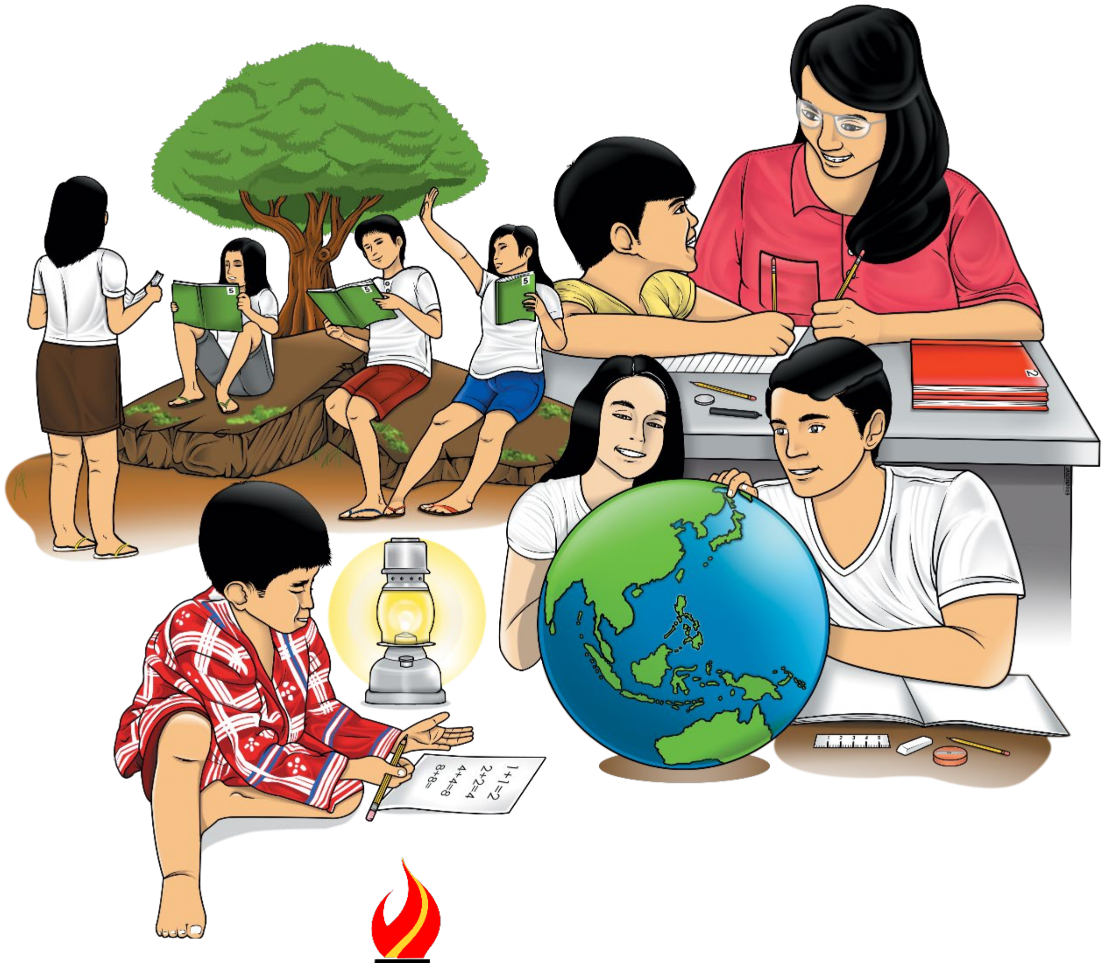
Senior High School

General Mathematics

Quarter 1 – Module 22:

Solving Real-life Problems Involving Exponential Functions, Equations, and Inequalities



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Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

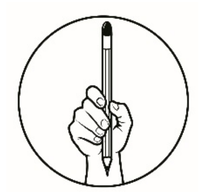
Thank you.

What I Need to Know

This module was designed and written with you in mind. It is here to help you solve real-life problems involving exponential functions, equations, and inequalities. Most of the time, students like you ask why you need to study Mathematics. Even though you know the answer, still you keep on asking this question because perhaps you did not realize how important it is to real-life situations.

This module hopes to help you make a wise decision in the future because it involves money matter problems.
After going through this module, you are expected to:

1. recall how to solve exponential functions, equations, and inequalities; and
2. solve real-life problems involving exponential functions, equations, and inequalities.



What I Know

Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. Which of the following situations can you apply solving exponential functions?
   a. calculating the area of rectangular field given the dimensions
   b. finding the age of your father if he is 15 more than thrice your age
   c. finding the number of bacteria with a growth rate of 25% after a certain period given the initial number
   d. getting the probability of a discrete random variable

2. If ₱1,000.00 is invested at the rate of 5% compounded quarterly, at the end of the year it is equal to __________________.
   a. ₱1,000.00+0.05x 4
   b. $(\text{₱}1000(1.05)^4$
   c. $(\text{₱}1000 + 0.05)^4$
   d. $\left(\text{₱}1000(1 + \frac{0.05}{4}\right)^4$

3. Which of the following formulas can be used to find the exponential growth of the population?
   a. $A = P(1 + \frac{r}{n})^{nt}$
   b. $A = P(1 + r)^t$
   c. $A = Pe^{rt}$
   d. $A = \pi r^2$

4. A bank offers you a time deposit with 7% interest compounded annually; give an exponential model for the offer if you wanted to invest ₱100,000.00 under this investment.
   a. A = ₱100,000$(1.07)^t$
   b. A = ₱100,000(1.07)t
   c. A = ₱100,000$(.07)^t$
   d. A = ₱100,000(.07)t

5. Using item no. 4 how much would your money be after 15 years?
   a. ₱250,000.00
   b. ₱275,000.00
   c. ₱275,903.15
   d. ₱375,903.15

For numbers 6 – 7, refer to the problem below:

Andy deposited an amount of ₱1,000.00 in the bank which offers 3% interest compounded annually and forgot about it due to his busy life. After 5 years, he remembered that he has money in the bank and checked his balance.

6. Which of the following is the formula to determine the total amount on his passbook after 5 years?
   a. $A = P(1 + r)^n$
   b. $A = P(1 - r)^n$
   c. $A = (1 - r)^n$
   d. $A = n(1 + r)^P$

7. How much money did Andy have on his account after 5 years?
   a. ₱1,010.96
   b. ₱1,129.24
   c. ₱1,159.27
   d. ₱1,231.05

For numbers 8 – 9, refer to the problem below:

The half-life of a radioactive substance is 3,000 years, with an initial amount of substance of 500 grams.

8. Give an exponential model of the amount remaining after t years.
   a. y = $500(1/2)^{t/3000}$
   b. y = $5{,}000(1/2)^{t/300}$
   c. y = $500(1/4)^{t/3000}$
   d. y = $5{,}000(1/4)^{t/300}$

9. What amount of substance remains after 2,000 years?
   a. 198.43 g
   b. 314.98 g
   c. 200 g
   d. 320 g

10. In the year 2010, Barangay Santolan has a population of 3,200. Its rate increases 1.05% every year. What is the population of the barangay after 3 years? (Use $P = P_0 e^{rt}$)
    a. 3,860
    b. 3,680
    c. 3,423
    d. 3,303

11. A car bought for ₱1,500,000.00 depreciates by 20% per year. After how many years can one buy the car at about half of its original price?
    a. 5 years
    b. 6 years
    c. 7 years
    d. 8 years

12. The half-life of a radioactive substance is 20 days and there are 5 grams initially. Determine the amount of substance left after 80 days.
    a. 5 g
    b. 3 g
    c. 1.25 g
    d. 0.3125 g

For numbers 13 – 14, refer to the problem below.

Gerson Joseph opened a savings account and deposited ₱15,000.00. Each year, the account increases by 5%.

13. Which of the following equations best represents the situation?
    a. $A_t = 15000(1 + 0.05)^t$
    b. $A_t = 15000(1 - 0.5)^t$
    c. $A_t = 15000(1 + 0.05)^{\frac{n}{t}}$
    d. $A_t = 15000(1 - 0.05)$

14. How many years will it take the account to reach ₱20,101.43?
    a. 5
    b. 6
    c. 7
    d. 8

15. The growth of a culture of bacteria is defined by the formula $y = 5000e^{0.03t}$, where t is the time (in days). How many bacteria will there be after two weeks?
    a. 8000
    b. 7805
    c. 7610
    d. 6705

Lesson 1 Solving Real-life Problems Involving Exponential Functions, Equations, and Inequalities

Exponential growth and decay are the common applications of the exponential functions. The population growth is modeled by an exponential function, which includes the growth of investment under a compound interest, the increase in the number of bacteria as time passes by and a lot more. In the previous module, you already learned how to represent the exponential functions to real-life situations. This module will help you gain a deeper understanding of the application of exponential functions, equations, and inequalities.

What's In

Before we proceed in solving real-life problems involving exponential functions, equations, and inequalities, let us first recall how to solve exponential equations and inequalities.

To solve exponential equations and inequalities, you should be familiar with the one-to-one property of exponential equations which state that if $x_1 \neq x_2$, then $b^{x_1} \neq b^{x_2}$. Conversely, if $b^{x_1} = b^{x_2}$ then $x_1 = x_2$. Also, you should know the property of exponential equalities:

If $b > 1$, then the exponential function $y = b^x$ is increasing for all x, which means that $b^x < b^y$ if and only if $x < y$.

If $0 < b < 1$, then the exponential function $y = b^x$ is decreasing for all x, which means that $b^x > b^y$ if and only if $x < y$.

Example 1

Solve the equation $2^{4x+2} = 64$.

Solution:

$2^{4x+2} = 2^6$

$4x + 2 = 6$

$4x = 6 - 2$

$4x = 4$

$x = 1$

Example 2

Solve the equation $\left(\frac{1}{10}\right)^{3x-1} = 1000$.

Solution:

$\left(\frac{1}{10}\right)^{3x-1} = 10^3$

$\left(\frac{1}{10}\right)^{3x-1} = \left(\frac{1}{10}\right)^{-3}$

$3x - 1 = -3$

$3x = -3 + 1$

$3x = -2$

$x = -\frac{2}{3}$

Example 3

Solve the inequality $5^x < 5^{2(x+1)}$.

Solution:

$$\boldsymbol{x < 2(x+1)}$$
$$\boldsymbol{x < 2x + 2}$$
$$\boldsymbol{-2 < 2x - x}$$
$$\boldsymbol{-2 < x}$$
$$\boldsymbol{x > -2}$$

Example 4

Solve the inequality $\left(\frac{343}{125}\right)^{x+2} \geq \frac{25}{49}$.

Solution:

$$\boldsymbol{\left(\frac{7}{5}\right)^{3(x+2)} \geq \left(\frac{5}{7}\right)^{2}}$$
$$\boldsymbol{3(x+2) \geq -2}$$
$$\boldsymbol{3x + 6 \geq -2}$$
$$\boldsymbol{3x \geq -8}$$
$$\boldsymbol{x > -\frac{8}{3}}$$

For the four examples given, I do hope you remember what you have learned in your previous module.

What's New

Read and analyze the problem below to answer the questions that follow.

What a Surprise!

Today is Alexa's birthday. Her parents want to give her a surprise, it is a savings account passbook with her name as the account holder. Her parents deposited an amount of ₱20,000.00 on the account at the time she was born. They think that it is about time for Alexa to manage her account. If you were Alexa, what would be your reaction if the passbook will be given to you as a birthday gift? Now that she is 18 years old, how much money will be in her savings account, if the money was invested with an interest of 3% compounded quarterly since the time it was deposited?

What is It

The previous activity is an example of real-life situations involving exponential function. If you doubt your answer or if you do not know what you are going to do to answer the problem above, that's okay. Well, you may read first the examples here, and you may go back to the activity after you fully understand how to solve involving compound interest. The following are the applications of exponential functions, equations, and inequalities to real-life problems.

Real-Life Problems Involving Exponential Function
Compound Interest

Example 1:

Danielle deposited ₱5,000.00 in an account that offers 6% interest compounded semi-annually. How much money is in his account at the end of three years?

The formula for compound interest is $A = P\left(1+\frac{r}{n}\right)^{nt}$
where A = final amount, P = principal or the initial amount, r = interest rate, n = number of times interest is compounded in one year, t = number of years.

Solution:

Given: $P = 5000$ $r = 6\%\ or\ 0.06$ $n = 2\ (semi-annually)$ $t = 3$

Find A.

$A = 5000\left(1+\frac{0.06}{2}\right)^{2(3)}$

$A = 5000(1+0.03)^6$

$A = 5000(1.03)^6 = 5000(1.194) = 5970.26$

Therefore, after three years, the amount of money in Danielle's account is ₱5,970.26

Note: If interest is compounded annually, $n = 1$.
If interest is compounded semi-annually, $n = 2$.
If interest is compounded quarterly, $n = 4$.
If interest is compounded monthly, $n = 12$.

Looking at this example, I believe that you are now ready to check your answer on the What's a Surprise Problem. Do you think you got it right? I believe you do.

Population Growth and Decay

In the module entitled Representing Real-Life Situations Using Exponential Functions, you encountered problems like population growth and decay. This time, you will encounter the population once again but with the concept of the natural exponential function. The natural exponential function is the function $f(x) = e^x$. (If you want to know about this number, you can read the book "e: The Story of a Number", by Eli Maor.)

Example 2:

A certain bacterium, given favorable growth conditions, grow continuously at a rate of 5.4% a day. Find the bacterial population after twenty-four hours, if the initial population was 500 bacteria.

When you read a problem that suggests growth continuously, you should be thinking "continuously-compounded growth formula". For this situation, the formula is $A = P_0e^{rt}$

where A = population after a certain period

P_0 = initial population

r = rate of change (growth rate but sometimes it is called decay rate)

t = time (growth/decay rates in contexts might be measured in minutes, hours, days, etc.)

Solution:

Given: $P = 500$ $r = 5.4\%\ or\ 0.054$ $t = 1$ day

Find A.

Note: 24 hours is converted to 1 day because the growth rate was expressed in terms of a given percentage *per day*. Thus,

$A = 500e^{0.054(1)}$

$A = 527.74$

Therefore, there will be about 528 bacteria after twenty-four hours.

Real-Problems involving Exponential Equation and Inequalities

Exponential equations and inequalities are equations and inequalities in which one or both sides involve a variable exponent. They are useful in situations involving repeated multiplication, especially when being compared to a constant value, such as in the case of interest. For instance, exponential inequalities can be used to determine how long it will take to double one's money based on a certain rate of interest.

Example 3:

Suppose that a population of a colony of bacteria increases exponentially, at the start of the experiment, there are 1000 bacteria. One hour later, the population has increased to 1200 bacteria. How long will it take for the population to reach 5000 bacteria? Round your answer to the nearest hour.

Solution.

Given: $A = 6000$ $P = 1000$ $r = \frac{200}{1000} = 0.2$

Find t.

$6000 = 1000e^{(0.2)t} \rightarrow$ (This is an exponential equation)

$\frac{6000}{1000} = \frac{1000e^{(0.2)t}}{1000}$ Multiplication Property of Equality

$6 = e^{(0.2)t}$

$\ln 6 = ln\, e^{(0.2)t}$ Changing exponential to logarithm

$\ln 6 = 0.2t$ Property of logarithm

$t = \frac{\ln 6}{0.2}$

$t = 8.96$

Therefore, it will take 8.96 hours to reach 5000 bacteria.

Example 4:

Michael owns ₱15,000.00 and he wants to invest his money into an account that will double his money. He is thinking of a financial institution that can make his dream come true. He is considering to invest his money in a lending company which offers a 15% interest compounded quarterly. For how long, will he invest his money in that company to earn at least twice as much as he has now?

Given: $A \geq 2(15000)$ (to earn at least twice as much as he has now)

$P = 15000$

$r = 15\%\ or\ 0.15$

$n = 4$

Find t. Why?

$2(15000) \geq 15000(1 + \frac{0.15}{4})^{4t} \rightarrow$ (This is an exponential inequality)

$3000015000 \geq (1 + 0.0375)^{4t}$ Simplify

$\frac{30000}{15000} \geq \frac{15000(1.0375)^{4t}}{15000}$ Multiplication Property of Equality

$2 \geq (1.0375)^{4t}$

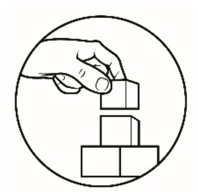
$4t \geq \log_{1.0375} 2$ Changing exponential to the logarithm

$\frac{\log 2}{\log 1.0375} = 18.83$ Change-of-base formula $log_b x = \frac{\log x}{\log b}$

$4t \geq 18.83$ Substitution

$t \geq 4.71$ Multiplication Property of Equality

Therefore, after at least 4.71 years Michael's money will be ₱30,000.00



What's More

Analyze the given problem and answer the questions that follow:

Activity 1.1

"I – Predict Mo"

1. In 2015, a certain municipality in Quezon Province has a population of 45, 300. Each year, the population increases at a rate of about 5%.

   a. What is the growth factor of the municipality?
   b. Determine an equation to represent the problem.
   c. What is the population of the municipality in 2020? Use the equation in letter (b).
   d. If the population continues to increase at the same rate, what is the population in 2025.

2. In the early stages of the COVID-19 epidemic in the Philippines, there were 50 persons infected but each day the number rose by 5%. After how many days would about 300 persons be infected?

Activity 1.2

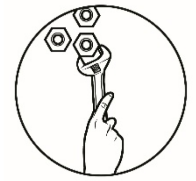
"Let's Invest!"

1. Jeanelle has ₱5,000.00 in a bank which is below her required maintaining balance. As a penalty, her money decreases at the rate of 5% every month. How much money will she have after 1 year?
2. If ₱20,000.00 is invested at 10% compounded quarterly, when will the amount of investment be tripled?
3. Mr. Jolo deposited an amount of ₱20,000.00 in a bank that gives 3% annual interest compounded monthly. How much money will he have in the bank after 4 years?

What I Have Learned

Reflect and answer the following:

1. What are the common application of exponential functions, equations, and inequalities to real-life situations?
2. In solving real-life problems involving exponential functions, equations, and inequalities, what do you think are the important skills that you should have to solve the problems? Enumerate the different steps that you should consider in solving real-life problems involving exponential functions, equations, and inequalities.



What I Can Do

Exponential Decay

Some things "decay" (get smaller) exponentially.

Example: Atmospheric pressure (the pressure of air around you) decreases as you go higher. It decreases about 12% for every 1000 m: an **exponential decay**. The pressure at sea level is about 1013 hPa (depending on weather).

Note: hPa stands for hectopascal (100 x 1 pascal) pressure

If the model that represents the situation is $y(t) = ae^{rh}$

where: a =(the pressure at sea level =1013 hPa)

$h =$ is in meters (distance, not time, but the formula still works)

$y(t) = y(1000)$ (It is a 12% reduction on 1013 $hPa = 891$ hPa)

a. Find the pressure on the roof of Grand Hyatt Hotel Manila (one of the tallest buildings in the Philippines) if its height is 1,043 feet.
b. Find the pressure on the top of Mount Pulag which is 2,922 meters tall.

Assessment

Choose the letter of the correct answer and write it on a separate sheet of paper.

1. Which of the following depicts the increase in number or size at a constantly growing rate?
    a. exponential decay
    b. exponential growth
    c. half-life
    d. time elapsed

2. Which of the following statements is best modeled by exponential growth?
    a. the cost of pencils as a function of the number of pencils
    b. the distance when a stone is dropped as a function of time
    c. the distance of a swinging pendulum bob from the center as a function of time
    d. the compound interest of an amount as a function of time

3. Lino invested ₱5,000.00 into an account that has a 5.5% annual increasing rate. What equation best describes this investment after ***t*** years?
    a. $A = 5000\ (0.055)^t$
    b. $A = 5000\ (1.055)^t$
    c. $A = 5000\ (1.55)^t$
    d. $A = 5000\ (5.5)^t$

4. If the population in 1995 of Barangay Manggahan is 1,500, and is increasing at a rate of 2.3% every 5 years, what is the projected population of the town in 2025?
    a. 2,967
    b. 1,681
    c. 1,722
    d. 1,759

For numbers 5-6, refer to the following:

Ms. Juana Care plans to invest her ₱1,000,000.00 in a company that offers 8% interest compounded annually.

5. Define an exponential model for this situation.
    a. $A = 1000000(1.08)^t$
    b. $A = 1,000,000(1.08)(t)$
    c. $A = 1000000(1.08)^{t+1}$
    d. $A = 1000000(1.08)(t)+1$

6. How much is the investment after 5 years?
    a. ₱5,400,000.00
    b. ₱5,400,001.00
    c. ₱1,586,874.32
    d. ₱1,469,328.08

7. The half-life of Zn-71 is 4.25 minutes. At t = 0, there were y_0 grams of Zn-71, but only 1/64 of this amount remains after some time. How much time has passed?
    a. t = 20.5
    b. t = 19.5
    c. t = 21.
    d. t = 25.5

8. Which of the following situations ***does not*** describe an exponential decay?
    a. the number of rabbits doubles every month
    b. the amount of substance decreases every 10 minutes
    c. the atmospheric pressure decreases as you go higher
    d. the value of a car depreciates every year

9. A photocopier is purchased for ₱15,200.00 and depreciates in value by 15% per year. Which equation best describes the value of the photocopier in ***x*** years?
    a. $y = 15200\ (0.15)^x$
    b. $y = 15200\ (0.85)^x$
    c. $y = 15200\ (1.15)^x$
    d. $y = 15200\ (1.85)^x$

10. Suppose ₱4000.00 is invested at 6% interest compounded annually. How much money will there be in the bank at the end of 5 years?
    a. ₱5,352.90
    b. ₱5,325.90
    c. ₱5,253.90
    d. ₱5,235.90

11. In 2012 the population of schoolchildren in a city was 90,000. This population increases at a rate of 5% each year. What will be the population of school children in year 2022?
    a. 148,385 school children.
    b. 150, 625 school children
    c. 165, 373 school children
    d. 190, 428 school children

For items 12-13, refer to the following:

The population of Lucena City is estimated to increase by 1.49% per year. According to 2015 census, the population of the city is 266, 248.

12. Which of the following best modelled the situation?
    a. $A = 266248e^{(0.0149)(t)}$
    b. $A = 266248(1.0149)^t$
    c. $A = 266248(\frac{1.0149}{n})^t$
    d. $A = 266248e^{(1.0149)(t)}$

13. What will be the population ten years from now?
    a. 309,027
    b. 332,929
    c. 350,456
    d. 402, 123

14. Joana earned ₱1500 last summer. If she deposited the money in a bank account that earns 5% interest compounded yearly, how much money will she have after five years?
    a. ₱2,015.35
    b. ₱1,914.42
    c. ₱1,846.48
    d. ₱3,560.15

15. If the population of a town doubles in 30 years, when will it be quadruple?
    a. in 45 years
    b. in 60 years
    c. in 90 years
    d. in 100 years

Additional Activities

Solve the following problems.

1. How much money will you have after 5 years, if you invest ₱2,000.00 at the rate of 2% compounded monthly?
2. At the start of the experiment, there are 400 bacteria. If the bacteria follow an exponential growth pattern with $r = 0.03$, what will be the population after 6 hours? How long will it take for the population to double?
3. Consider a population of bacteria that grows according to the function $f(t) = 500e^{0.05t}$, where t is measured in minutes. How many bacteria are present in the population after 4 hours?

Answer Key

What I Know

1. c
2. d
3. c
4. a
5. c
6. a
7. c
8. a
9. b
10. a
11. d
12. d
13. a
14. b
15. c

What's More

ACT 1.1:

a. 5%

b. $P = 45300e^{0.05t}$

c. 58,167

d. 74,688

ACT 1.2:

a. times 3

b. $y = y_0(3)^{\frac{t}{3}}$

c. 31,749

Activity 1.3

1. ₱2701.80

2. The amount will be tripled in 12 years

3. The money will be ₱22,546.56 in 4 years.

Assessment

1. b
2. d
3. c
4. c
5. a
6. d
7. d
8. a
9. b
10. a
11. a
12. a
13. b
14. b
15. a

References

Faylogna, Frelie T., Calamiong, Lanilyn L., Reyes, Rowena D., *General Mathematics.*Sta. Ana. Manila Philippines: Vicarish Publications and Trading. Inc. Copyright 2017. Reprinted 2018

General Mathematics Learner's Material. First Edition. 2016. pp. 77-98

ExponentialFunctions: The "Natural" Exponential "e". https://www.purplemath.com/modules/expofcns5.htm

Exponential Inequalities. https://brilliant.org/wiki/exponential-inequalities/?quiz=exponential-inequalities-same-base#_=_

Exponential Growth and Decay. https://www.mathsisfun.com/algebra/exponential-growth.html

Lucena City Population. https://www.citypopulation.de/php/philippines-luzon-admin.php?adm2id=045624

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