term, -2 must be added to the preceding term.

15. I was advised by my physician to walk each day in the morning as my daily exercise. On my first day, I walked 40m. On the second and third day, I walked 60m and 80m, respectively, and so on. Which of the following is the distance I walked on the 10th day if I continue the pattern in my daily walk?

a. 180 m b. 200 m c. 220 m d. 240 m

Lesson 1 Illustrates An arithmetic Sequence

What's In

In the previous module, you have learned that a sequence is an arrangement of objects, numbers or even figures which follows a certain pattern. Also, you have learned about the processes in finding patterns of any sequence.

Look at the sequences below. Can you see the specific pattern they follow?

2, 4, 6, 8, …	3, 6, 12, 24, …	5, 10, 15, 20, …

Let us try to give emphasis on the differences you observed as we proceed with this module.

What's New

Look at the following sequences. What is the pattern you observed in each sequence?

	Sequences	**Pattern**
1.	2, 4, 6, 8, …	A term is obtained by adding 2 to the preceding term.
2.	3, 6, 12, 24, …	A term is obtained by multiplying 2 to the preceding term.
3.	5, 10, 15, 20, …	A term is obtained by adding 5 to the preceding term.

What is It

Observe the following sequences:

1. 4, 7, 10, 13, ...
2. 33, 38, 43, 48, ...
3. -2, -6, -10, -14, ...
4. 100, 98, 96, 94, ...
5. $\frac{1}{2}$, 1, 1 $\frac{1}{2}$, 2, ...

Can you give the next two terms of the above sequences? How did you get those terms?

If you get 16 and 19 in item 1, then you are correct. Notice that a constant number 3 is added to the preceding term to get the next term. To get the next term in item 2, 5 is added to the preceding term. To get the next term in items 3, 4 and 5, the numbers -4, -2, and $\frac{1}{2}$ are added to the preceding term, respectively.

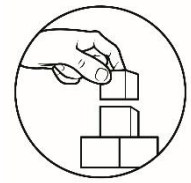
Notice that to get the next term in each of the sequences above, a constant or a common number is **added** to the preceding term or the number before it. The constant number being added is called the common difference and we represent it as *d.* All these sequences are called ***arithmetic sequences.***

To find the **common difference (*d*),** you can simply subtract

- the second term (a_2) by the first term (a_1),
  a_2 - a_1, or
- the third term $(a_3)$by the second term (a_2),
  a_3 - a_2 , or
- the fourth term $(a_4)$by the third term (a_3),
  a_4- a_3, or
- in general, a term (a_n) by its preceding term (a_{n-1})
  $d = a_n - a_{n-1}$.

Arithmetic Sequences

A sequence in which term after the first is formed by adding a fixed number to the preceding term is called ***arithmetic sequence***. The fixed number or constant is called the common difference denoted by ***d.***



What's More

Activity 1:

From the discussion on arithmetic sequence earlier, solve the problems that follows.

1. Determine if the sequence is arithmetic or not. If it is, find the common difference and the next three terms of the sequence.

   -4, 3, 10, 17, ...

   Solution:

   a. To find out if the sequence is arithmetic, there must be a common difference between any two consecutive terms in the sequence.

   $a_2 - a_1 = 3 - (-4)$
   $= 7$

   $a_3 - a_2 = 10 - (3)$
   $= 7$

   $a_4 - a_3 = 17 - 10$
   $= 7$

   Because there is a common difference between consecutive terms, the sequence is arithmetic.

   b. Since the common difference is 7, the next three terms are obtained by adding 7 to the preceding term.

   $a_5 = a_4 + 7$
   $= 17 + 7$
   $= 24$

   $a_6 = a_5 + 7$
   $= 24 + 7$
   $= 31$

   $a_7 = a_6 + 7$
   $= 31 + 7$
   $= 38$

   Thus, the common difference is 7 and the next three terms are 24, 31, 38.

2. Write the first five terms of the arithmetic sequence with 5 as the first term and with a common difference of -2.

   Solution:

   First term: $a_1 = 5$

   Second term: $a_2 = a_1 + (-2)$
   $= 5 - 2$
   $= 3$

Third term: $a_3 = a_2 + (-2)$
$= 3 - 2$
$= 1$

Fourth term: $a_4 = a_3 + (-2)$
$= 1 - 2$
$= -1$

Fifth term: $a_5 = a_4 + (-2)$
$= -1 - 2$
$= -3$

To get a term, we added the common difference or the constant to the preceding term. So, the first five terms of the sequence are 5, 3, 1, -1, and -3.

3.Find the common difference in an arithmetic sequence whose $a_2 = 1$, $a_5 = 7$.
Solution:

a. Remember that to find a term we add the common difference d, to the preceding term. For instance,
$a_3 = a_2 + d$ Equation 1
$a_{4=} a_3 + d$ Equation 2
$a_5 = a_4 + d$ Equation 3.

b. Substitute a_3 in Equation 1 to Equation 2
$a_{4=} a_3 + d$
$a_{4=} (a_2 + d) + d$
$a_{4=} a_2 + 2d$ Equation 4

c. Substitute a_4 in Equation 4 to Equation 3
$a_5 = a_4 + d$
$a_5 = (a_2 + 2d) + d$
$a_5 = a_2 + 3d$ Equation 5

d. Substitute the given in Equation 5
$a_5 = a_2 + 3d$
$7 = 1 + 3d$ Equation 6

e. Solve for d in Equation 6.
$7 = 1 + 3d$
$6 = 3d$
$2= d$

So, the common difference, d is 2.

4. Find the common difference of the arithmetic sequence
3a -1, 3a, 3a + 1, …

Solution:
a. We must remember that in an arithmetic sequence, the common difference d is a term minus the preceding term. Thus
$d = a_2 - a_1$ Equation 1 or
$d = a_3 - a_2$ Equation 2

b. If we use Equation 1 and substitute the given, we have

$$d = 3a - (3a - 1)$$

c. If we simplify, we obtain

$$d = 1.$$

So, the common difference is 1.

5. Find the value of **a** to make 3**a** +1, 4**a**, 6**a** + 1, … an arithmetic sequence?

Solution:

a. We must again remember that in an arithmetic sequence, the common difference d is a term minus the preceding term. Thus

$$d = a_2 - a_1 \text{ Equation 1 and}$$

$$d = a_3 - a_2 \text{ Equation 2}$$

b. If we substitute the given in Equation 1 and Equation 2,

$$d = 4\mathbf{a} - (3\mathbf{a} + 1) \quad \text{Equation 3}$$

$$d = (6\mathbf{a} + 1) - 4\mathbf{a} \quad \text{Equation 4}$$

c. Equate Equation 3 and Equation 4

$$4\mathbf{a} - (3\mathbf{a} + 1) = (6\mathbf{a} + 1) - 4\mathbf{a}$$

d. Combine similar terms

$$\mathbf{a} - 1 = 2\mathbf{a} + 1$$

e. Further simplify

$$-2 = \mathbf{a}$$

Thus, the value of **a** is -2.

6. I was advised by my physician to walk each day in the morning as a daily exercise. On the first day, I walked 40m. On the second and third day, I walked 60m and 80m, respectively, and so on. Which of the following is the distance I walked on the 10th day if I continue the pattern in my daily walk?

Solution:

a. If we analyze the problem, 40m, 60m, 80m, … follows an arithmetic sequence since there is a common difference which is equivalent to 20. Dropping the unit, the given could be written as follows

40 , 60, 80, …

b. Continuing this pattern until the 10th term, we have

40 , 60, 80, 100, 120, 140, 160, 180, 200, 220, …

Thus, in the 10th day, the distance travelled is 220 m.

Activity:

Find the common difference and the next three terms of each arithmetic sequence. Write your answer on your answer sheet.

Common Difference		***Next 3 Terms***
_____	1. 24, 14, 4, ___, ___, ___	
_____	2. 6, 10, 14, ___, ___, ___	

_____ 3. -7, 4, 15, ___, ___, ____

_____ 4. 21, 15, 9, ___, ___, ___

_____ 5. -8, -6, -4, ___, ___, ___

_____ 6. 5, -1, -7, ___, ___, ___

_____ 7. 4.1, 11.1, 18.1, ___, ___, ___

_____ 8. -1, -8, -15, ___, ___, ___

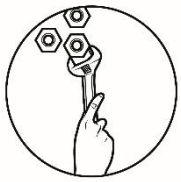
_____ 9. -3x, -10x, -17x ___, ___, ___

_____ 10. 3a -1, 3a, 3a + 1, _____, _____, _____

What I Have Learned

Answer the questions that follows on your answer sheet.

a. What is an arithmetic sequence?
b. How do we get the next terms of an arithmetic sequence?



What I Can Do

There is Math Around Us

Arithmetic sequence can be observed around us. Like the following fare rate for first 4 kms of a modernized PUJ under General Community Quarantine released by LTFRB last April 24, 2020.

Distance	Fare
First kilometer	11.00
Second kilometer	12.50
Third kilometer	14.00
Fourth kilometer	15.50

If we compute the increase of fare for every increase of kilometer distance, they are all equivalent to 1.50. With this, the fare rate is an example of an arithmetic sequence.

Aside from examples involving money, identify three situations or three things that you see or observe in your surroundings that illustrate an arithmetic sequence. Write your answer in your answer sheet.

Assessment

Choose the letter that you think best answers the question. Write your answer on your answer sheet.

1. Which of the following describes an arithmetic sequence?
   a. A sequence in which a term is formed by adding any number to the preceding term.
   b. A sequence in which there is an equal difference between consecutive terms.
   c. A sequence in which a term minus the preceding term is always positive.
   d. A sequence in which terms follows a pattern.

2. Which of the following is the common difference in the sequence
   0, 4, 8, 12, ...?
   a. 1 b. 2 c. 3 d.4

3. Which of the following is the common difference in the sequence
   3, -2, -7, ...?
   a.1 b. -5 c. 5 d. -1

4. Which of the missing term in this arithmetic sequence:
   23, 18, 13, 8, 3, _____, -7, -12,... ?
   a.-2 b. 2 c. -5 d. 5

5. Which of the following is the common difference in the sequence:
   -7, -4, -1, 2, 5,...?
   a.-3 b. 3 c. 4 d. -4

6. Which of the following is the common difference in the arithmetic sequence: 3, $\frac{13}{4}$, $\frac{7}{2}$, $\frac{15}{4}$, ... ?
   a. $\frac{1}{4}$ b. $\frac{3}{4}$ c. $\frac{5}{2}$ d. 4

7. Which of the following is an arithmetic sequence?
   a.1, 2, 3, 5, 7, 9... c. 1, -1, -3, -5...
   b.1, 10, 20, 30... d. 7, -7, 7, -7 ...

8. Which of the following is the common difference of an arithmetic sequence if a_3 = 4 and a_5 = 14?
   a.6 b. 5 c. 4 d. 3

9. Which of the following is the common difference of the arithmetic sequence 7p + 2, 5p +12, 3p + 22, ...?
   a.2p b. -2p +10 c. 2p - 10 d. 6p

10. Which of the following is the first positive term of the arithmetic sequence: -11, -8, -5, ...?
    a. -4 b. 3 c. -2 d. 1

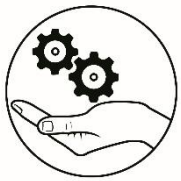
11. Which of the following is the value of p so that the terms,
    7p + 2, 5p +12, 2p – 1,... form an arithmetic sequence?
    a. -8 b. -5 c. -13 d. -23

12. Which of the following is NOT an arithmetic sequence?
    a. -5, -2, 1, 4
    b. 11, 14, 17, 20
    c. 1, 4, 7
    d. 3, 7, 12, 18

13. Which of the following is not an arithmetic sequence?
    a.1, 2, 3, 4, 5, …
    b.3, 9, 27, 81, …
    c. 4.5, 5.0, 5.5, 6.0, …
    d. 13, 2, -9, -20, -31, …

14. Which is NOT true about the arithmetic sequence: 25, 32, 39, 46, …?
    a. The common difference is 7.
    b. The 7^{th} term is 60.
    c. the 6^{th} term is 60.
    d. the 8^{th} term is 74.

15. During a free-fall rappelling, a skydiver jumps 36 feet, 48 feet, and 60 feet on the first, second, and third fall, respectively. If he continues to jump at this pattern, how many feet will he have jumped during the tenth fall?
    a.144 b. 156 c. 132 d. 140



Additional Activity

Determine whether the following situations illustrate an arithmetic sequence. If yes, then give what is asked. Write your answer on your answer sheet.

1. I was advised by my physician to walk each day in the morning as my daily exercise. On my first day, I walked 40m. On the second and third day, I walked 60m and 80m, respectively, and so on. What is the distance I walked on the 10^{th} day if I continue the pattern in my daily walk?

2. The number of works done by a backhoe in a certain area doubles every 2 hours. If there are N number of works to start with, find the number of works done in 14 hours.

Answer Key

Assessment:

1. A
2. D
3. B
4. A
5. B
6. A
7. C
8. B
9. B
10. D
11. D
12. D
13. B
14. B
15. A

Additional Activity

1. YES-
   In my 10th day of walking, I would have walked 220m.
2. No

What I Can Do

Depends on student's responses.

What's More

1.d= -10 -4,-14, -24

2.d= 4 18, 22, 26

3.d= 11 26, 37, 48

4.d=-6 3, -3, -9

5.d=2 -2, 0, 2

6.d=-6 -13, -19, -25

7.d= 7 25.1, 32.1, 39.1

8.d= -7 -22, -29, -36

9.d= -7x -24x,-31x,-38x

10.d= 1 3a+2, 3a+3, 3a+4

What I Have Learned

1.Arithmetic sequence - A sequence in which term after the first is formed by adding a fixed number to the preceding term.

2. By adding a fixed number or constant from the preceding term.

What I Know

1. C
2. D
3. C
4. D
5. C
6. D
7. A
8. C
9. D
10. B
11. A
12. C
13. D
14. A
15. D

References

Melvin M. Callanta, et al. *Mathematics Learner's Module*. Pasig City, 2015.

Gladys C. Nivera and Minie Rose C. Lapinid, *Grade 10 Mathematics: Patterns and Practicalities*. Makati City, Don Bosco Press, 2015.

Land Transportation Franchising and Regulatory Board. *Guidelines for Public Transportation for Areas Under General Community Quarantine.*2020. http://ltfrb.gov.ph/wp-content/uploads/2020/04/MC-2020-017-RE-GUIDELINES-FOR-PUBLIC-TRANSPORTATION-AREAS-UNDER-GCQ-min.pdf

For inquiries or feedback, please write or call:

Department of Education - Bureau of Learning Resources (DepEd-BLR)

Ground Floor, Bonifacio Bldg., DepEd Complex
Meralco Avenue, Pasig City, Philippines 1600

Telefax: (632) 8634-1072; 8634-1054; 8631-4985

Email Address: blr.lrqad@deped.gov.ph * blr.lrpd@deped.gov.ph: