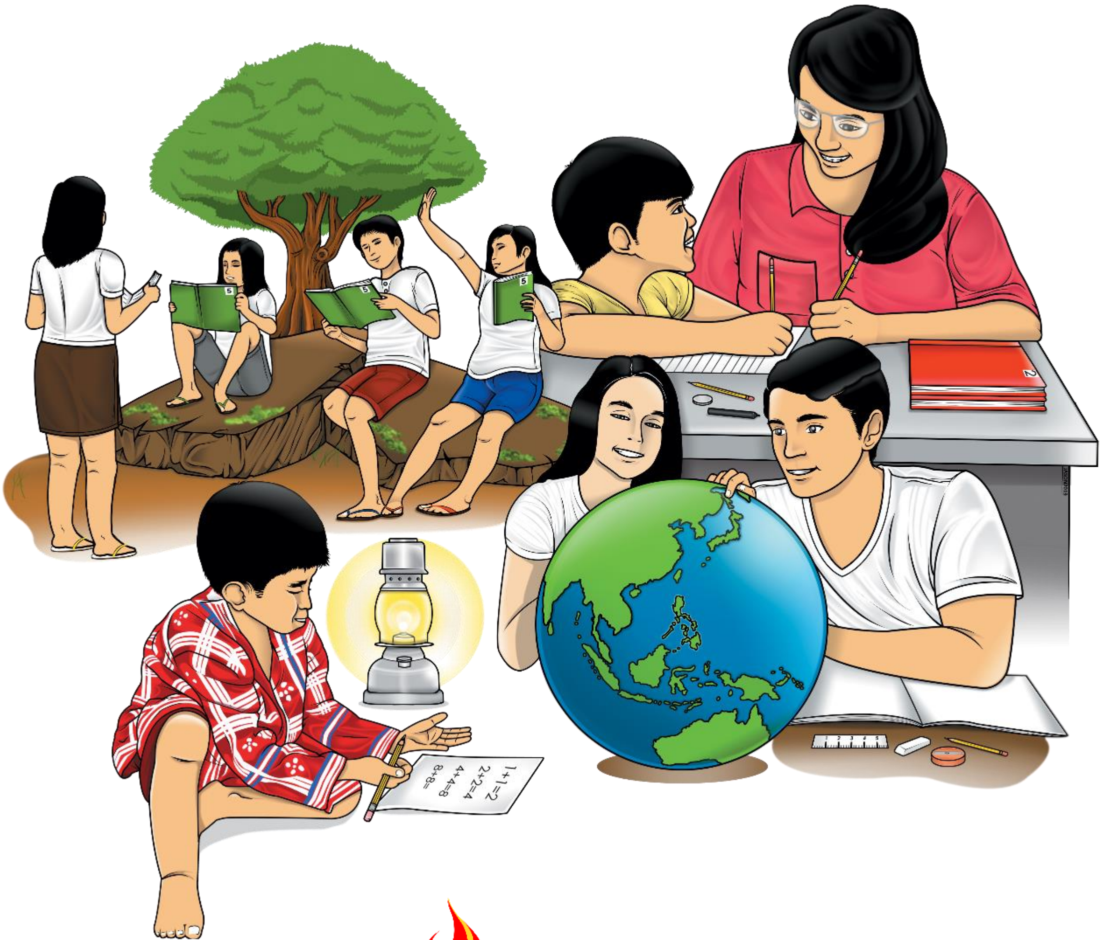
10

Mathematics

Quarter 1 – Module 10: Polynomial Equation



ALTERNATIVE DELIVERY MODE

Mathematics – Grade 10
Alternative Delivery Mode
Quarter 1 – Module 10: Polynomial Equation
First Edition, 2020

Published by the Department of Education
Secretary: Leonor Magtolis Briones
Undersecretary: Diosdado M. San Antonio

Development Team of the Module

Writer's Name: Rose Celia K. Hangdaan and Selalyn B. Maguilao

Editor's Name: Heather G. Banagui; Laila B. Kiw-isen

Reviewer's Name: Bryan A. Hidalgo, RO EPS for Mathematics

Management Team:

May B. Eclar
Marie Carolyn B. Verano
Carmel F. Meris, CES-CLMD
Ethielyn E. Taqued
Edgar H. Madlaing
Soraya T. Faculo
Francisco C. Copsiyan

Printed in the Philippines by:

Department of Education – Cordillera Administrative Region

Office Address: DepEd-CAR Complex, Wangal, La Trinidad, Benguet
Telefax: (074) 422-4074
E-mail Address: car@deped.gov.ph

10

Mathematics

Quarter 1 – Module 10: Polynomial Equation

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

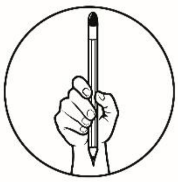
Thank you

What I Need to Know

This module was designed and written with you in mind. It is here to help you define and identify a polynomial equation, classify a polynomial equation according to its degree, define root (solution) of a polynomial equation, prove rational root theorem, find the roots of any polynomial equation using the rational root theorem, and solve problems involving polynomial equation. The scope of this module permits it to be used in many different learning situations. The language used recognizes the diverse vocabulary level of students. The lessons are arranged to follow the standard sequence of the course but the order in which you read and answer this module is dependent on your ability.

This module contains Lesson 1: Illustrating a Polynomial Equation and Lesson 2: Finding the Roots of Polynomial Equation. After going through this module, you are expected to:

- define a polynomial equation
- identify a polynomial equation
- classify a polynomial equation according to its degree.
- define root (solution) of a polynomial equation,
- prove rational root theorem,
- find the roots of any polynomial equation using the rational root theorem, and
- solve problems involving polynomial equation.



What I Know

Are you ready? You are task to answer the following questions before we proceed with our lesson. Do not worry, we only want to know how knowledgeable are you with the topics that we will be discussing in this module.

DIRECTION: Read and analyze each item carefully. Write the letter of the correct answer on the blank provided for.

_____1. Which of the following is an example of a polynomial equation?

A. $14x = 0$ C. $5x^3 - 4\sqrt{2x} + x = 0$

B. $x = \frac{1}{2X}$ D. $x^{-4} + 4x^{-3} = 0$

_____2. Which of the following is the degree of the polynomial equation $2x^4 – x^2 – 4 = 0$?

A. 0 B. -4 C. 2 D. 4

_____3. What value of m will make $2m – 1$ equal to zero?

A. 2 B. $\frac{1}{2}$ C. 0 D. $\frac{-1}{2}$

_____4. Find a quadratic polynomial with roots 1 and -2.

A. $a^2 - a + 2 = 0$ C. $a^2 + a - 2 = 0$

B. $a^2 - a - 2 = 0$ D. $a^2 + a + 2 = 0$

_____5. Find a cubic polynomial equation with roots -2, 2 and 4.

A. $m^3 + 4m^2 – 4m - 16 = 0$ C. $m^3 – 4m^2 - m + 16 = 0$

B. $10m^3 - m^2 - m + 16 = 0$ D. $m^3 – 4m^2 – 4m + 16 = 0$

_____6. According to Rational Root Theorem, which of the following is NOT a possible root of the polynomial $6b^3 – 3b^2 + 2b – 4 = 0$?

A. $\frac{3}{2}$ B. $\frac{1}{3}$ C. $\frac{4}{3}$ D. – 2

_____7. Using Rational Root Theorem, list all the possible rational roots of the polynomial equation $4x^4 + 31x^3 - 4x^2 - 89x + 22 = 0$.

A. $\pm22, \pm11, \pm2, \pm1$ C. $\pm1, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm2, \pm11, \pm\frac{11}{2}, \pm\frac{11}{4}, \pm22$

B. $\pm4, \pm2, \pm1$ D. $\pm1, \pm\frac{1}{2}, \pm\frac{1}{11}, \pm2, \pm4, \pm\frac{2}{11}, \pm\frac{4}{11}, \pm\frac{1}{22}$

_____8. Which of the following is NOT a root of $x(x + 3)(x + 3)(x – 1)(2x + 1) = 0$?

A. 0 B. 1 C. 3 D. $-\frac{1}{2}$

_____9. If 7 is a rational root of the polynomial $2x^3 - 14x^2 + x - 7 = 0$, which of the following is true?

A. (x + 7) is a factor of the polynomial
B. (x – 7) is a factor of the polynomial
C. 7 is a factor of the polynomial
D. -7 is a factor of the polynomial

_____10. How many possible real roots does the polynomial equation $5x^3 + 4x^2 - 31x + 6 = 0$ have?

A. at most 3 C. at most 2
B. at least 3 D. at least 2

_____11. If -1 is a root of $x^2 - 3kx + 3k - 7 = 0$, then the value of k is ___.

A. $\frac{7}{3}$ B. 3 C. 7 D. 1

_____12. Which of the following polynomial equations has −3 as a root?

A. $a^3 + 3a^2 - a + 3 = 0$ C. $a^3 - 3a^2 + a + 3 = 0$
B. $a^3 - 3a^2 - a + 3 = 0$ D. $a^3 + 3a^2 - a - 3 = 0$

_____13. Identify all real roots of $4x^4 + 31x^3 - 4x^2 - 89x + 22 = 0$.

A. $-2, \frac{1}{4}$ C. $\pm 1, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm 2, \pm 11, \pm\frac{11}{2}, \pm\frac{11}{4}$
B. $-2, \frac{1}{4}, -3 + 2\sqrt{5}, -3 - 2\sqrt{5}$ D. $\pm 2, \pm\frac{1}{4}, \pm\sqrt{5}, -3 - 2\sqrt{5}$

For items 14 -15. The volume of a rectangular solid is 750 cubic units. The width (w) is 7 units more than the height (h) and the length ($\mathcal{l}$) is 1 unit more than 8 times the height.

_____ 14. Which correctly represents the length of the solid?

A. (7w) ($\mathcal{l}$) (3+2h) B. 8h + 1 C. h + 7 D. 8h

_____ 15. What is the working equation to find the dimensions of the solid?

A. 750 = h(h + 7)(8h + 1) C. v = (8h + 1)(h)
B. 750 = (7w)($\mathcal{l}$)(8h) D. v = (7w)($\mathcal{l}$)(8h)

Lesson 1	Illustrates Polynomial Equations

What's In

We will start this lesson by reviewing how to

- identify if an expression is a polynomial or not,
- classify the degree of any given polynomial and,
- differentiate an expression from an equation.

Activity 1: Polynomials or NOT?

Direction: Put a check mark on the square before an expression if it is a polynomial ☑, otherwise put an X mark ☒.

☐ 1. $2x^3 + 3x - 2$

☐ 2. $2x^{-3} + 3x + 2$

☐ 3. $2^x - 3x + 2$

☐ 4. $2x + \frac{3}{x} + 2$

☐ 5. $2x^3 + \sqrt{3x} - 2$

How did you identify any given expression as a polynomial or not?

Activity 2: Can you identify my degree?

Direction: Write the degree of the following polynomial expressions.

Polynomials	Degree
1. $x^4 - 2x^3 + x$	
2. $x + 1$	
3. $x^3 - 5$	
4. $3x^5$	
5. 10	

How did you identify the degree of each expression?

Activity 3: Expression or equation?

Can you give at least 3 examples of an expression and an equation?

Expressions	Equations

How did you identify an expression? How about an equation?

What's New

You are now equipped with previously discussed concepts that will help you understand polynomial equation. Try answering the following questions.

A. If a **polynomial** is an algebraic expression involving only nonnegative-integer powers of one or more variable and containing no variable in the denominator and an **algebraic equation** is a mathematical statement stating that two algebraic expressions are equal, then, what do you think is the definition of a polynomial equation?

B. If a polynomial can be classified according to the degree of the term in the polynomial, do you think we can also classify a polynomial equation according to the degree of the polynomial involved? If yes, how?

Do you think you obtain a correct answer? To verify, let's continue with the next part.

What is It

POLYNOMIAL EQUATION

In defining a polynomial equation in the previous activity, probably you combined the definition of a polynomial and an equation. Your answer might be equivalent to the following possible definition of polynomial equation. A polynomial equation is a mathematical statement stating that two algebraic expressions are equal.

To have a more complete definition of a polynomial equation, we consider the following definitions.

- A **polynomial of degree n in x** is an algebraic expression consisting of terms with non-negative powers of x. Its general form is

$$a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_2x^2 + a_1x + a_0$$

where n is a non-negative integer, the coefficients $a_n, a_{n-1}, a_{n-2}, a_2, a_1, a_0$ are constants and x is a <u>variable</u>.

- An **algebraic equation** is a mathematical statement stating that two algebraic expressions are equal.

- Therefore, we can define **polynomial equation of degree n in x** as a mathematical statement consisting of terms and non-negative powers of x. Its general form is

$$a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_2x^2 + a_1x + a_0 = 0$$

where n is a non-negative integer, the coefficients $a_n, a_{n-1}, a_{n-2}, a_2, a_1, a_0$ are real numbers and $a_n \neq 0$ and x is a variable.

To further illustrate, observe the following examples.

Polynomial equations in x	Non-Polynomial equations in x
1. $x^3 + 12x^2 - 8x - 10 = 0$ 2. $\frac{1}{2}x^2 + x + 20 = 0$ 3. $3x - 1 = 2x^2$	1. $ax^2 + xy - 3 = 0$ 2. $x^3 + 2x$ 3. $2x^6 + 3x^{-2} = 4$ 4. $5x - \frac{1}{x} = 0$ 5. $6x^{\frac{2}{3}} = 3$

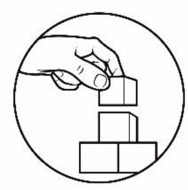
Note: x is used only as a variable, it can also make use of other letters in the English alphabet aside from x.

Aside from knowing how to identify if an equation is a polynomial equation or not, we can also classify its degree. The **degree of a polynomial equation** is the same as the degree of the term with the highest degree in the polynomial.

Observe the following examples.

Polynomial Equations	Degree
1. $8a - 10 = 0$	1. Degree 1
2. $b^2 + b = 20$	2. Degree 2
3. $\frac{1}{2}x^2 + x + 20 = 0$	3. Degree 2
4. $2d^3 - 1 = d^2$	4. Degree 3
5. $h^4 + 4h^2 - 1 = 0$	5. Degree 4
5. $k^7 + k^3 - 2k + 1 = 0$	6. Degree 7

Generally, first-degree polynomials in one variable are called linear equations, second-degree polynomials as quadratic equations, third-degree polynomials as cubic polynomials, fourth-degree polynomials as quartic polynomials, and fifth-degree polynomials as quintic polynomials. There are more degree classifications for polynomials, but these are by far the most commonly used.



What's More

Knowing the definition and classification of polynomial Equations, I know that you can do the following activity on your own.

Activity 4: Do I belong?

Direction: Given the following equations, group them as polynomial equations and NOT Polynomial equations.	
1. $2x^4 + 2x^3 + 10x = 11$	5. $\sqrt{x^7 + 3x^6 - 4x} = 0$
2. $2a^{-3} + 3a^2 + 5a - 3 = 0$	6. $4x^5 - 2x^3 + 5x - \frac{1}{x}$
3. $k^3 + 3k^2 + 9k - 3 = 0$	7. $15x = 0$
4. $p^2 + 3p + \frac{5}{p} = 0$	8. $2m^4 + 3m^3 + 2m + 1 = 0$

Activity 5: What's my degree?

Direction: For each polynomial equation, write the degree in the corresponding cell in the table.

Polynomial Equations	Degree
1. $x^3 - 15x - 18 = 0$	
2. $-15x - 18 = 0$	
3. $3x^4 + 1 = 0$	
4. $-18x = 0$	
5. $x^6 - 9 = 0$	

What I Have Learned

Finally, do the following to summarize the concepts discussed.

Activity 6

Direction: List down criteria to identify a polynomial equation.

Lesson 2 Finding the Roots of a Polynomial Equation

What's In?

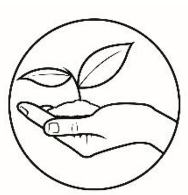
Let's review our previous lesson on synthetic division, remainder theorem and factor theorem by answering the following activity:

Activity 1

A) Fill in the blanks with words and symbols that will best complete the statements given below.

1) If p(x) is divided by (x+1) using synthetic division, the value of ***r*** to be used will be equal to ________?
2) If p(x) is divided by (2x -1) using synthetic division, the value of ***r*** to be used will be equal to ________?
3) If p(a) is divided by (a+1), then the remainder is __________.
4) If p(a) is divided by (a+1), and the remainder is equal to zero, then (a+1) is a __________ of p(a).

B) Divide ($2a^3 - 3a^2 – 4a – 17$) by ($a – 3$) using synthetic division.



What's New?

Let us recall that when a variable in an equation is replaced by a specific number, the resulting statement may be either true or false. If it is true, then that number is called a **solution** (or **root**) of the equation. The set of all solutions is called the **solution set** of the equation. A number that is a solution is said to satisfy the equation.

Examples:

1) -4 is a root of the equation $x + 4 = 0$ because when -4 is substituted to x, it makes the equation true.
2) 2 is NOT a root of the equation $x^2 + 4 = 0$ because when 2 is substituted to x, it makes the equation false.

I think you are ready for the activity below.

Activity 2

Match the polynomial equation on the left to its corresponding roots or solution sets on the right.

Polynomial Equation	**Roots / Solution Sets**
1) $a + 5 = 3$	A) $\{-3, -\frac{1}{2}, 2\}$
2) $b^2 - b - 6 = 0$	B) $\{1, 2, 3\}$
3) $(a + 3)(a^2 - 4) = 0$	C) $\{-2, 3\}$
4) $(m + 3)(2m + 1)(m - 2) = 0$	D) $\{-3, -2, 2\}$
5) $(x - 1)^2 (x - 2)(x - 3) = 0$	E) $\{-2\}$

What is it?

How did you find activity 2? Did you answer the activity correctly?

To help you solve for the roots of any polynomial equation that is not written in factored form is for you to apply your skill in synthetic division, factoring and the remainder theorem that we will be discussing below.

However, it is also important to note the following theorems:

a) in searching for complex roots:

Fundamental Theorem of Algebra
Every polynomial equation $P(x)$ of degree n has at least one complex root.

b) in searching for real roots:

The number of real roots:
A polynomial equation cannot have more real roots than its degree.

That is, an n^{th} degree polynomial equation can have at most n real roots. Example: a quadratic equation can have at most 2 real roots.

c) in searching for rational roots:

Rational Root Theorem

If a polynomial equation $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0 = 0$ has integer coefficients, then every rational root of the polynomial equation has the form $\frac{p}{q}$, where p and q have no common factors other than 1 and where p is an integer factor of the constant term, a_0 and q is an integer factor of the leading coefficient, a_n.

Before we will use these theorems in solving the roots of a polynomial equation, let us prove first the Rational Root Theorem. Study the proof below:

Proof (Rational Root Theorem):

A) Prove that p is a factor of a_0.

1) Since $\frac{p}{q}$ is a root of the polynomial equation, then:

$$a_n\left(\frac{p}{q}\right)^n + a_{n-1}\left(\frac{p}{q}\right)^{n-1} + \ldots + a_2\left(\frac{p}{q}\right)^2 + a_1\left(\frac{p}{q}\right) + a_0 = 0$$

2) Multiply both sides of the equation by q^n.

$$q^n\left[a_n\left(\frac{p}{q}\right)^n + a_{n-1}\left(\frac{p}{q}\right)^{n-1} + \ldots + a_2\left(\frac{p}{q}\right)^2 + a_1\left(\frac{p}{q}\right) + a_0\right] = (0)q^n$$

$$a_n p^n + a_{n-1}p^{n-1}q + \ldots + a_2 p^2 q^{n-2} + a_1 p q^{n-1} + a_0 q^n = 0$$

3) Add $-a_o q^n$ to both sides of the equation.

$$a_n p^n + a_{n-1}p^{n-1}q + \ldots + a_2 p^2 q^{n-2} + a_1 p q^{n-1} + a_0 q^n + \left(-a_0 q^n\right) = 0 + \left(-a_0 q^n\right)$$

$$a_n p^n + a_{n-1}p^{n-1}q + \ldots + a_2 p^2 q^{n-2} + a_1 p q^{n-1} = -a_0 q^n$$

4) Factor out p on the left side of the equation.

$$p\left(a_n p^{n-1} + a_{n-1}p^{n-2}q + \ldots + a_2 p q^{n-2} + a_1 q^{n-1}\right) = -a_0 q^n$$

5) Since p is a factor of the left side of the equation, then p must also be a factor of the right side.
6) Since p and q have no common factor except 1, then p must be a factor of a_0 (*and not* q^n). This proves the first part of the rational root theorem.

Now it's your turn to do the second part of the proof which is to prove that q is a factor of a^n. Use the following steps as your guide.

Activity 3

Prove that q is a factor of a_n.

1) Since $\frac{p}{q}$ is a root of the polynomial equation, then:

    ____

2) Multiply both sides of the equation by q^n.

    ____

3) Add $-a_n p^n$ to both sides of the equation.

    ____

4) Factor out q on the left side of the equation.

    ____

5) Since q is a factor of the left side of the equation, then

    ____

6) Since p and q have no common factor except 1, then _________________________. This proves the second part of the rational root theorem.

This time, we are now ready to find the roots of a polynomial equation. The following procedures will guide you on how to find for roots.

1) Use the rational root theorem to list all possible rational roots of the polynomial equation.

2) Use synthetic division to evaluate each possible root, $\frac{p}{q}$, listed in step 1. If the remainder is equal to zero, then $\frac{p}{q}$ is a root of the polynomial equation. Repeat synthetic division until the remaining quotient is a quadratic polynomial. Then use factoring or quadratic formula to find the other roots.
3) List down the roots of the polynomial equation.

Example 1. Find the roots of $x^3 + 2x^2 - 5x - 6 = 0$.

Step 1**.** List all the possible roots of the equation using the rational root theorem**.**

a) List factors of the constant term, $a_0 = -6$: $p = \pm 1, \pm 2, \pm 3, \pm 6$

b) List factors of the leading coefficient, $a_0 = 1$: $q = \pm 1$

c) List possible rational roots, $\frac{p}{q}$: $\frac{p}{q} = \frac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1}$

Therefore, the possible rational roots are $\pm 1, \pm 2, \pm 3, \pm 6$.

Step 2. Using these possible roots, divide the polynomial through synthetic division and look for a zero remainder.

a) Let start with 1,

$$\begin{array}{r|rrrr} 1 & 1 & 2 & -5 & -6 \\ & & 1 & 3 & -2 \\ \hline & 1 & 3 & -2 & -8 \end{array} \longrightarrow$$

since the remainder is not zero, we will try another value.

b) Let's try -1

$$\begin{array}{r|rrrr} -1 & 1 & 2 & -5 & -6 \\ & & -1 & -1 & 6 \\ \hline & 1 & 1 & -6 & 0 \end{array} \longrightarrow$$

since the remainder is zero, then $(x + 1)$ is factor of the polynomial.

$x^2 + x - 6 \longrightarrow$ since the quotient is a quadratic, then we can easily find the other factors.

$(x + 3)\ (x - 2)$

c) Therefore, the polynomial equation in factored form is given by $(x + 1)\ (x + 3)\ (x - 2) = 0$.

Step 3. List down the roots of the polynomial equation.

$$(x + 1)\ (x + 3)\ (x - 2) = 0$$

a) To solve for the roots, apply the zero product property. Equate each factor to zero and solve for x.

$x + 1 = 0$ $x + 3 = 0$ $x - 2 = 0$

$x = -1$ $x = -3$ $x = 2$

b) Therefore, the roots are: { $\mathbf{-1, -3, 2}$ }.

Example 2. Find the roots of $2m^4 + 3m^3 - 7m^2 - 12m - 4 = 0$.

Step 1.
a) $a_0 = -4$ $p = \pm 1, \pm 2, \pm 4$
b) $a_n = 2$ $q = \pm 1, \pm 2$
c) possible roots $\frac{p}{q} = \pm 1, \pm 2, \pm 4, \frac{1}{2}$

Step 2.

$$\begin{array}{r|rrrrr} -1 & 2 & 3 & -7 & -12 & -4 \\ & & -2 & -1 & 8 & 4 \\ \hline 2 & 2 & 1 & -8 & -4 & 0 \\ & & 4 & 10 & 4 & \\ \hline & 2 & 5 & 2 & 0 & \end{array}$$

(m + 1) is a factor.

(m - 2) is another factor.

$2m^2 + 5m + 2$ quotient is a quadratic

$(2m + 1)(m + 2)$ factor quadratic polynomial

The polynomial equation in factored form is given by
$(m + 1)(m - 2)(2m + 1)(m + 2) = 0$.

Step 3. Therefore, the roots are: { **-1, 2,** $-\frac{1}{2}$ **, -2** }.

Example 3. Find the roots of $k^3 + 8k^2 + 18k + 12 = 0$.

Step 1.
a) $a_0 = 12$ $p = \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$
b) $a_n = 1$ $q = \pm 1$
c) possible roots $\frac{p}{q} = \pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

Step 2.

$$\begin{array}{r|rrrr} -2 & 1 & 8 & 18 & 12 \\ & & -2 & -12 & -12 \\ \hline & 1 & 6 & 6 & 0 \end{array}$$

(k + 2) is a factor.

$k^2 + 6k + 6$ quotient is quadratic and not factorable.

The polynomial equation in factored form is given by

$(k + 2)(k^2 + 6k + 6) = 0$.

Step 3. List the roots of the polynomial equation.

$(k + 2)\ (k^2 + 6k + 6) = 0.$

a) Since the other factor is quadratic and not factorable, we will use quadratic formula to solve for the other roots.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(6)}}{2(1)}$$

$$x = \frac{-6 \pm \sqrt{36 - 24}}{2}$$

$$x = \frac{-6 \pm \sqrt{12}}{2}$$

$$x = \frac{-6 \pm 2\sqrt{3}}{2}$$

$$x = -3 \pm \sqrt{3}$$

b) Therefore, the roots are: **{ -2, $-3 \pm \sqrt{3}$ }.**

Example 4. Find the roots of $b^3 - 4b^2 + 7b - 4 = 0$.

Step 1. a) $a_0 = -4$ $\quad p = \pm 1, \ \pm 2, \pm 4$

b) $a_n = 1$ $\quad q = \pm 1$

c) possible roots $\quad \frac{p}{q} = \pm 1, \pm 2, \pm 4$

Step 2.

$$\begin{array}{r|rrrr} 1 & 1 & -4 & 7 & -4 \\ & & 1 & -3 & 4 \\ \hline & 1 & -3 & 4 & 0 \end{array}$$

(b - 1) is a factor.

$\Downarrow$

$b^2 - 3b + 4$ quotient is a quadratic and not factorable.

The polynomial function in factored form is given by

$(b - 1)\ (b^2 - 3b + 4) = 0.$

Step 3. List the roots of the polynomial equation.

$(b - 1)\ (b^2 - 3b + 4) = 0.$

a) Since the other factor is quadratic and not factorable, we will use quadratic formula to solve for the other roots.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(4)}}{2(1)}$$

$$x = \frac{3 \pm \sqrt{9-16}}{2}$$

$$x = \frac{3 \pm \sqrt{-7}}{2}$$

b) Therefore, the roots are: $\left\{1, \frac{3 \pm \sqrt{-7}}{2}\right\}$

What's More?

Now, your turn.

Activity 4:
Find the roots of the following polynomial equation using the rational root theorem. Use the listed steps as your guide.

1) $b^3 + 2b^2 - 19b - 20 = 0$

a) List all the possible rational roots:

i) $a_0 =$ _______ $p =$ _____________

ii) $a_n =$ ______ $q =$ _____________

iii) possible roots $\frac{p}{q} =$ ______________

b) Use synthetic division and write the polynomial equation in factored form:

c) List the roots of the polynomial:

2) $b^3 + 5b^2 + 7b + 3 = 0$

a) List all the possible rational roots:

i) $a_0 =$ ______ $p =$ __________

ii) $a_n =$ ______ $q =$ __________

ii) possible roots $\frac{p}{q} =$ __________

b) Use synthetic division and write the polynomial equation in factored form:

c) List the roots of the polynomial:

Activity 5.

Find all the real roots of the following polynomial equation:

1) $(3x + 2)(x^2 - 25) = 0$

2) $2x^3 - 3x^2 - 8x + 12 = 0$

3) $3x^4 + 5x^3 - 28x^2 + 4x + 16 = 0$

4) $x^5 - 2x^4 + x^3 - 2x^2 - 2x + 4 = 0$

5) $x^4 - 4x^3 + 4x^2 - 36x - 45 = 0$

6) $x^4 + 10x + 9 = 0$

7) $x^3 + 9x^2 + 22x + 12 = 0$

8) $4x^3 - 18x^2 + 10x - 1 = 0$

What I Have Learned

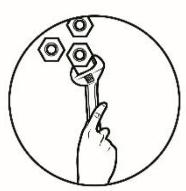
Summing up, let us list down what we have learned in our discussion.

Activity 6
Fill in the blanks with words and symbols that will best complete the statements given below.

1) Every polynomial equation *P(x)* of degree n has ______________ complex root.
2) An n^{th} degree polynomial equation can have _________________ real roots.
3) The theorem that gives a list of possible rational roots of a polynomial equation is called: _____________________________.

4 - 5)

Every rational root of a polynomial equation is in the form $\frac{p}{q}$, where p is an integer factors of __________________ and q is an integer factor of __________________.



What I can do

In this part of the module, we will apply the concepts of solving polynomial equation in solving word problems. Consider the problem below.

Problem: A rectangular box is to be made from a piece of cardboard 6 cm wide and 14 cm long by cutting out squares of the same size from the corners and turning up the sides. If the volume of the box is 40 cm^3, what should be the length of the side of the square to be cut out be?

a) Identify the given values:

Width of the cardboard: 6 cm   Length of the cardboard: 14 cm
Volume of the box: 40 cm^3

b) Represent the unknowns:

x = be the length of the side of the square to be cut out from the corners of the cardboard.
6 – 2x = width of the box
14 – 2x = length of the box

c) Formulate the equation:

Volume of a box = (Length)(width)(height)

$$40 = x(6 - 2x)(14 - 2x)$$

Solution:

$$x(6 - 2x)(14 - 2x) = 40$$

$$6x - 2x^2 (14 - 2x) = 40$$

$$\frac{4x^3 - 40x^2 + 84x - 40 = 0}{4}$$

$$x^3 - 10x^2 + 21x - 10 = 0$$

$$\begin{array}{r|rrrr} 2 & 1 & -10 & 21 & -10 \\ & & 2 & -16 & 10 \\ \hline & 1 & -8 & 5 & 0 \end{array} \longrightarrow \text{2 is one root.}$$

$$x^2 - 8x + 5 = 0$$

Use the quadratic formula to solve for the other roots.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \longrightarrow x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(5)}}{2(1)}$$

$$\therefore x = 4 \pm \sqrt{11}$$

Therefore the possible lengths of the square to be cut out be is equal to 2 cm, 7.32 cm and 4.63 cm.

Congratulations, I know that you are ready to apply what you had learned in this module.

Activity 7

Problem Solving: Solve the following problem. Identify what is being asked by choosing your answer in the answer box below.

A rectangular box has dimensions 12 in, 4 in, and 4 in. If the first two dimensions is decreased and the last dimension is increased by the same amount, a second box is formed, and its volume is five-eighths of the volume of the first box. Determine the dimensions of the second box.

1. Dimensions of the first box.	
2. Volume of the first box.	
3. Volume of the second box. (Hint: five-eighths of the volume of the first box)	
4. First dimension of the second box.	

(Note: use variables to indicate decrease)	
5. Second dimension of the second box. (Note: use variables to indicate decrease)	
6. Third dimension of the second box. (Note: use variables to indicate increase)	
7. Working equation for the dimension of the second box.	
8. Dimensions of the second box.	

Answer Box

(12 – x) in	(x + 4) in	10 in x 2 in x 4 in	14 in^3
(x – 4) in	(4 – x) in	(12-x) (4–x) (4+x)= 192	120 in^3
(x-12) (x–4) (x+4)= 20	20 in^3	192 in^3	(x – 12) in
12 in x 4 in x 4 in	16 in^3	(12-x) (4–x) (4+x)= 120	(4 + x) in

Assessment

DIRECTION: Let us determine how much you have learned from this module. Read and analyze each item carefully. Write the letter of the correct answer on the blank provided for.

_____1. Which of the following is NOT a polynomial equation?
A. $x^3 - 2x^2 + 3x - 2 = 0$ C. $x^{-2} + x = 0$
B. $-3x + 5x^{14} - 3 = 0$ D. $5x = 0$

_____2. Which of the following is the degree of the polynomial equation x=9?
A. 0 B. 1 C. 3 D. 9

_____3. What value of m will make 2m + 1 equal to zero?
A. 2 B. $\frac{1}{2}$ C. 0 D. $\frac{-1}{2}$

_____4. Find a quadratic polynomial with roots -1 and 2.
A. $a^2 - a + 2 = 0$ C. $a^2 + a - 2 = 0$
B. $a^2 - a - 2 = 0$ D. $a^2 + a + 2 = 0$

_____5. Find a cubic polynomial equation with roots -2, 2 and -4.

A. $m^3 + 4m^2 - 4m - 16 = 0$ C. $m^3 - 4m^2 - m + 16 = 0$

B. $10m^3 - m^2 - m + 16 = 0$ D. $m^3 - 4m^2 - 4m + 16 = 0$

_____6. According to Rational Root Theorem, which of the following is a possible zero of the polynomial $p(b) = 6b^3 - 3b^2 + 2b - 4$?

A. $\frac{3}{4}$ B. $\frac{5}{3}$ C. $\frac{3}{2}$ D. $\frac{1}{2}$

_____7. Using Rational Root Theorem, list all the possible rational roots of t the polynomial equation $4x^4 + 31x^3 - 4x^2 - 89x + 22 = 0$.

A. $\pm22, \pm11, \pm2, \pm1$ C. $\pm1, \pm\frac{1}{2}, \pm\frac{1}{11}, \pm2, \pm4, \pm\frac{2}{11}, \pm\frac{4}{11}, \pm\frac{1}{22}$

B. $\pm4, \pm2, \pm1$ D. $\pm1, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm2, \pm11, \pm\frac{11}{2}, \pm\frac{11}{4}, \pm22$

_____8. If 7 is a rational zero of the polynomial $p(x) = 2x^3 - 14x^2 + x - 7$, which of the following is NOT true?

A. $(x + 7)$ is a factor of $p(x)$ C. 7 is a possible zero of $p(x)$

B. $(x - 7)$ is a factor of $p(x)$ D. -7 is not a possible zero of $p(x)$

_____9. How many possible real roots does the polynomial equation $4x^3 + 4x^2 - 31x + 5 = 0$ have?

A. at most 4 B. at most 3 C. at least 4 D. at least 3

_____10. If 4 is a root of $x^2 - 3kx + 3k - 7 = 0$, then the value of k is ___.

A. $\frac{7}{4}$ B. 1 C. 4 D. 7

_____11. Which of the following polynomial equations has 3 as a root?

A. $a^3 + 3a^2 - a + 3 = 0$ C. $a^3 - 3a^2 + a + 3 = 0$

B. $a^3 + 3a^2 - a - 3 = 0$ D. $a^3 - 3a^2 - a + 3 = 0$

_____12. Identify all real roots of $4x^4 + 31x^3 - 4x^2 - 89x + 22 = 0$.

A. $-2, \frac{1}{4}$ C. $\pm1, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm2, \pm11, \pm\frac{11}{2}, \pm\frac{11}{4}$

B. $\pm2, \pm\frac{1}{4}, \pm\sqrt{5}, -3 - 2\sqrt{5}$ D. $-2, \frac{1}{4}, -3 + 2\sqrt{5}, -3 - 2\sqrt{5}$

For items 13 -14. The volume of a rectangular solid is 750 cubic units. The width (w) is 7 units more than the height (h) and the length (l) is 1 unit more than 8 times the height.

_____ 13. Which correctly represents the width of the solid?

A. (7w) (l) (3+2h) B. 8h + 1 C. h + 7 D. 8h

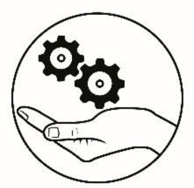
_____ 14. What is the working equation to find the dimensions of the solid?

A. v = (7w)(l)(8h) C. 750 = (h + 7)(8h + 1)

B. v = (8h + 1)(h) D. 750 = h(h + 7)(8h + 1)

_____15. In Philippine National High School, the length and the width of the building is (x + 9) m and (x – 4) m respectively. If its area is 500m^2, which of the following mathematical statements can show the relationship between the dimension of the school building?

A. (x + 9) (x - 4) = 500 C. $x^2 + 5x – 36 = 500$
B. $x^2 + 5x – 36 = 1000$ D. (x + 9) (x - 4) = 100



Additional Activity

Activity 8.

Answer the following problems:

1) Find k so that the binomial (x – 1) is a factor of $-3x^4 + kx^3 + 6x^2 - 9x + 3$.

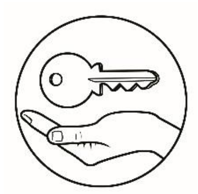
2) Find the value of b so that -2 is a root of $3x^3 + bx^2 + 5x – 27 = 0$.

3) Find a cubic polynomial equation with roots 2, −2, & 4.

4) Which of the following has no rational roots:

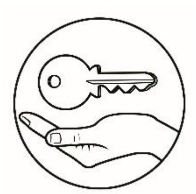
$x^4 + 5x^2 + 4 = 0$ or $x^3 + 4x^2 – 5 = 0$?

5) Find three rational numbers such that their product is 210. The second number is 5 less than the first. The third number is 1 more than twice the first.



Answer Key (Lesson 1)

What I know?	What's In?			What's More?	What I have Learned
	Activity 1	**Activity 2**	**Activity 3**	**Activity 4**	**Activity 6**
1) A 2) D 3) B 4) C 5) D 6) A 7) C 8) C 9) B 10) A 11) D 12) D 13) B 14) B 15) A	1. √ 2. X 3. X 4. X 5. X	1. 4 2. 1 3. 3 4. 5 5. 0	Answers may vary Note: An equation expresses equality of two terms, thus it has an equal sign. An expression on the other hand, does not express an equality of two terms, thus it does not have an equal sign	Polynomial Equations (1, 3, 7, 8) NOT Polynomials (2, 4, 5, 6) **Activity 5** 1. 3 2. 1 3. 4 4. 1 5. 6	A polynomial equation of degree n in x is a mathematical statement consisting of terms: • with non-negative powers of x, • are not inside a radical sign and • variables are not found in the denominator if expressed as rational numbers.



ANSWER KEY (Lesson 2)

<table>
<tr>
<td>Activity 1
A.
1) -1
2) $\frac{1}{2}$
3) p(-1)
4) factor

B)
$2a^2 + 3a + 5 - \frac{2}{a-3}$</td>
<td rowspan="2">Activity 3

Prove that q is a factor of a_n.

1) Since $\frac{p}{q}$ is a root of the polynomial equation, then:

$$a_n\left(\frac{p}{q}\right)^n + a_{n-1}\left(\frac{p}{q}\right)^{n-1} + \ldots + a_2\left(\frac{p}{q}\right)^2 + a_1\left(\frac{p}{q}\right) + a_0 = 0$$
2) Multiply both sides of the equation by q^n.

$$q^n\left[a_n\left(\frac{p}{q}\right)^n + a_{n-1}\left(\frac{p}{q}\right)^{n-1} + \ldots + a_2\left(\frac{p}{q}\right)^2 + a_1\left(\frac{p}{q}\right) + a_0\right] = (0)q^n$$

$$a_n p^n + a_{n-1}p^{n-1}q + \ldots + a_2 p^2 q^{n-2} + a_1 p q^{n-1} + a_0 q^n = 0$$
3) Add $-a_n p^n$ to both sides of the equation.</td>
</tr>
<tr>
<td>Activity 2
1. e
2. c
3. d
4. a
5. b</td>
</tr>
</table>

$$a_n p^n + a_{n-1} p^{n-1} q + \ldots + a_2 p^2 q^{n-2} + a_1 p q^{n-1} + a_0 q^n + \left(-a_0 p^n\right) = 0 + \left(-a_n p^n\right)$$

$$a_{n-1} p^{n-1} q + \ldots + a_2 p^2 q^{n-2} + a_1 p q^{n-1} + a_0 q^n = -a_n p^n$$

4) Factor out q on the left side of the equation.

$$q\left(a_{n-1} p^{n-1} + \ldots + a_2 p^2 q^{n-3} + a_1 q^{n-2}\right) = -a_n p^n$$

5) Since q is a factor of the left side of the equation, then then q must also be a factor of the right side.

6) Since p and q have no common factor except 1, then q must be a factor of a_n (*and not* p^n). This proves the second part of the rational root theorem.

Activity 4

1) $b^3 + 2b^2 - 19b - 20 = 0$

a) List possible roots:

$\pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$

b) Synthetic Division

$$\begin{array}{r|rrrr} -1 & 1 & 2 & -19 & -20 \\ & & -1 & -1 & 20 \\ \hline & 1 & 1 & -20 & 0 \end{array}$$

$b^2 + b - 20$

$(b + 5)(b - 4)$

Factored form:

$(b + 1)(b + 5)(b - 4) = 0$

c) List down roots:.

{-1, -5, 4}

2) $b^3 + 5b^2 + 7b + 3 = 0$

a) List possible roots:

$\pm 1, \pm 3$

b) Synthetic Division

$$\begin{array}{r|rrrr} -1 & 1 & 5 & 7 & 3 \\ & & -1 & -4 & -3 \\ \hline & 1 & 4 & 3 & 0 \end{array}$$

$b^2 + 4b + 3$

$(b + 3)(b + 1)$

Factored form:

$(b + 1)^2 (b + 3) = 0$

c) List down roots.

{-1, -3}

Activity 5

1) $-\frac{2}{3}, -5, 5$
2) $-2, \frac{3}{2}, 2$
3) $-4, -\frac{2}{3}, 1, 2$
4) $-1, 1, 2$
5) $-1, 5$
6) no real solution
7) $-3 \pm \sqrt{5}, -3$
8) $\frac{1}{2}, \frac{4 \pm \sqrt{14}}{2}$

Activity 6

1) at least one
2) at most n
3) Rational Root Theorem
4) the constant term
5) the leading term

Activity 7

1. 12 in x 4 in x 4 in
2. 192 in^3
3. 120 in^3
4. (12 – x) in
5. (4 – x) in
6. (4 + x) in
7. (12 – x)(4 – x)(4 + x) =120
8. 10 in x 2 in x 6 in

Assessment:

1. C
2. B
3. D
4. B
5. A
6. D
7. D
8. A
9. B
10. B
11. D
12. D
13. C
14. D
15. A

Additional Activity:

1) 3
2) $\frac{61}{4}$
3) $x^3 - 4x^2 - 4x + 16 = 0$
4) $x^4 + 5x^2 + 4 = 0$
5) 2, 7, 15

References

Callanta, Melvin M. et al. (2015). Mathematics Learners Module

Capul, Erist A. et.al. (2015). Next Generation Math. Diwa Learning Systems, Inc.

Chavez, Chariston D. et al. (2016). Integrated Math for Grade 10. New Horizon Publications

Leithold, (1992). College Algebra and Trigonometry. Addison-Wesley Publishing Company

Find zeros of a polynomial function College Algebra, Retrieved from https://courses.lumenlearning.com/ivytech-collegealgebra/chapter/find-zeros-of-a-polynomial-function/

For inquiries or feedback, please write or call:

Department of Education - Bureau of Learning Resources (DepEd-BLR)

Ground Floor, Bonifacio Bldg., DepEd Complex
Meralco Avenue, Pasig City, Philippines 1600

Telefax: (632) 8634-1072; 8634-1054; 8631-4985

Email Address: blr.lrqad@deped.gov.ph * blr.lrpd@deped.gov.ph