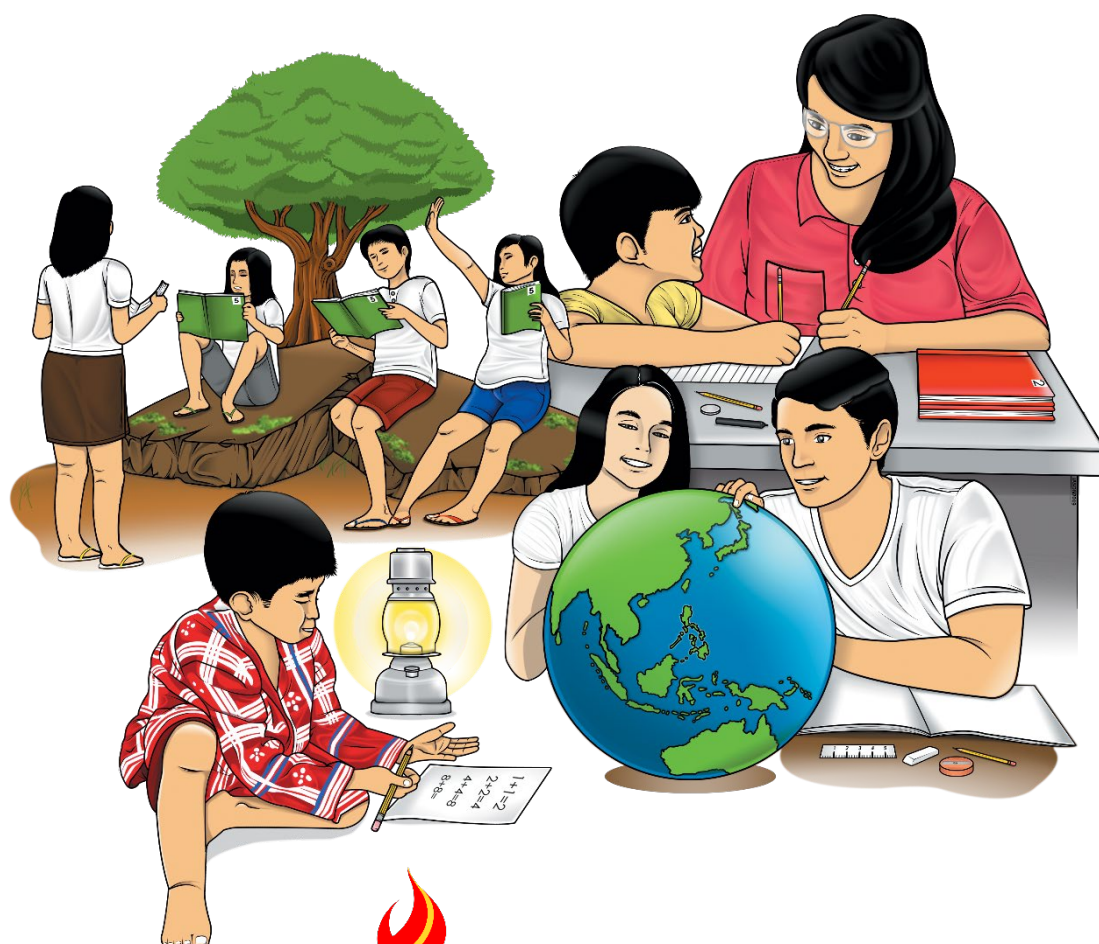


General Mathematics

Quarter 1 – Module 26: Domain and Range of Logarithmic Functions



General Mathematics
Alternative Delivery Mode
Quarter 1 – Module 26: Domain and Range of Logarithmic Functions
First Edition, 2021

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General Mathematics

Quarter 1 – Module 26:

Domain and Range of Logarithmic Functions

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

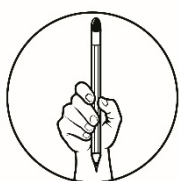


What I Need to Know

This module was written for students to understand the concept of domain and range of a logarithmic function. The topic to be discussed in this module includes finding the domain and range of a logarithmic function algebraically. The language used in this module is appropriate to the diverse communication and language ability of the learners.

After going through this module, you are expected to:

1. define domain and range;
2. understand the properties of logarithmic function; and
3. determine the domain and range of the logarithmic function.



What I Know

Choose the letter of the best answer. Write your chosen letter on a sheet of paper.

1. What is the inverse of the function $x = b^y$?
a. $b = y$
b. $x = y$
c. $y = x$
d. $y = b$
2. What is the domain of the logarithmic function $f(x) = \log_b x$?
a. $(0, \infty)$
b. $(0, -\infty)$
c. $(-\infty, \infty)$
d. $(\infty, -\infty)$
3. What is known as the possible values of the independent variable x ?
a. domain
b. inputs
c. outputs
d. range
4. What is the range of the function $f(x) = x$ where $b < 1$?
a. $(0, \infty)$
b. $(-\infty, 0)$
c. $(\infty, -\infty)$
d. $(-\infty, \infty)$
5. What is the domain of the function $f(x) = \log_2(3x - 6)$?
a. $(2, \infty)$
b. $(-2, \infty)$
c. $(-\infty, 2)$
d. $(-\infty, 2)$
6. The range of the function are the corresponding values of the independent variable y which are often called _____.
a. domain
b. inputs
c. outputs
d. range

7. What is a set of all y values?
- domain
 - inputs
 - outputs
 - range
8. What is the domain of the function $f(x) = \log_5(x - 1)$?
- $(-1, \infty)$
 - $(1, \infty)$
 - $(-\infty, 1)$
 - $(-\infty, -1)$
9. What is a set of all x values?
- domain
 - inputs
 - outputs
 - range
10. What is the domain of the function $f(x) = \log(5 - 3x)$?
- $(-\infty, \frac{3}{5})$
 - $(-\infty, 5)$
 - $(-\infty, \frac{5}{3})$
 - $(-\infty, 3)$
11. What is the inverse of the logarithmic function?
- exponential
 - linear
 - polynomial
 - quadratic
12. What is the range of the function $f(x) = \log_2(x - 2)$?
- $(0, \infty)$
 - $(-\infty, 0)$
 - $(\infty, -\infty)$
 - $(-\infty, \infty)$
13. Which of the following is NOT an exponential function?
- $y = 1^x$
 - $y = 2^x$
 - $y = 3^x$
 - $y = 4^x$
14. What is the inverse of $y = \log_2 x$?
- $y = x^2$
 - $y = 2^x$
 - $2^y = x$
 - $x = y^2$
15. What is the domain of the function $y = \log_3(x - 5) + 2$?
- $(-5, \infty)$
 - $(5, \infty)$
 - $(2, \infty)$
 - $(-2, \infty)$

Lesson

1

Domain and Range of Logarithmic Functions

The domain of a function is the set of possible values of the independent variable. The range is the set of the resulting values that the dependent variable can have as x varies throughout the domain. This module focuses on the domain and range of a logarithmic function.



What's In

To fully understand this topic and for you to easily grasp the next lesson, let us recall some concepts from the previous lessons on exponential function and its relationship with its inverse function, the logarithmic function.

Let us start with the definition of an exponential function which is a function of the form

$$f(x) = b^x, \text{ where } b > 0 \text{ and } b \neq 1.$$

Examples are as follows: $f(x) = 3^x$, $f(x) = \left(\frac{1}{2}\right)^x$, $g(x) = 2^{-x}$, and $y = \left(\frac{1}{5}\right)^{-2x}$. Its domain is a set of real numbers while its range is a set of all positive real numbers.

Let us also recall that if a function is a one-to-one function, then an inverse function exists denoted by f^{-1} having the following properties:

- f^{-1} is a one-to-one function
- domain of f^{-1} is the range of f
- range of f^{-1} is the domain of f

If the position of x and y in $y = b^x$ are interchanged and then y is solved for the resulting equation, the rule of correspondence of the inverse of the exponential function is obtained. This rule is denoted by the symbol:

$$y = \log_b x$$

Since the exponential function is one-to-one, its inverse must also be a function. Just like in the exponential function, where $b > 0$, and $b \neq 1$.

Let us also recall lessons regarding domain and range of a function by answering the following:

a. $f(x) = 2x + 5$

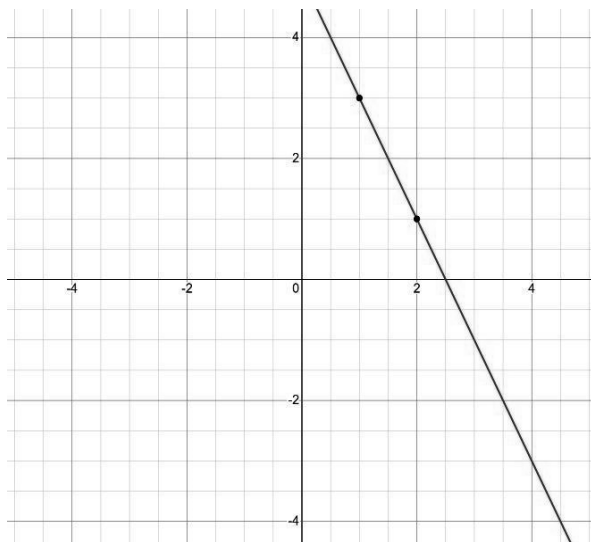
b. $f(x) = \sqrt{x - 3}$

c. $f(x) = \frac{x+3}{x-2}$

Solution:

a. $f(x) = 2x + 5$

Since the linear function $-2x+5$ is a polynomial function, and based on its graph, its domain is $\{x|x \text{ is a real number}\}$ and generally its range is $\{y|y \text{ is a real number}\}$.



b. $f(x) = \sqrt{x - 3}$

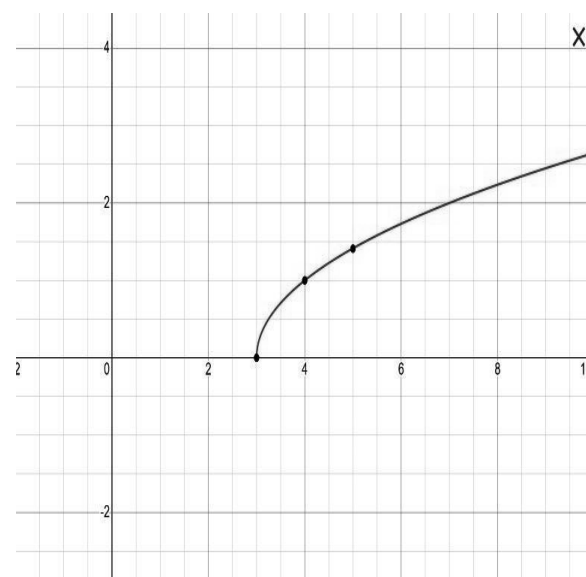
Solving for the domain and range:

$$x - 3 \geq 0$$

$$x \geq 3$$

Therefore, the domain is $\{x|x \geq 3\}$ and the range is $\{y|y \geq 0\}$ as you can see from the graph.

Take note that the number under a square root sign must be positive.



c. $f(x) = \frac{x+3}{x-2}$

In a rational function, the denominator of a fraction cannot be zero, therefore, the domain is $\{x|x \neq 2\}$.

For the range, interchange the variable of the given function, then solve for y.

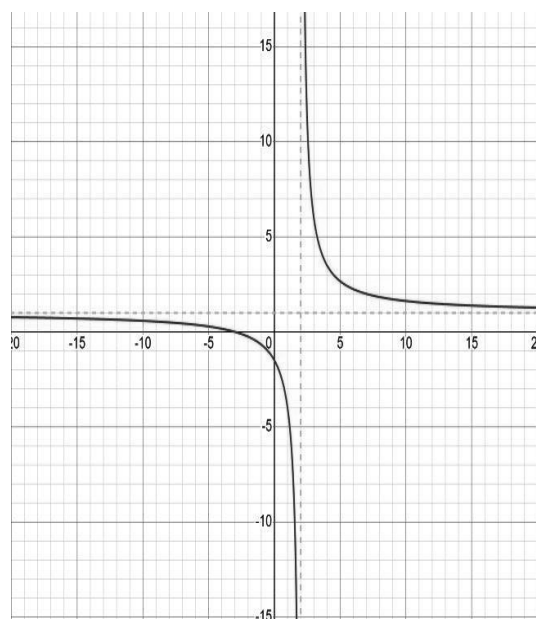
$$x(y-2) = y+3$$

$$xy - 2x = y + 3$$

$$xy - y = 2x + 3$$

$$y(x-1) = 2x+3$$

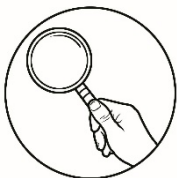
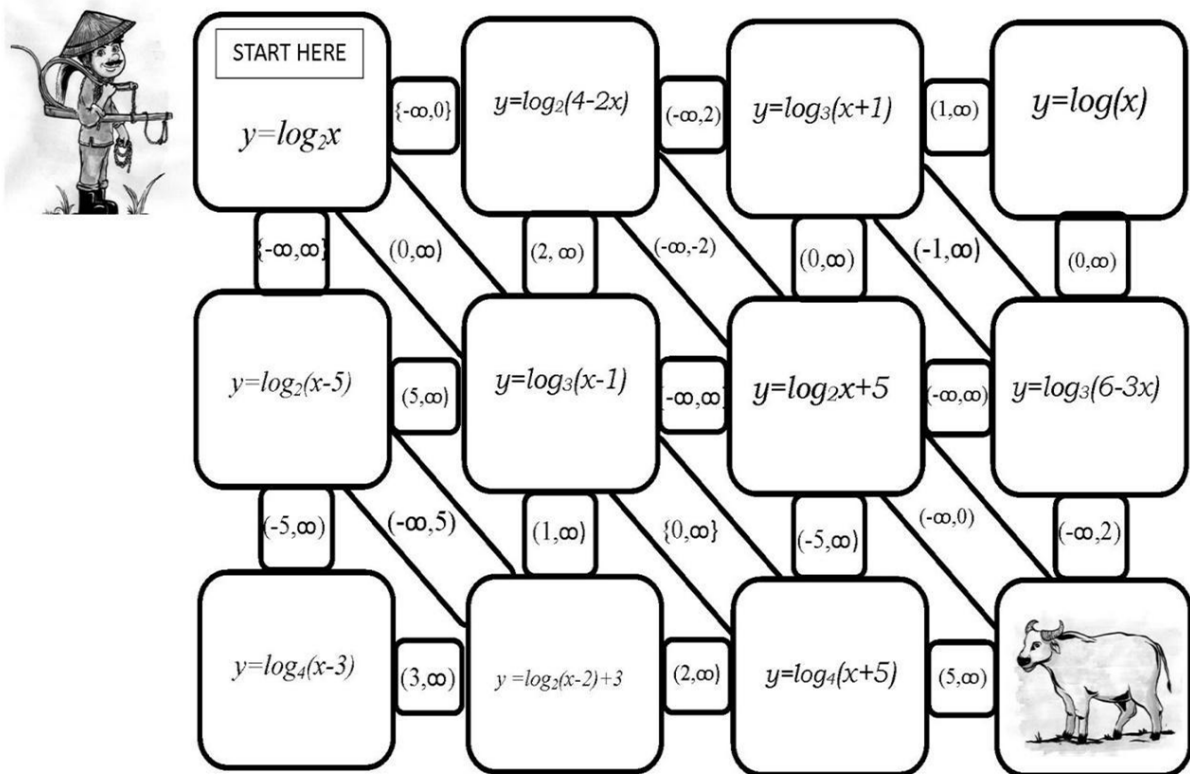
Therefore, the range is $\{y|y \neq 1\}$.





What's New

Let us help Mang Kulas find his lost carabao by going through the maze. Find the domain of the given logarithmic functions to get to the carabao.



What is It

In the previous activity, you need to help Mang Kulas find his lost carabao by going through the maze. In doing so, you need to answer by finding the domain of the logarithmic functions given.

After you finish the activity, reflect to the following questions:

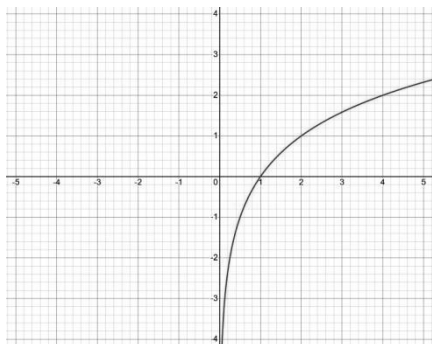
1. Do you find difficulty in finding the domain of a function?
2. How do you find the domain of the logarithmic function?
3. How can you define domain and range?

If you think that the activity is difficult, that is okay because after you read more about domain and range of logarithmic function, you can go back to the activity and help Mang Kulas find his carabao. The discussion below will help you understand more the domain and range of the logarithmic function.

Domain and Range of Logarithmic Function

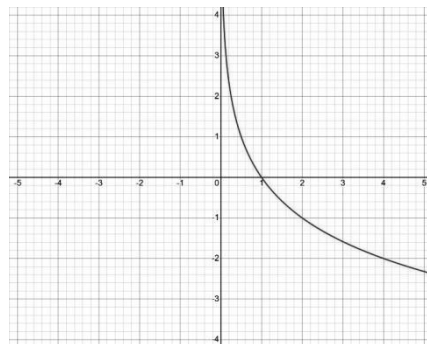
The domain of a function is the set of all possible values of the independent variable x . The possible values of the independent variable x are often called **inputs**. The range of the function are the corresponding values of the dependent variable y . The corresponding values of the dependent variable y are often called **outputs**.

In the case of a logarithmic function, its domain is defined as a set of all positive real numbers while its range is a set of real numbers.



$f(x) = \log_b x$, where $b > 1$

Domain: $(0, \infty)$
Range: $(-\infty, \infty)$



$f(x) = \log_b x$, where $b < 1$

Domain: $(0, \infty)$
Range: $(-\infty, \infty)$

Transformation of the parent function $f(x) = x$ either by shift, stretch, compression, or reflection changes the domain of the parent function. When finding the domain of a logarithmic function, it is important to remember that the domain consists *only of positive real numbers*. That is, the argument of the logarithmic function must be greater than zero.

Example 1. Find the domain and range of $f(x) = \log_5(2x - 4)$

Solution

$2x - 4 > 0$ Set up an inequality showing an argument greater than zero.

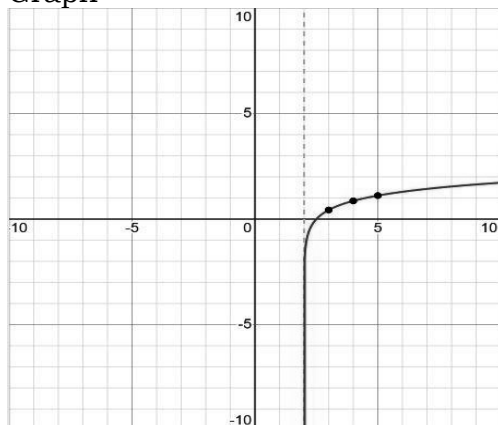
$2x > 4$ Solve for x .

or 2

Domain: $(2, \infty)$ Write the domain in interval notation.

Range: $(-\infty, \infty)$

Graph



From the graph of the function, it can be seen that the curve is asymptotic at $x = 2$. Therefore, the domain and range are as follows:

Domain: $(2, \infty)$

Range: $(-\infty, \infty)$

Example 2. Find the domain and range of $f(x) = \log_2(x - 3) + 2$.

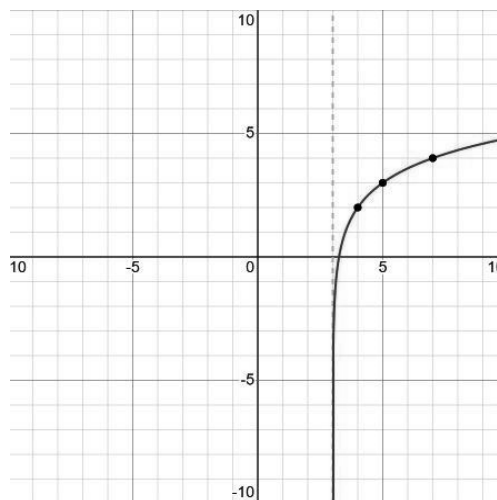
Solution:

Graph

$$\begin{aligned} x-3 &> 0 \\ x-3+3 &> 0+3 \\ x &> 3 \end{aligned}$$

Domain: $(3, \infty)$

Range: $(-\infty, \infty)$



Example 3. Find the domain and range of $f(x) = \log(3 - 2x)$.

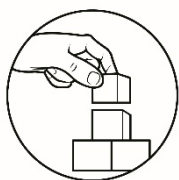
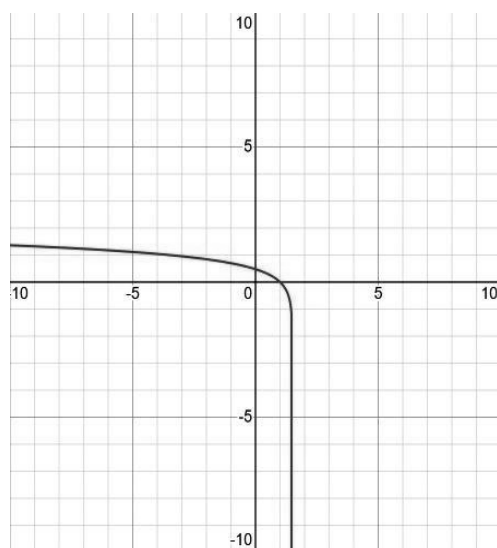
Solution:

Graph

$$\begin{aligned} 3-2x &> 0 \\ -3+3-2x &> 0-3 \\ -2x &> -3 \end{aligned}$$

Domain: $(-\infty,)$

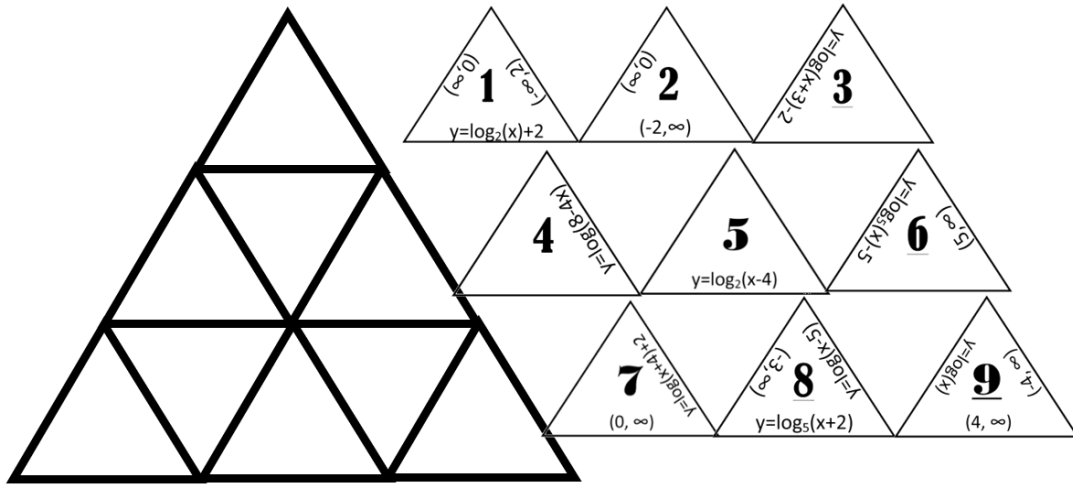
Range: $(-\infty, \infty)$



What's More

Activity 1.1

Arrange the small triangles to fit into the larger triangle accordingly. Make sure that the given function corresponds to its right domain. Write the number of your answer inside the triangle to form a larger triangle.



Activity 1.2

MATCH IT: Match column A with column B by drawing a line to connect them.

Column A

1. $y = \log_2(x - 1)$ ●
2. $y = \log_3 x - 1$ ●
3. $y = \log_5(x + 2) - 5$ ●
4. $y = \log_2(x - 3)$ ●
5. $y = \log_{0.5}(x + 5) - 3$ ●

Column B

1. Domain: $(-2, \infty)$, Range: $(-\infty, \infty)$ ●
2. Domain: $(3, \infty)$, Range: $(-\infty, \infty)$ ●
3. Domain: $(-5, \infty)$, Range: $(-\infty, \infty)$ ●
4. Domain: $(1, \infty)$, Range: $(-\infty, \infty)$ ●
5. Domain: $(0, \infty)$, Range: $(-\infty, \infty)$ ●

Activity 1.3

Determine the domain and range of the following:

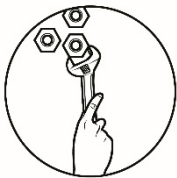
1. $f(x) = \log_2(3 - 2x)$
2. $f(x) = \log(x) - 5$
3. $f(x) = \log_3(x + 2) + 4$
4. $f(x) = \log_2(x + 1) - 2$
5. $f(x) = \log_2(4 - 7x)$



What I Have Learned

Complete the following statements with the correct word/s.

1. The _____ of a function is the set of all possible values of the independent variable _____. These possible values are often known as _____.
2. The _____ of a function is the set of all possible values of the dependent variable _____. These possible values are often known as _____.
3. The _____ $f(x) = \log_b x$ is the inverse of _____ $f(x) = b^x$.
4. The domain of the function $f(x) = \log_b x$ is _____ while its range is _____.



What I Can Do

Answer the problem given below.

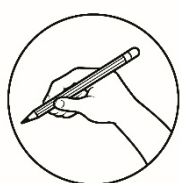
Loudness is measured in decibels. The formula for the loudness of a sound is given by " $\text{dB} = 10 \frac{\log \log I}{I_0}$ " where I_0 is the intensity of "threshold sound", or sound that can barely be perceived. Other sounds are defined in terms of how many times more intense they are than threshold sound. For instance, a cat's purr is about 316 times as intense as threshold sound, for a decibel rating of:

$$\begin{aligned}
 \text{dB} &= 10 \frac{\log \log I}{I_0} \\
 &= 10 \frac{\log \log (316 I_0)}{I_0} \\
 &= 10 \log [316] \\
 &= 24.9968708262\dots, \text{or } 25 \text{ decibels.}
 \end{aligned}$$

Considering that prolonged exposure to sounds above 85 decibels can cause hearing damage or loss, and considering that a gunshot from a .22 rimfire rifle has an intensity of about $I = (2.5 \times 10^{13})I_0$, should you follow the rules and wear ear protection when relaxing at the rifle range? If yes, how will you encourage others to follow the rules in wearing ear protection at the rifle range?

Rubric for rating this activity

20	The problem is correctly answered applying the concept and properties of logarithmic functions. The model is used appropriately.
15	The problem is correctly answered applying the concept and properties of logarithmic function. The model is used with some misrepresentation.
10	The problem is partially answered by applying a different solution. The model is not used at all.
5	The problem is not correctly answered applying the concept and properties of logarithmic functions. The model is not used at all.



Assessment

Multiple Choice: Choose the letter of the best answer. Write your answer in your notebook.

- Range is a set of all _____ values.
 - w
 - x
 - y
 - z
- What is the range of the exponential function $y = b^x$?
 - $(0, \infty)$
 - $(-\infty, 0)$
 - $(-\infty, \infty)$
 - $(\infty, -\infty)$
- What is the domain of the function $f(x) = \log_3(3x - 2)$?
 - $(\frac{3}{2}, \infty)$
 - $(-\infty, \frac{3}{2})$
 - $(-\infty, \frac{2}{3})$
 - $(\frac{2}{3}, \infty)$
- What is known as the possible values of the independent variable x?
 - domain
 - inputs
 - outputs
 - range
- What is the inverse of the function $f(x) = \log_b x$?
 - $b = y^x$
 - $b = x^y$
 - $x = b^y$
 - $y = b^x$
- What is a set of all x values?
 - domain
 - inputs
 - outputs
 - range

7. What is the domain of the function $f(x) = \log(7 - 4x)$?
- $(-\infty, \frac{4}{7})$
 - $(\frac{4}{7}, \infty)$
 - $(\frac{7}{4}, \infty)$
 - $(-\infty, \frac{7}{4})$
8. Outputs are the possible values of the _____ variable y.
- constant
 - dependent
 - independent
 - real
9. What is the domain of the function $f(x) = \log_2(x + 1)$?
- $(-\infty, 1)$
 - $(-\infty, -1)$
 - $(-1, \infty)$
 - $(1, \infty)$
10. What is the range of the function $f(x) = \log_b x$?
- $(-\infty, \infty)$
 - $(-\infty, 0)$
 - $(0, \infty)$
 - $(1, \infty)$
11. What is the inverse of an exponential function?
- linear
 - logarithmic
 - polynomial
 - quadratic
12. What is the range of the function $f(x) = \log_2(x + 2)$?
- $(-\infty, 2)$
 - $(-\infty, -2)$
 - $(2, \infty)$
 - $(-2, \infty)$
13. Which of the following is the correct logarithmic form of $16^{\frac{1}{4}} = \frac{1}{8}$?
- $\frac{1}{8} = \frac{3}{4}$
 - $\frac{3}{4} = \frac{1}{8}$
 - $16 = \frac{3}{4}$
 - $16 = \frac{1}{8}$
14. Which of the following is an exponential function?
- $y = 2x^4$
 - $y = x^4$
 - $y = 4^x$
 - $y = x^{-4}$
15. What is the domain of the function $f(x) = \log_3(x + 6) - 2$?
- $(-\infty, 6)$
 - $(-\infty, -6)$
 - $(6, \infty)$
 - $(-6, \infty)$



Additional Activities

Graph the following logarithmic functions using an online graphing calculator then find its domain and range.

1. $y = \log_2(x - 2)$

2. $y = \log_3(x + 3)$

3. $y = \log_4(x - 1)$

4. $y = \log_{\frac{1}{2}}(x + 2)$

5. $y = \log_{\frac{1}{3}}(x + 1)$

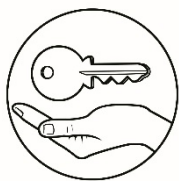
6. $y = \log_2 x - 2$

7. $y = \log_3 x + 3$

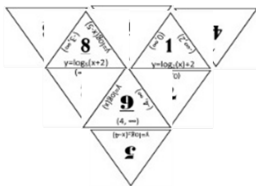
8. $y = \log_3 x - 1$

9. $y = \log_{\frac{1}{2}} x + 2$

10. $y = \log_{\frac{1}{3}} x + 1$



Answer Key

Assessment 1. c 2. a 3. d 4. b 5. c 6. a 7. d 8. b 9. c 10. a 11. b 12. d 13. a 14. c 15. d	What's More Activity 1.1  Activity 1.2 1. D 2. E 3. A 4. B 5. C Activity 1.2 1. $D = (-\infty, 3)$ $R = (-\infty, \infty)$ 2. $D = (0, \infty)$ $R = (-\infty, \infty)$ 3. $D = (-2, \infty)$ $R = (-\infty, \infty)$ 4. $D = (-1, \infty)$ $R = (-\infty, \infty)$ 5. $D = (-\infty, \frac{7}{4})$ $R = (-\infty, \infty)$	What I Know 1. c 2. a 3. b 4. d 5. a 6. c 7. d 8. b 9. a 10. c 11. a 12. d 13. a 14. c 15. b
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