Senior High School

General Mathematics

Quarter 1 – Module 9:

Intercepts, Zeroes and Asymptotes of Rational Functions

ALTERNATIVE DELIVERY MODE

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Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

What I Need to Know

This module was designed and written to help you determines the intercepts, zeroes and asymptotes of rational functions. Knowing what a rational function is, you are now ready to learn its other properties. It includes finding the intercepts, zeroes and asymptotes. These will be your guide to easily determine the behavior of a rational function and will prepare you for graphing rational function. The scope of this module permits it to be used in many different learning situations. The language used recognizes the diverse vocabulary level of students. The lesson is arranged to follow the standard sequence of the course.

In this module you will determine the intercepts, zeroes and asymptotes of rational functions.

The module consists of one lesson namely: Intercepts, Zeroes, and Asymptotes of Rational Functions.

After going through this module, you are expected to:

1. recall the meaning of intercepts, zeroes and asymptotes;
2. identify the intercepts, zeroes and asymptotes of rational functions;
3. solve for the intercepts, zeroes and asymptotes of rational functions.

What I Know

In this part, let us see how much you know about the lesson by answering the questions in pre-assessment below. If you obtain 100% or a perfect score, skip the module and immediately move to the next module. While in the event you missed a point, please proceed on the module as it will enrich your knowledge in finding the intercepts, zeroes and asymptotes of rational functions. Let’s get started!

I. Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. Which of the following is the set of all values that the variable x can take?
    a. Range
    b. Intercept
    c. Domain
    d. Zeroes

2. What is the domain of f(x) =$\frac{x-3}{x+3}$?
   a. All real numbers
   b. All real numbers except – 3
   c. All real numbers except 3
   d. Cannot be determined

3. What value/s of x that will make the function meaningless f(x) = $\frac{x-1}{x}$?
   a. x = -1
   b. x = 0
   c. x = 1
   d. All real numbers

4. Complete the sentence: The real numbers zeroes are also ___________ of the graph of the function.
   a. Asymptote
   b. x – intercepts
   c. y – intercepts
   d. Range

5. Which of the following is the set of all values that f(x) can take?
   a. Range
   b. Intercept
   c. Domain
   d. Zeroes

6. What is the range of f(x) =$\frac{1}{x}$?
   a. R =$\{y|y = 1\}$
   b. R =$\{y|y = 0\}$
   c. R = $\{y|y \neq 1\}$
   d. R = $\{y|y \neq 0\}$

7. Which of the following is a true statement?
   a. A rational function is a quotient of functions.
   b. Asymptotes are a common characteristic of rational functions.
   c. An asymptote is a line that a graph approaches, but does not touch.
   d. All of the above.

8. If the degree of the leading coefficient of the numerator is equal to the degree of the leading coefficient of the denominator of a rational function, which of the following statements has to be true?
   a. The graph has no asymptote
   b. The graph of the function has slant asymptote
   c. The graph of the function has a horizontal asymptote
   d. None of the above

9. What is the horizontal asymptote of $f(x) = \frac{x+5}{3x^2}$?
   a. y = 3
   b. y = 0
   c. y = - 2
   d. y = - 3

10. What is the vertical asymptote of $f(x) = \frac{3x+1}{x-5}$?
   a. x = 5
   b. x = 3
   c. x = 1
   d. x = 0

11. What is the oblique asymptote of $f(x) = \frac{x^2-3x}{x+3}$?
   a. y = 3x
   b. y = x – 6
   c. y = x -3
   d. y = 3x + 6

12. Oblique asymptote occurs when there is no horizontal asymptote, the statement is ___________.

   a. Always true
   b. Sometimes true
   c. Never true
   d. Cannot be determined

13. How will you describe the horizontal asymptote of $f(x) = \frac{3}{3+x}$?
   a. does not exist
   b. approaching at x = 3
   c. approaching at y = -3
   d. approaching at y = 0

14. If the x – intercept of a rational function is at x = 5, what is the zero of the function?

   a. x = 5
   b. x = 0
   c. x = - 5
   d. cannot be determined

15. What is the y – intercept of $f(x) = \frac{2x^2+x+3}{2x^2+3x+1}$?
   a. 3
   b. 0
   c. – 3
   d. – 6

Lesson 1 Intercepts, Zeroes, and Asymptotes of Rational Functions

In the previous lesson, you learned how to find domain and range of a rational function. In this particular lesson, determining intercepts, zeroes and asymptotes of rational functions will be done. Knowing fully the concept of the different properties of rational function will be your guide to easily determine the behavior of a rational function and it will prepare you for the next topic which is about graphing rational function.

What's In

Let's recall first what you have learned from the previous lesson by answering the following questions:

A. Which of the following is an example of rational function?

1. F(x) = $\frac{3x^2+1}{x-1}$ 2. $\frac{x}{3} = \frac{8}{3}$ 3. $\frac{1}{3x-1} + 3 < 0$

B. Find the domain and range of the functions.

1. F(x) = $\frac{x}{x+3}$ 2. f(x) = $\frac{3}{x-4}$ 3. g(x) = $\frac{x+1}{x^2-1}$

Let us see if you got the correct answer in the activity, if your answer in question A is number 1, you got it right you have a clear understanding of the concept of rational function but if you are incorrect allow me to help you recall what a rational function is, *when two polynomial functions are expressed as a quotient and can be written in the form* $f(x) = \frac{p(x)}{q(x)}$ *and q(x) is a not the zero function it is called a rational function.* Numbers 2 and 3 are not examples of rational function, it is a rational equation and rational inequality, respectively. Number 1 is written as the quotient of two polynomial functions, so it is a rational function.

For activity B, let us review the meaning of domain and range of the function. **Domain** is the set of first coordinates of a relation and it is the value of x that will not make the denominator of the function equal to zero while **Range** is the set of second coordinates. To determine the domain of rational function, simply equate the denominator to zero and then solve for x, this value should be avoided so that the function will not give an undefined or a meaningless function. Example find the domain of F(x) = $\frac{x}{x+3}$, equating the denominator to zero, we have x + 3 = 0, so the value of x = -3, so the domain of the function are all real numbers except -3 remember we will avoid value/s that will make our denominator equal to zero, so if we will substitute -3 to our x in the denominator it will result to 0 and it will give us an undefined function. **In notation, D=** $(-\infty, -\mathbf{3}) \cup (-\mathbf{3}, \infty)$

To find the range of the function, change f(x) to y then, solve for x; remember range are real values of y that will make a real value for the function. For example, find the range of F(x) = $\frac{x}{x+3}$;

Changing F(x) to y, the new function is ⟹ $y = \frac{x}{x+3}$

By doing cross multiplication we have ⟹ y(x+3) = x

Distributing y we now have ⟹ xy + 3y = x

Simplifying the equation will give ⟹ xy – x = 3y

Factoring the left side of the equation ⟹ x(y – 1) = 3y

Dividing the equation by (y – 1) ⟹ $\frac{x\cancel{(y-1)}}{\cancel{(y-1)}} = \frac{3y}{(y-1)}$

Removing common factor, the value of x ⟹ $x = \frac{3y}{y-1}$

Since we are looking for the value of y that will give a real value for the function so we need to find value/s for y that will not make the denominator equal to 0.

Equating the denominator to zero ⟹ y – 1 = 0

So, y = 1.

The range of the function F(x) = $\frac{x}{x+3}$ is all real values of y except 1. In notation,

R= $(-\infty, \mathbf{1}) \cup (\mathbf{1}, \infty)$**.**

The following are the answers to Activity B

1. Domain = $\{x/x\neq -3\}$ or $(-\infty, \mathbf{-3}) \cup (\mathbf{-3}, \infty)$
   Range = $\{y/y \neq 1\}$ or $(-\infty, \mathbf{1}) \cup (\mathbf{1}, \infty)$

2. Domain = $\{x/x\neq 4\}$ or $(-\infty, \mathbf{4}) \cup (\mathbf{4}, \infty)$
   Range = $\{y/y \neq 0\}$ or $(-\infty, \mathbf{0}) \cup (\mathbf{0}, \infty)$

3. Domain = $\{x/x\neq -1 \text{ or } x \neq 1\}$ or $(-\infty, \mathbf{-1}) \cup (\mathbf{-1}, \mathbf{1}) \cup (\mathbf{1}, \infty)$
   Range = $\{y/y \neq 0\}$ or $(-\infty, \mathbf{0}) \cup (\mathbf{0}, \infty)$

How is your review of the rational function? I believed you got it all correct. Are you ready to learn new things about rational functions? Let's do the next activity.

Notes to the Teacher

The teacher may say that "the domain refers to the set of possible input values and range is the set of possible output values" is related to the saying "you saw what you reap". Like in our day-to-day activities if we show good deeds to others, in return we will receive the same treatment.

What's New

Activity

I – Connect Mo!

Connect the given statement/phrase in column A with the answer in column B to complete the statement/phrase in column A. Write the letter of your answer in a separate sheet of paper.

COLUMN A	COLUMN B
1. The intercepts of the graph of a rational function …	M. the x - intercepts
2. To find the x – intercept of a function …	A. let x = 0
3. The zeroes of the function is also …	G. rational function
4. To find the y – intercept of a function …	I. are the points of intersection of its graph and an axis
5. The function of the form $f(x) = \frac{g(x)}{h(x)}$, where g(x) & h(x) are polynomials	C. let y = 0

How was the activity? I believed that you connected it right. In the activity you were asked to find the intercepts and zeroes of rational function. Just like in our lives we are looking for something to achieve our goal in life such as harmonious family relationship, good grades, finish senior high school and college to find a good job all these things are related to each other that will make us happy and contented. So, in this lesson, you will know how to identify intercepts, zeroes and asymptotes of rational function.

What is It

INTERCEPTS AND ZEROES OF RATIONAL FUNCTIONS

The intercepts of the graph of a rational function are the points of intersection of its graph and an axis.

The *y-intercept* of the graph of a rational function r(x) if it exists, occurs at r(0), provided that r(x) is defined at x = 0. To find y-intercept simply evaluate the function at x = 0.

The *x-intercept* of the graph of a rational function r(x), if it exists, occurs at the zeros of the numerator that are not zeros of the denominators. To find x – intercept equate the function to 0.

The *zeroes of a function* are the values of x which make the function zero. The numbered zeroes are also x-intercepts of the graph of the function.

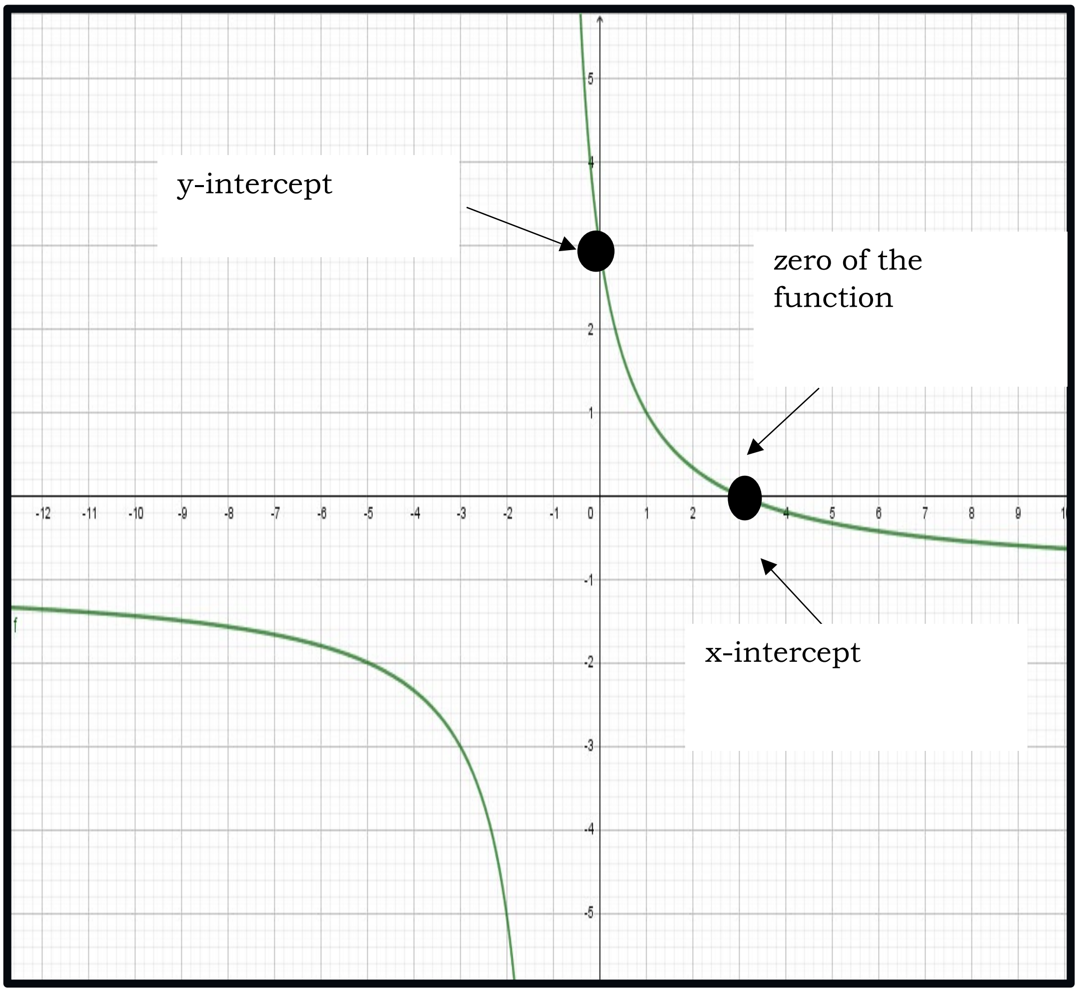


Figure 1. x and y intercepts using GeoGebra

EXAMPLES.

1. Find the x- and y – intercepts, of the following rational functions:

    1. f(x) = $\frac{3-x}{x+1}$

    2. b. f(x) = $\frac{3x}{x+3}$

    3. f(x) = $\frac{x^2-3x+2}{x^2-4}$

2. Determine the zeroes of the following rational functions:

   a. g(x) = $\frac{x-2}{x+6}$

   b. h(x) = $\frac{x-3}{x^2-9}$

   c. G(x) = $\frac{x^2+x-2}{x^2-4}$

SOLUTIONS.

1. To find x – intercept equate the function to 0.

$$f(x) = \frac{3-x}{x+1}$$

$0 = \frac{3-x}{x+1}$	Equate the function to 0.
$\frac{3-x}{x+1} = 0$	By Symmetric Property of Equality.
$3 - x = 0$	Multiply both sides by (x + 1).
$3 + (-3) - x = 0 + (-3)$	By Addition Property of Equality (APE).
$-x = -3$	Simplify.
$(-1)(-x) = (-1)(-3)$	By Multiplication Property of Equality (MPE).
$x = 3$	

So, the x – intercept is (3, 0).

By analyzing the example, we can say that to find the x – intercept simply equate the numerator of the function to 0.

To find the y – intercept, change the x value of the function to 0.

$f(x) = \frac{3-x}{x+1}$	Substitute 0 to x values of the function.
$f(x) = \frac{3-0}{0+1}$	Simplifying the fraction.
$f(x) = \frac{3}{1} = 3$	Value of f(x) or y.

So, the y – intercept is 3 or (0, 3).

2. f(x) = $\frac{3x}{x+3}$

To find the x – intercept, simply equate the numerator to 0,

0 = 3x	Equate the numerator to 0.
3x = 0	By Symmetric Property of Equality.
$\frac{\cancel{3}x}{\cancel{3}} = \frac{0}{3}$	Simplifying the fraction by multiplying both sides by 1/3.
x = 0	

So, the x – intercept is 0 or (0, 0).

To find the y – intercept, change the x value of the function to 0.

$f(x) = \frac{3x}{x+3}$	Substitute 0 to x values of the function.
$f(x) = \frac{3(0)}{0+3}$	Simplifying the fraction.
$f(x) = \frac{0}{3} = 0$	The value of $f(x)$ or y – intercept.

So, the y – intercept is 0 or (0, 0).

3. $f(x) = \frac{x^2-3x+2}{x^2-4}$

$x^2 - 3x + 2 = 0$		Equate the numerator to 0.
(x – 2) (x – 1) = 0		By factoring.
x – 2 = 0 x = 2	x – 1 = 0 x = 1	Solve for x, by Zero product property.

So, the x – intercepts are x = 2 and x = 1. But by looking at the denominator of the original function if we substitute 2 to the value of x,

$$x^2 - 4 = (2)^2 - 4 = 0,$$

The denominator will become 0, the function becomes meaningless.
So, we will only accept x – intercept at **x = 1 or (1, 0).**

To find the y – intercept:

$f(x) = \frac{x^2-3x+2}{x^2-4}$, change the x value of the function to 0.

$f(x) = \frac{(0)^2-3(0)+2}{(0)^2-4}$ Simplify the fraction.

f(x) = $\frac{2}{-4}$ Reduce the fraction to lowest term.

$f(x) = -\frac{1}{2}$ The value of f(x) or y.

So, the y – intercept is $-\frac{1}{2}$ or (0, $-\frac{1}{2}$).

2. Determine the zeroes of the following rational functions:

a. g(x) = $\frac{x-2}{x+6}$ b. H(x) = $\frac{x-3}{x^2-9}$ c. G(x) = $\frac{x^2+x-2}{x^2-4}$

To find the zeroes of a rational function, equate the function to 0 or solve for the x – intercept of the function by equating the numerator to 0.

a. g(x) = $\frac{x-2}{x+6}$,

x – 2 = 0 Equate the numerator to 0.

x = 2 Solve for x.

Thus, the zero of g(x) is 2.

b. $H(x) = \frac{x-3}{x^2-9}$

$H(x) = \frac{x-3}{x^2-9}$ Simplify by factoring the denominator.

$\frac{\overset{1}{\cancel{x-3}}}{\cancel{(x-3)}(x+3)}$ Remove common factors.

$\frac{1}{x+3} = 0$ Equate the numerator to 0.

1 = 0 False statement.

So, there is **<u>no zero</u>** of the function. Which means that no point on the graph touches the x – axis.

c. G(x) = $\frac{x^2+x-2}{x^2-4}$

$G(x) = \frac{\cancel{(x+2)}(x-1)}{\cancel{(x+2)}(x-2)}$ Simplify by factoring both the numerator and denominator.

$G(x) = \frac{x-1}{x-2}$ Remove common factors.

x -1 = 0 Equate the numerator to 0.

x = 1 Solve for x.

Thus, the zero of G(x) = 1.

ASYMPTOTES

An asymptote is an imaginary line to which a graph gets closer and closer as the x or y increases or decreases its value without limit.

Kinds of Asymptote

- Vertical Asymptote
- Horizontal Asymptote
- Oblique / Slant Asymptote

VERTICAL ASYMPTOTE

The vertical line $x = a$ is a vertical asymptote of a function f if the graph increases or decreases without bound as the x values approach ***a*** from the right or left. See illustration below.

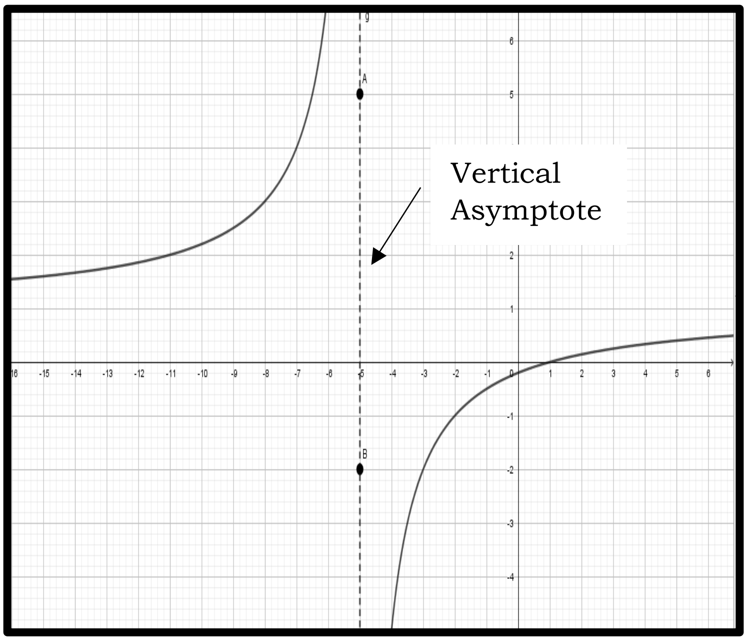


Figure 2. Illustration of Vertical Asymptote using geogebra

By looking at the illustration we can see that the graph of the function is approaching at x = - 5 represented by the dotted line and as we can see the graph is getting closer and closer to x = - 5 but it will not pass or intersect the line. So, the vertical asymptote of the graph is at x =- 5. We can find vertical asymptote even without looking at the graph of the function.

Finding Vertical Asymptote

To determine the vertical asymptote of a rational function, first reduce the given function to simplest form then find the zeroes of the denominator that are not zeros of the numerator.

Examples

Determine the vertical asymptote of each rational function.

a. F(x) = $\frac{(x-1)}{(x+5)}$
b. f(x) = $\frac{x+2}{(x+1)(x-4)}$
c. c. g(x) = $\frac{2x^2-x+1}{x^2-6x+9}$

Solutions

a. The zero of the numerator is 1 and the zero of the denominator is -5. The vertical asymptote for F(x) = $\frac{(x-1)}{(x+5)}$ is x = - 5. The value is zero of the denominator but not of the numerator.

b. The zero of the numerator is -2 and the zeroes of the denominator are -1 and 4. The vertical asymptote for f(x) = $\frac{x+2}{(x+1)(x-4)}$ are x = - 1 and x = 4. These values are zeroes of the denominator but not of the denominator.

c. Since the function is in quadratic form, reduce it to simplest form. The simplest form of g(x) = $\frac{2x^2-x-1}{x^2-5x+6}$ is g(x) = $\frac{(2x+1)(x-1)}{(x-3)(x-2)}$. The zeroes of the numerator are -1/2 and 1. The zeroes of the denominator are 3 and 2. The vertical asymptote for g(x) =$\frac{(2x+1)(x-1)}{(x-3)(x-2)}$ are x = 2 and x = 3. These values are zeroes of the denominator but not of the denominator.

HORIZONTAL ASYMPTOTE

The horizontal line $y=b$ is a horizontal asymptote of the function *f* if *f(x)* gets closer to b as x increases or decreases without bound.

Looking at the graph, we can see that the graph of the function is approaching a line in the y – axis, that line is called the horizontal asymptote. In the graph we can see that it is getting closer and closer at y = 1 but it only approaches but never touches or intersects y = 1. So, the horizontal asymptote of the function is at y = 1. We can determine horizontal asymptote arithmetically by comparing the degree of the leading coefficient of the numerator and denominator of the function.

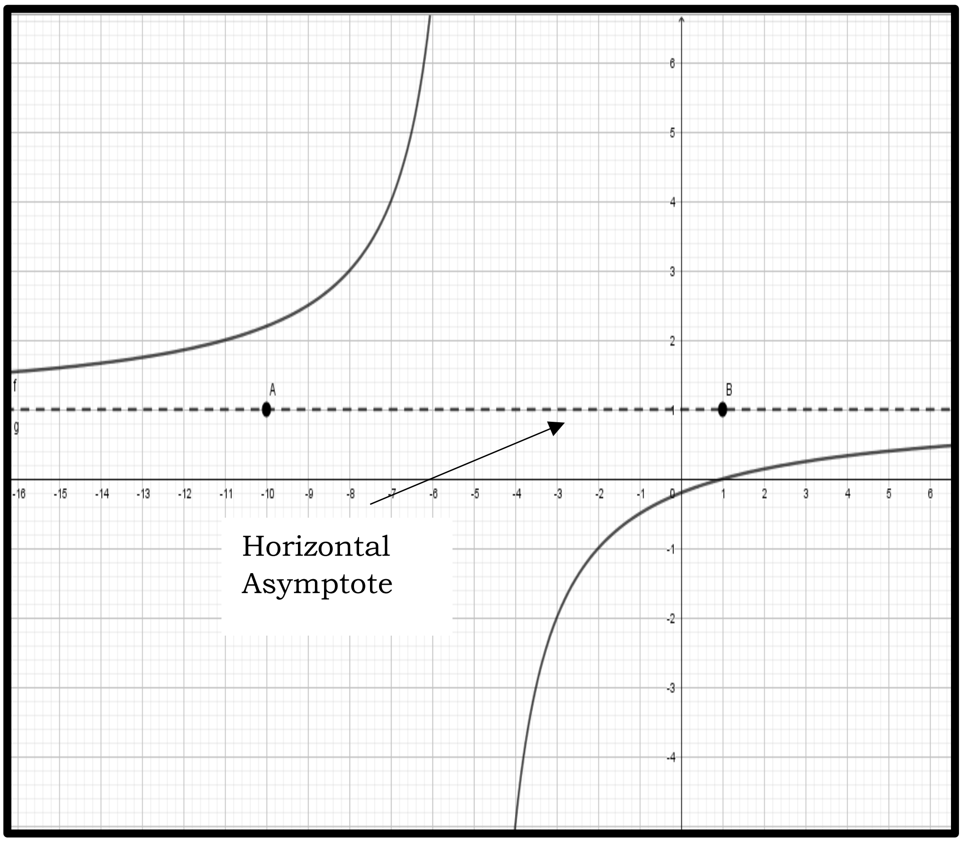


Figure 3. Illustration of Vertical Asymptote using GeoGebra

Finding the horizontal asymptote of a rational function

To determine the horizontal asymptote of a rational function, compare the degree of the numerator n and the degree of the denominator *d.*

- If n < d, the horizontal asymptote is y= 0
- If n = d, the horizontal asymptote y is the ratio of the leading coefficient of the numerator a, to the leading coefficient of the denominator b. That is $y = \frac{a}{b}$.
- If n > d, there is no horizontal asymptote.

Note: A rational function may or may not cross its horizontal asymptote. If the function does not cross the horizontal asymptote y=b, then b is not part of the range of the rational function.

EXAMPLES

Determine the horizontal asymptote of each rational function.

a. F(x) = $\frac{3x+8}{x^2+1}$
b. b. f(x) = $\frac{3 + 8x^2}{x^2+1}$
c. c. g(x) = $\frac{8x^3-1}{1-x^2}$

SOLUTIONS

a. The degree of the numerator 3x + 8 is less than the degree of the denominator $x^2 + 1$. Therefore, the horizontal asymptote is y = 0.

b. The degree of the numerator $3 + 8x^2$ and that of the denominator $x^2 + 1$ are equal. Therefore, the horizontal asymptote y is equal to the ratio of the leading coefficient of the numerator 8 to the leading coefficient of the denominator 1. That is $y = \frac{8}{1}$ = 8.

c. The degree of the numerator $8x^3 - 1$ is greater than the degree of the denominator $1 - x^2$. Therefore, there is no horizontal asymptote.

Aside from vertical and horizontal asymptote, a rational function can have another asymptote called oblique or slant. It occurs when there is no horizontal asymptote or when the degree of the numerator is greater than the degree of the denominator.

SLANT / OBLIQUE ASYMPTOTE

An oblique asymptote is a line that is neither vertical nor horizontal. It occurs when the numerator of $f(x)$ has a degree that is one higher than the degree of the denominator.

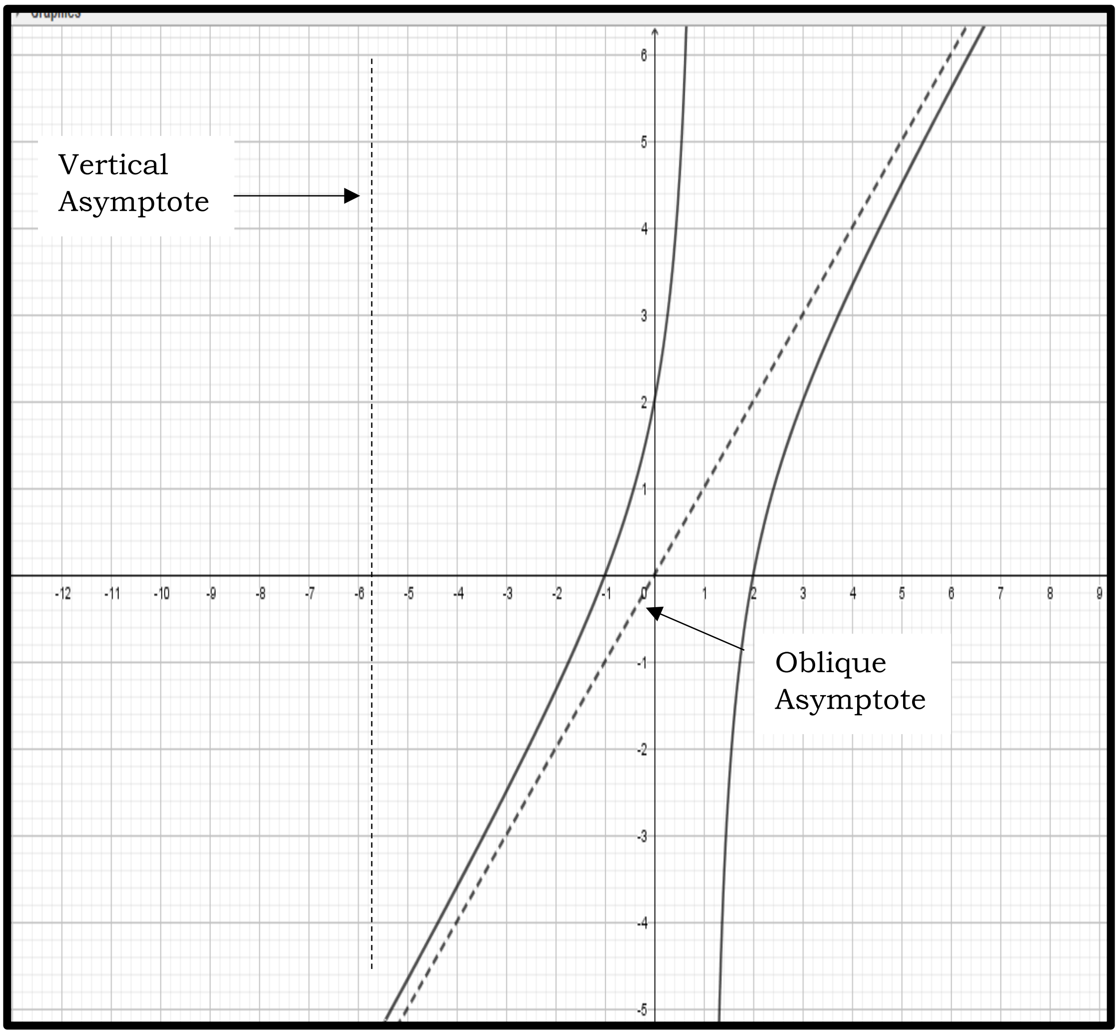


Figure 4. Illustration of Oblique Asymptote using geogebra

Looking at the graph we can see that there is vertical asymptote and there is no horizontal asymptote. In this case, oblique or slant asymptote occurs. We can determine the oblique / slant asymptote using your knowledge of division of polynomials.

Finding Oblique or Slant Asymptote

To find slant asymptote simply divide the numerator by the denominator by either using long division or synthetic division. The oblique asymptote is the quotient with the remainder ignored and set equal to y.

EXAMPLES

Consider the function $h(x) = \frac{x^2+3}{x-1}$. Determine the asymptotes.

By looking at the function, h(x) is undefined at x = 1, so the vertical asymptote of h(x) is the line at x = 1.

There is no horizontal asymptote because the degree of the numerator is greater than the degree of the denominator.

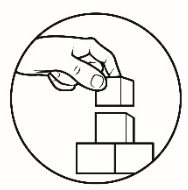
If the numerator and denominator of h(x) are divided, we get

$$h(x) = \frac{x^2+3}{x-1}$$

$$
\begin{array}{rl}
 & \qquad x + 1 \text{ r. } 4 \\ \cline{2-2}
= \quad x - 1 & \big)\, \cancel{x^2} + 0x + 3 \\
 & -\cancel{x^2}\ \cancel{(-)}\ x \\
 & \quad + \\ \hline
 & \qquad \cancel{x} + 3 \\
 & \quad -\ \ \cancel{x}\ \cancel{(-)}\ 1 \\
 & \qquad\qquad + \\ \hline
 & \qquad\qquad 4
\end{array}
$$

So, the quotient is x + 1 + $\frac{4}{x-1}$.

Thus, the line y = x + 1 is the oblique asymptote of $h(x) = \frac{x^2+3}{x-1}$.



What's More

Now it's your turn.

Independent Practice 1

Given the rational function f(x) = $\frac{2x+6}{X-3}$, answer the following questions:

1. What are the two functions used to form the rational function?
2. What is the x-intercept of the function? Which function did you use to determine the x-intercept? Why?
3. What is the y – intercept of the function? How did you get the y – intercept?
4. What is the zero of the function?

Try This!

Independent Assessment 1

Complete the table below by giving the intercepts and zeroes of rational function.

Rational Function	x - intercept	y - intercept	Zeroes of the function
1. $f(x) = \frac{x-9}{x+3}$			
2. $f(x) = \frac{x^2-10x+25}{x+5}$			
3. $f(x) = \frac{x^2+9}{x^2-3}$			

Independent Practice 2

True or False. Tell whether each of the following is **true** or **false.** If the statement is wrong change the underlined word to make it correct. Write your answer on the space provided before each number.

__________ 1. An intercept is a line (or a curve) that the graph of a function gets close to but does not touch.

__________ 2. If $n > d$, there is no horizontal asymptote.

__________ 3. To determine the vertical asymptote of a rational function, find the zeroes of the numerator.

__________ 4. If $n < d$, the vertical asymptote is $y = 0$.

__________ 5. The horizontal asymptote of $f(x) = \frac{x}{x^2-1}$ is $y = 1$.

__________ 6. The vertical asymptote of $f(x) = \frac{(x-1)(x+3)}{x^2-1}$ are $x = 1$ and $x = 2$.

Remember Me!

- An asymptote is an imaginary line to which a graph gets closer and closer as the x or y increases or decreases its value without limit.
- To find vertical asymptote of a rational function, first reduce the given function to simplest form then find the zeroes of the denominator that are not zeros of the numerator.
- To determine the horizontal asymptote of a rational function, compare the degree of the numerator n and the degree of the denominator d.

  - *If n < d,* the horizontal asymptote *is y= 0*
  - *If n = d,* the horizontal asymptote y is the ratio of the leading coefficient of the numerator a, to the leading coefficient of the denominator *b. That is* $y = \frac{a}{b}$.
  - *If n > d,* there is no horizontal asymptote.

- An oblique asymptote is a line that is neither vertical nor horizontal. It occurs when the numerator of $f(x)$ has a degree that is one higher than the degree of the denominator. Divide the numerator by the denominator by either using long division or synthetic division. The oblique asymptote is the quotient with the remainder ignored and set equal to y.

Independent Assessment 2

Determine the vertical and horizontal asymptotes of the following rational functions.

Vertical Asymptote Horizontal Asymptote

1. $f(x) = \frac{2}{2x+5}$ ____________________ ____________________

2. $f(x) = \frac{x+3}{x+7}$ ____________________ ____________________

3. $f(x) = \frac{(x+3)(x-2)}{(x+5)(x-4)}$ ____________________ ____________________

4. $g(x) = \frac{2 + 3x}{x^2+3x-4}$ ____________________ ____________________

5. $g(x) = \frac{x-3}{2x^2-8}$ ____________________ ____________________

What I Have Learned

Let us summarize what you have learned from this module by completing the following statements. Write the correct word/s in a separate sheet of paper.

1. _____________ of the graph of a rational function are the points of intersection of its graph and an axis.

2. _____________ of a function are the values of x which make the function zero. The numbered zeroes are also _____________ of the graph of the function.

3. _______________ of the graph of a rational function r(x), if it exists, occurs at the zeros of the numerator that are not zeros of the denominators. To find ___________ equate the function to __________.

4. _______________of the graph of a rational function r(x) if it exists, occurs at r(0), provided that r(x) is defined at x = 0. To find ______________ simply evaluate the function at x = ___________.

5. An _____________ is an imaginary line to which a graph gets closer and closer as the x or y increases or decreases its value without limit.

6. To find ________________of a rational function, first reduce the given function to simplest form then find the zeroes of the denominator that are not zeros of the numerator.

7. To determine the ______________ of a rational function, compare the degree of the numerator n and the degree of the denominator d.
    - If n < d, the horizontal asymptote is __________

- If n = d, the horizontal asymptote y is the ratio of the leading coefficient of the numerator a, to the leading coefficient of the denominator b. That is y = ________.
- If n > d, there is ____________ horizontal asymptote.

8. An oblique asymptote is a line that is ______________________. To determine oblique asymptote, divide the numerator by the denominator by either using long division or synthetic division. The oblique asymptote is the quotient with the remainder ignored and set equal to y.

What I Can Do

Let's apply what you have learned from the lesson.

The concentration (C) of a given substance in a mixture is the ratio of the amount of substance to the total quantity. In symbols,

$$C = \frac{S}{Q}$$

where C is the concentration, S is the amount of substance, and T is the total quantity. If 8 ounces of punch contains 4 ounces of pure orange juice, the concentration of orange juice in the punch is 4/8 or 50%. The punch is 50% orange juice. Consider the problem where we begin that 8 ounces of punch that is 50% orange juice and want to write a function that gives the orange juice concentration after x ounces of pure orange juice are added.

Questions:

a. How much orange juice do you begin with? Write an expression for the amount of orange juice present after x ounces has been added.

b. Write an expression for the total amount of punch present after x ounces has been added.

c. Using the answers in (a) and (b), write a rational function defining the pineapple juice concentration as a function of x.

d. Give the x and y - intercepts of the rational function.

e. What is the equation of the vertical asymptote and of the horizontal asymptote?

Assessment

Let's Do This!

1. Which of the following is the set of all values that $f(x)$ take?
   a. Range
   b. Intercept
   c. Domain
   d. Zeroes

2. What is the y-intercept of $f(x) = \frac{x-3}{x+3}$?
   a. 0
   b. – 1
   c. – 3
   d. – 5

3. What is the x – intercept of $f(x) = \frac{x-1}{x}$?
   a. x = -1
   b. x = 0
   c. x = 1
   d. All real numbers

4. Complete the sentence: The x- intercept of rational function is also _________ of the graph of the function.
   a. asymptote
   b. range
   c. zero
   d. domain

5. Which of the following are the points of intersection of the graph and the axes?
   a. Range
   b. Intercept
   c. Domain
   d. Zeroes

6. What is the domain of f(x) $=\frac{3}{x}$?
   a. D = $\{x|x = 1\}$
   b. D =$\{x|x = 0\}$
   c. D = $\{x|x \neq 1\}$
   d. D = $\{x|x \neq 0\}$

7. Which of the following is **not** a true statement?
   a. A rational function is a quotient of functions.
   b. Asymptotes are a common characteristic of rational functions.
   c. An asymptote is a line that a graph approaches but does not touch.
   d. Domain and Range of rational functions are always equal

8. If the degree of the leading coefficient of the numerator is less than to the degree of the leading coefficient of the denominator of a rational function, which of the following statements has to be true?
   a. The graph has no asymptote
   b. The graph of the function has slant asymptote
   c. The graph of the function has a horizontal asymptote
   d. None of the above

9. What is the zero of $f(x) = \frac{x+5}{3x^2}$?
   a. x = 5
   b. x = 0
   c. x = - 3
   d. x = - 5

10. What is the horizontal asymptote of $f(x) = \frac{3x+1}{x-5}$?
   a. y = 5
   b. y = 3
   c. y = 1
   d. y = 0

11. What is the y - intercept of $f(x) = \frac{x^2-3x}{x+3}$?
   a. y = 3
   b. y = 1
   c. y = 0
   d. y = - 2

12. When the degree of the leading coefficient of the denominator of a rational function is greater than the degree of the leading coefficient of the numerator, the horizontal asymptote is at $y = \frac{a_n}{a_d}$ the statement is ____________.
   a. Always true
   b. Sometimes true
   c. Never true
   d. Cannot be determined

13. How will you describe the vertical asymptote of $f(x) = \frac{(x-3)(x-2)(x+5)}{(x-1)(x-3)(x-2)}$?

a. does not exist
b. approaching at x = 1
c. approaching at x = -1
approaching at x = 0

14. What is the x – intercept of $f(x) = \frac{x^2-2x-15}{x^2-25}$?

a. x = 5
b. x = 3
c. x = - 3
d. x = - 5

15. What is the horizontal asymptote of $f(x) = \frac{2x^2+x+3}{2x^2+3x+1}$?

a. y = 3
b. y = 2
c. 1
d. 0

Additional Activities

To deepen your knowledge on finding the intercepts, zeroes and asymptotes of rational function you can visit the following websites, https://youtu.be/gDC7XflNbQl and https://youtu.be/GgdGpjiJmkl.

For those who don't have online connections you can answer the following questions to deepen your understanding about the lesson.

Analyze the given function and determine:

a. x – and y- intercepts
b. zeros
c. Asymptotes

1. $f(x) = \frac{x+1}{x-4}$

2. $f(x) = \frac{x^2-4x+5}{x-4}$

Answer Key

Independent Assessment 1

x-intercept	y-intercept	Zeroes of f(x)
9	-3	9
5 (multiplicity 2)	5	5(multiplicity 2)
none	-3	none

Independent Practice 1

1. 2x + 6 and x – 3
2. x = - 3 or (-3, 0)
   2x + 6, in getting the x-intercept use the numerator of the function
3. y = -2 or (0, -2)
   substitute 0 to the x value of the function
4. The zero is at
   x = -3

What I Know

1. I
2. C
3. M
4. A
5. G

What I Know

1. c
2. b
3. b
4. b
5. a
6. d
7. d
8. c
9. b
10. a
11. b
12. a
13. d
14. a
15. a

Independent Assessment 2

	Vertical Asymptote	Horizontal Asymptote
1	x = -5/2 or -2.5	y = 0
2	x = - 7	y = 1
3	x = -5 & x = 4	y = 1
4	x = -4 & x = 1	y = 0
5	x = -2 & x = -2	y = 0

Application

1. 4, 4 + x
2. 8 + x
3. C(x) = (4+x)/(8+x)
4. x = - 4 or (-4, 0)
   y = 0.5 or (0, 0.5)
5. y = 1

Independent Practice 2

1. Asymptote
2. True
3. Denominator
4. Horizontal asymptote
5. y = 0
6. x = -1

Additional Activity

1. a. $x = -1$ and $y = -\frac{1}{4}$
   b. $x = -1$
   c. VA at $x = 4$
   HA at $y = 1$
   SA = none

2. a. x = none and
   $y = -1.25$
   b. none
   c. VA at $x = 4$
   HA none
   SA none

Post - Assessment

1. a
2. b
3. c
4. c
5. b
6. d
7. d
8. c
9. d
10. b
11. c
12. c
13. b
14. c
15. c

References

DIWA Senior High School Series: General Mathematics, DIWA Learning Systems Inc, Makati City, 2016.

General Mathematics Learner's Materials. Pasig City, Philippines: Department of Education- Bureau of Learning Resources, 2016.

Orines, Fernando B., Next Century Mathematics 11 General Mathematics, Phoenix Publishing House, Quezon City, 2016.

Oronce, Orlando A., General Mathematics, 1st Edition, Rex Book Store, Inc., Sampaloc Manila, 2016.

Santos, Darwin C. and Ma. Garnet P. Biason, Math Activated: Engage Yourself and Our World General Math, Don Bosco Press, Makati City, 2016.

Young, Cynthia, Algebra and Trigonometry, John Wiley & Sons, Inc. New Jersey, 2010.

Internet Source:
https://youtu.be/gDC7XflNbQl
https://youtu.be/GgdGpjiJmkl.

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