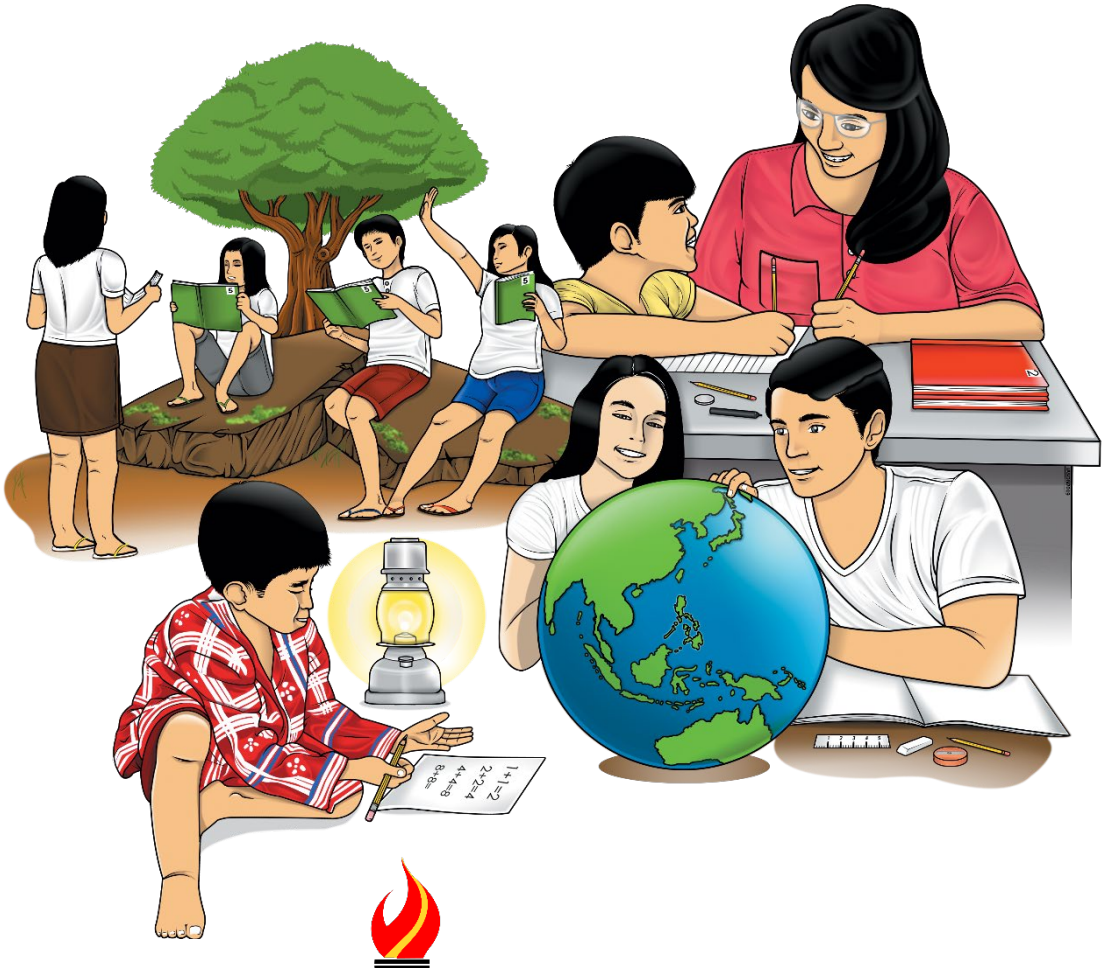
Senior High School

DepED
DEPARTMENT OF EDUCATION

General Mathematics

Quarter 1 – Module 27: Intercepts, Zeroes and Asymptotes of Logarithmic Functions



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Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

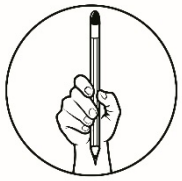
Thank you.

What I Need to Know

This module will help you determine the intercepts and zeroes of logarithmic functions using the algebraic solution and its asymptotes through its domain which are essentials in the next chapter. The topics to be discussed in this module will able you to prepare to solve real-life applications of logarithmic functions. The language used in this module is appropriate to a diverse communication and language ability of the learners.

After going through this module, you are expected to:

1. find the intercepts of logarithmic functions;
2. solve for the zeroes of logarithmic functions; and
3. determine the asymptotes of logarithmic functions.



What I Know

Choose the letter of the best answer. Write your chosen letter on a sheet of paper.

1. What is a line that the curve approaches, as it heads toward infinity?
   a. asymptote
   b. domain
   c. intercept
   d. range

2. It is where a function crosses the x or y-axis.
   a. asymptote
   b. domain
   c. intercept
   d. range

3. What is the x-intercept of $f(x) = \log_2(x - 4)$?
   a. 4
   b. -4
   c. -5
   d. 5

4. Logarithmic function is not defined for _____ numbers and zero.
   a. negative
   b. positive
   c. real
   d. whole

5. The graph of the function $f(x) = \log_b x$ has a vertical asymptote at _____.
    a. x =1
    b. x = -1
    c. x = 0
    d. x = 2

6. What is the inverse of the exponential function?
    a. logarithmic
    b. linear
    c. polynomial
    d. rational

7. What is known as the x-value that makes the function equal to 0?
    a. asymptote
    b. intercept
    c. range
    d. zeroes

8. What is a function of the form $f(x) = b^x$?
    a. exponential
    b. logarithmic
    c. linear
    d. polynomial

9. It is where the functions cross the x-axis and where the height of the function is zero.
    a. asymptote
    b. x-intercept
    c. y-intercept
    d. zeroes

10. What is the x-intercept of the function $f(x) = log\ (2x + 3)$?
    a. (1,0)
    b. (0,1)
    c. (0, -1)
    d. (-1,0)

11. What are the zeroes of the function $f(x) = \log_2 x^2$?
    a. x=0 and x=1
    b. x=1 and x=-1
    c. x=0 and x=-1
    d. x=2 and x=-2

12. The graph of the function $f(x) = \log_4(3x - 2)$ has a vertical asymptote at _____.
    a. $x = \frac{2}{3}$
    b. $x = \frac{3}{2}$
    c. x=2
    d. x=3

13. What is the x-intercept of the function $f(x) = ln\,(x - 3)$?
    a. (0,4)
    b. (0,-4)
    c. (-4,0)
    d. (4,0)

14. The graph of the function $f(x) = log\,x - 2$ has a vertical asymptote at _____.
    a. x=1
    b. x=0
    c. x=-1
    d. x=2

15. What is the inverse of $y = \log_2 x$?
    a. $y = x^2$
    b. $y = 2^x$
    c. $2^y = x$
    d. $x = y^2$

Lesson 1 Intercepts, Zeroes, and Asymptotes of Logarithmic Functions

This topic focuses on how to determine the intercept, zeroes, and asymptote of a logarithmic function. It is also about the concept of finding the intercept and zeroes of a logarithmic function applying the transformation of logarithmic function to exponential form and determining the asymptote of a logarithmic function using the idea of its domain.

What's In

Let us start our discussion by recalling some important topics that will guide you as you go along with this module.

It can be remembered that the logarithmic function $f(x) = x$ is the inverse of the exponential function $f(x) = b^x$ and since the logarithmic function is the inverse of the exponential function, the domain of the logarithmic function is the range of exponential function, and vice versa.

In general, the function $f(x) = x$ where $b, x > 0$ and $b \neq 1$ is a continuous and one-to-one function. Note that the logarithmic function is ***not defined*** for negative numbers or zero. The graph of the function approaches the y-axis as x tends to ∞, but never touches it. The function rises from -∞ to ∞ as x increases if b > 1 and falls from ∞ to -∞ as x increases if $0 < b < 1$.

Therefore, the domain of the logarithmic function $y = \log_b x$ is the set of positive real numbers and the range is the set of real numbers.

What's New

Decode It. Solve for the zero and asymptote of the given logarithmic functions. Blacken the circle that corresponds to your answer and write the letter in the appropriate box to decode the word.

1.) $y = log_{\frac{1}{4}}(x + 2)$

- **E** x=-1, VA: x=2
- **T** x=-2, VA: x=-1
- **H** x=-1, VA: x=-2

2.) $y = log_3 x - 1$

- **I** x=-3, VA: x=-0
- **B** x=3, VA: x=-0
- **D** x=0, VA: x=-3

3.) $y = log_2(x - 1)$

- **D** x=-1, VA: x=2
- **R** x=-2, VA: x=1
- **E** x=2, VA: x=-1

4.) $y = log_2(3x - 5)$

- **S** x=2, VA: x=-3/5
- **C** x=2, VA: x=-5/3
- **N** x=3/5, VA: x=2

5.) $y = log_3(2x - 6)$

- **R** x=7/2, VA: x=3
- **A** x=2/7, VA: x=3
- **P** x=7/2, VA: x=-3

6.) $y = log_2(4x + 5)$

- **A** x=-2, VA: x=-1
- **R** x=1, VA: x=-2
- **P** x=-1, VA: x=-2

The number 0 is originally called

4	2	6	1	3	5

What is It

In order to decode the activity above, you are going to solve the zero of the function and find its vertical asymptote. Then, you are going to blacken the circle that corresponds to your answer and from the letters of the word will be revealed to decode the answer.

After you go through the activity, reflect on the following questions:

1.) How do you find the activity?
2.) Did you decode the answer? What is your answer?
3.) What did you do to find the zero of the given logarithmic function? How about finding the vertical asymptote?

With the idea that you gained from the previous activity, you are now ready to learn the lesson. Let us begin.

Intercepts and Zeroes of Logarithmic Functions

An intercept in Mathematics is where a function crosses the x or y-axis. x-intercepts are where functions cross the x-axis. They are also called roots, solutions, and zeroes of a function. They are found algebraically by setting $y=0$ and solving for x. The zero of a function is the x-value that makes the function equal to 0, that is, $f(x) = 0$. In this section, our discussion will focus only on the x-intercept of a given logarithmic function.

Example 1. Find the intercept and zeroes of $f(x) = log\ (2x + 3)$.

To find the intercept, we let y = 0 then solve for x.

$f(x) = log\ (2x + 3)$
$0 = log\ (2x + 3)$
$10^0 = 2x + 3$ → change from logarithmic to exponential function
$1 = 2x+3$ → since $10^0 = 1$
$2x = 1-3$
$2x = -2$ → divide both sides by 2
$x = -1$

Therefore, the x-intercept is at (-1,0) and the zero of the function is -1.

Example 2. Find the intercept and zeroes of $f(x) = log_2 x^2$.

To find the intercept, we let y = 0 then solve for x.

$f(x) = x^2$

$0 = x^2$

$2^0 = x^2$ ⟶ change from logarithmic to exponential function

$1 = x^2$ ⟶ since $2^0 = 1$

$x = \pm\sqrt{1}$

$x = \pm 1$

Therefore, the x-intercepts are at (1,0) and (-1,0) and the zeroes of the function are 1 and -1.

Example 3. Find the intercept and zeros of $f(x) = ln\ ln\ (x - 3)$

To find the intercept, we let y = 0 then solve for x.

$f(x) = ln\ (x - 3)$

$0 = ln\ (x - 3)$

$x - 3 = e^0$ ⟶ change from logarithmic to exponential function

$x-3 = 1$ ⟶ since $e^0 = 1$

$x = 1 + 3$

$x = 4$

Therefore, the x-intercept is at (4,0) and the zero of the function is 4.

Vertical Asymptote of Logarithmic Function

An asymptote is a line that a curve approaches, as it heads towards infinity. It is a vertical asymptote when as x approaches some constant value c (either from the left or from the right) then the curve goes towards ∞ or -∞.

In dealing with the vertical asymptote of a logarithmic function, it is a must to remember that logarithmic function is not defined for negative numbers or zero, and the domain of a logarithmic function $f(x) = x$ x is a set of positive real numbers. A logarithmic function will have a vertical asymptote precisely where its argument (i.e. the quantity inside the parentheses) is equal to zero.

Example 1. Find the vertical asymptote of the graph of $f(x) = log\ x - 2$.

Since the domain of the logarithmic function is *(0, ∞)*, the graph has a vertical asymptote at x = 0.

Example 2. Find the vertical asymptote of the graph of $f(x) = \log_4(3x - 2)$.

Set the argument *(3x-2)* equal to zero then solve for x, that is,

$3x - 2 = 0$

$3x = 2$ dividing both sides by 3

$x = \frac{2}{3}$

Since the logarithmic function is defined for $x > \frac{2}{3}$, the graph has a vertical asymptote at $x = \frac{2}{3}$.

Example 3. Find the vertical asymptote of the graph of $f(x) = \log_2(x + 3) + 2$.

Set the argument *(x+3)* equal to zero then solve for x, that is,

$x + 3 = 0$

$x = -3$

Since the logarithmic function is defined for $x > -3$, the graph has a vertical asymptote at $x = -3$.

What's More

Activity 1.1

Match It: Match column A with column B by drawing a line to connect them.

Column A		***Column B***
1. $y = 2\log_2 x$	●	● 1. VA: x=-2, int.: (-1,0) zero: -1
2. $y = \log_3 x - 1$	●	● 2. VA: x=0, int.: (0.125,0) zero: 0.125
3. $y = \log_{0.25}(x + 2)$	●	● 3. VA: x=0, int.: (1,0) zero: 1
4. $y = \log_2(x - 3)$	●	● 4. VA: x=3, int.: (4,0) zero: 4
5. $y = \log_{0.5}(x) - 3$	●	● 5. VA: x=0, int.: (3,0) zero:3

Activity 1.2

Unscramble the letters to find the correct answer then write your answers in the boxes provided before each number.

 (tysatomep)

 (narge)

 (atmlocgrihi)

 (oseerz)

 (ncprteite)

1. It is a line that the curve approaches but never touches it.
2. It refers to a set of all y-values.
3. It is the inverse of exponential function.
4. It refers to the x-value that makes the function equal to 0.
5. It is where a function crosses the x or y-axis

(moadni)

(oxetlapenni)

(atvneegi)

(ifev)

(lriectva)

6. It refers to the set of all x-values.
7. It is a function of the form $f(x)=b^x$.
8. Logarithmic function is not defined for _____ numbers and zero.
9. It is the x-intercept of $f(x)=\log_2(x-4)$.
10. The graph of the function $f(x)=\log_b x$ has a _____ asymptote at x = 0.

Activity 1.3

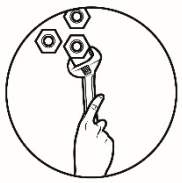
Determine the x-intercepts, zeroes, and vertical asymptotes of the following:

1. $f(x) = \log_2 x$
2. $f(x) = log\ (x) - 3$
3. $f(x) = \log_3(x-2) + 4$
4. $f(x) = \log_2(x+1) - 2$
5. $f(x) = \log_2(\log_{16} x) + 2$

What I Have Learned

Complete the following statement with the correct word/s.

1. The logarithmic function _____ is the inverse of $f(x) = b^x$.
2. An _____ is where the functions cross the x or y-axis and _____ is where the curve cross the x-axis.
3. An _____ is a line that a curve approaches as it approaches_____.
4. The _____ of a function is the x-value that makes the function equal to _____.
5. A logarithmic function is _____ on negative numbers and_____.



What I Can Do

Answer the problem given below.

pH Level In chemistry, the pH of a substance is defined as $pH = -log\,[H^+]$
where H+ is the hydrogen ion concentration, in moles per liter. Find the pH level of each substance.

SUBSTANCE	***HYDROGEN ION CONCENTRATION***
a. Pineapple juice	1.6×10^{-4}
b. Hair conditioner	0.0013
c. Mouthwash	6.3×10^{-7}
d. Eggs	1.6×10^{-8}
e. Tomatoes	6.3×10^{-5}

a.) How can you help to prevent water supplies such as rivers, lakes, and streams from developing an imbalance water pH level?

Rubric for rating this activity:

20	All questions are answered correctly using the model given in the problem.
15	4 questions are answered correctly using the model given in the problem.
10	2-3 questions are answered correctly using the model given in the problem.
5	0-1 questions are answered correctly using the model given in the problem.

Assessment

Choose the letter of the best answer. Write your answer in your notebook.

1. Intercept is where a function crosses the _____.
   a. x-axis
   b. x and y-axis
   c. y-axis
   d. y and z-axis

2. Logarithmic function is not defined for negative numbers and _____.
   a. one
   b. three
   c. two
   d. zero

3. What is the x-intercept of the function $f(x) = \log_3(3x - 2)$?
   a. x=1
   b. x=-1
   c. x=3
   d. x=2

4. The graph of $f(x) = \log_b x$ has a _____ at x=0.
   a. horizontal asymptote
   b. vertical asymptote
   c. x-intercept
   d. y-intercept

5. What is the zero of $f(x) = \log_2(x - 4)$?
   a. -4
   b. 4
   c. 5
   d. -5

6. Asymptote is a line that the curve approaches as it approaches _____,
   a. curve
   b. infinity
   c. one
   d. zero

7. What is the inverse of the function $y=b^x$?
   a. $y = b$
   b. $x = b$
   c. $y = x$
   d. $b = x$

8. What is the x-intercept of the function $f(x) = \log_2(2x + 5)$?
    a. (-2,0)
    b. (2,0)
    c. (1,0)
    d. (-1,0)

9. What is the zero of the function $f(x) = \log_2(x + 1)$?
    a. 2
    b. -1
    c. 0
    d. 1

10. The x-intercept is where the function crosses the x-axis and where the height of the function is _____.
    a. maximum
    b. negative
    c. one
    d. zero

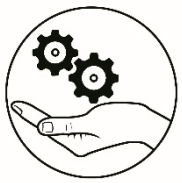
11. What is the inverse of a logarithmic function?
    a. exponential
    b. linear
    c. polynomial
    d. quadratic

12. What is the intercept of the function $f(x) = \log_2(x + 2)$?
    a. x=2
    b. x=-1
    c. x=-2
    d. x=1

13. What is the zero of the function $f(x) = ln\,(x - 3)$?
    a. 4
    b. -4
    c. 2
    d. -2

14. The graph of the function $f(x) = log\,(2x - 3)$ has a vertical asymptote at _____.
    a. x=2
    b. x=3
    c. x = $\frac{2}{3}$
    d. $x = \frac{3}{2}$

15. What is the intercept of the function $f(x) = \log_3(x + 6)$?
    a. x=5
    b. x=-5
    c. x=6
    d. x=-6



Additional Activities

Determine the intercept, zero, and vertical asymptote of the following logarithmic functions. Write your answer in a sheet of paper.

1. $y = \log_3(x + 3)$
2. $y = \log_{\frac{1}{3}} x + 1$
3. $y = \log_4(x - 1)$
4. $y = \log_{\frac{1}{3}}(x + 1)$
5. $y = \log_{\frac{1}{2}} x + 2$
6. $y = \log_2 x - 2$
7. $y = \log_2(x - 2)$
8. $y = \log_3 x + 3$
9. $y = \log_3 x - 1$
10. $y = \log_{\frac{1}{2}}(x + 2)$

Answer Key

What I Know

1. a
2. c
3. d
4. a
5. c
6. a
7. d
8. a
9. b
10. d
11. b
12. a
13. d
14. b
15. c

What's More

Activity 1.1

1. c
2. e
3. a
4. d
5. b

Activity 1.2

1. asymptote
2. range
3. logarithmic
4. zeroes
5. intercept
6. domain
7. exponential
8. negative
9. five
10. vertical

Activity 1.3

1. VA: x =0, Int. (1, 0)
   Zero: 1
2. VA: x = 0, Int.
   (1000,0) Zero: 1000
3. VA: x = 2, Int.
   (163/81,0) Zero:
   163/81
4. VA: ,x = -1 Int. (3, 0)
   Zero: 3
5. VA: x = 1, Int. (2, 0)
   Zero: 2

Assessment

1. b
2. d
3. a
4. b
5. c
6. b
7. c
8. a
9. c
10. d
11. a
12. b
13. a
14. d
15. b

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