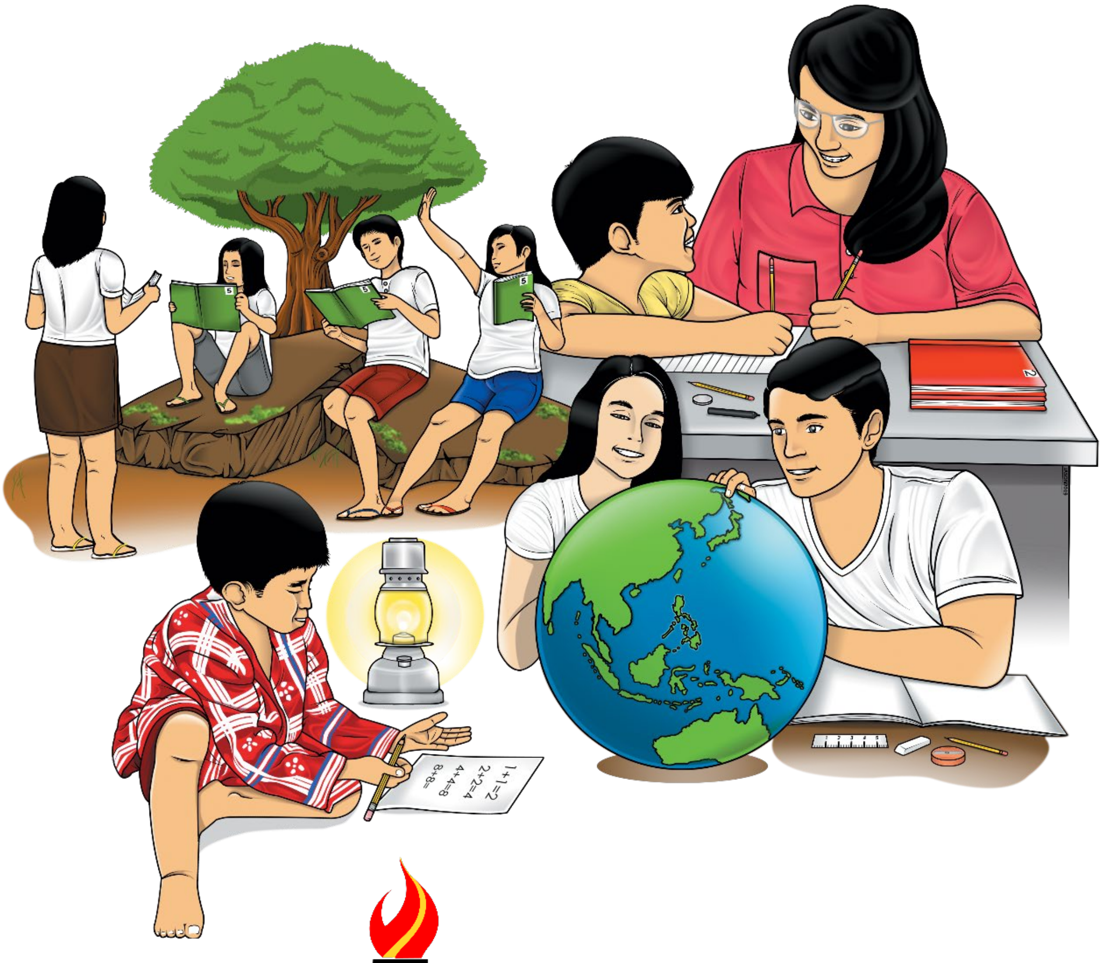
Senior High School

General Mathematics

Quarter 1 – Module 21: Intercepts, Zeroes and Asymptotes of Exponential Functions



ALTERNATIVE DELIVERY MODE

CO_Q1_General Mathematic SHS
Module 21

General Mathematics
Alternative Delivery Mode
Quarter 1 – Module 21: Intercepts, Zeroes and Asymptotes of Exponential Functions
First Edition, 2021

Published by the Department of Education
Secretary: Leonor Magtolis Briones
Undersecretary: Diosdado M. San Antonio

Development Team of the Module

Writers: Azalea A. Gallano, Maria Corazon C. Tolentino

Editors: Elizabeth D. Lalunio, Anicia J. Villaruel, Roy O. Natividad

Reviewers: Jerry Punongbayan, Diosmar O. Fernandez, Dexter M. Valle, Celestina M. Alba, Jerome A. Chavez, Emerita R. Marquez, Angelica P. Beriña, Divina O. Ella, Dennis E. Ibarrola, Erlene E. Barandino, Jadeth D. Bendal

Illustrators: Hanna Lorraine G. Luna, Diana C. Jupiter

Layout Artists: Sayre M. Dialola, Roy O. Natividad, Gil A. Alayan

Management Team: Francis Cesar B. Bringas, Job S. Zape, Jr.
Ramonito Elumbaring, Reicon C. Condes
Elaine T. Balaogan, Fe M. Ong-ongowan
Elias A. Alicaya Jr., Hermoneges M. Panganiban
Gregorio A. Co Jr., Phillip B. Gallendez
Gregorio T. Mueco, Herbert D. Perez
Lorena S. Walangsumbat, Josephine T. Natividad
Jee-Ann O. Borines, Anicia J. Villaruel
Asuncion C. Ilao, Dexter M. Valle

Printed in the Philippines by ________________________

Department of Education – Region 4A CALABARZON

Office Address: Gate 2 Karangalan Village, Brgy. San Isidro, Cainta, Rizal
Telefax: 02-8682-5773/8684-4914/8647-7487
E-mail Address: lrmd.calabarzon@deped.gov.ph

Senior High School

General Mathematics

Quarter 1 – Module 21: Intercepts, Zeroes and Asymptotes of Exponential Functions

DepED
DEPARTMENT OF EDUCATION

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.

What I Need to Know

This module was designed and written with you in mind. It is here to help you master the different ways to determine the zeroes, intercepts, and asymptotes of exponential functions. The scope of this module permits it to be used in many different learning situations. The language used recognizes the diverse vocabulary level of students. The lessons are arranged to follow the standard sequence of the course. But the order in which you read them can be changed to correspond with the textbook you are now using.

After going through this module, you are expected to:

1. determine zeroes of an exponential function; and
2. determine intercepts and asymptotes of an exponential function given the graph of an exponential function.

What I Know

Test yourself on the topics to be discussed in this module. Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. Simplify the expression $3^{(x+3)}$.
   a. $3^{(x+3)} = 3^x \bullet 3^3 = 27(3^x)$
   b. $3^{(x+3)} = 3^{2x} \bullet 3^3 = 27(3^{3x})$
   c. $3^{(x+3)} = 3^{x+3} \bullet 3^3 = 27(3^{x+3})$
   d. $3^{(x+3)} = 3^{x/3} \bullet 3^3 = 27(3^{x/2})$

2. Where should the y-intercept of the graph of the function $b^x a^x$ be?
   a. The y-intercept is at (0,b)
   b. The y-intercept is at (0,0)
   c. The y-intercept is at (b,0)
   d. The y-intercept is at (b,b)

3. For what values of x is the function $f(x) = 3^x$ less than 1?
   a. $f(x) < 1$ for all $x < 1$
   b. $f(x) > 1$ for all $x < 1$
   c. $f(x) > 1$ for all $x < -1$
   d. $f(x) < 1$ for all $x < -1$

4. Where do the graphs of $y = a^x$ and $y = a^{-x}$ intersect?
    a. They intersect at the point (0,0)
    b. They intersect at the point (1,0)
    c. They intersect at the point (1,1)
    d. They intersect at the point (0,1)

5. Which describes the functions $f(x) = 3^x$ and $f(x) = (\frac{1}{3})^x$?
    a. $f(x) = (\frac{1}{3})^x$ decreases as x decreases and increases as x increases.
       $f(x) = (3)^x$ increases as x decreases and decreases as x increases.
    b. $f(x) = (\frac{1}{3})^x$ increases as x decreases and decreases as x increases.
       $f(x) = (3)^x$ decreases as x decreases and increases as x increases.
    c. $f(x) = (\frac{1}{3})^x$ increases as x decreases and decreases as x increases.
       $f(x) = (3)^x$ does not decrease as x increases and decreases as x increases.
    d. $f(x) = (\frac{1}{3})^x$ does not increase as x decreases and does not decrease as x increases.
       $f(x) = (3)^x$ increases as x decreases and decreases as x increases.

6. If $3^x = 3^8$, what is x?
    a. -4
    b. -2
    c. 6
    d. 8

7. Find x if $2^{x-1} = 8$.
    a. 4
    b. 3
    c. 2
    d. 1

8. Find the zero of $h(x) = 2^{x-3} – 4$.
    a. -3
    b. 0
    c. 5
    d. 7

9. What value of x will make the function $y = 2^{3x} – 1$ equal to 0?
    a. 2
    b. 1
    c. 0
    d. -1

10. Determine the zeroes of the exponential function $f(x) = 2^x$.
    a. (0,-1)
    b. (0,-2)
    c. (0,2)
    d. no zero

11. The graph of a function of the form y = a^x passes through which of the following points?
  a. (-1, 0)
  b. (1, 0)
  c. (0, 1)
  d. (0, -1)

12. Which of the statements is best modeled by exponential growth?
  a. The cost of pencils as a function of the number of pencils.
  b. The compound interest of an amount as a function of time.
  c. The distance when a stone is dropped as a function of time.
  d. The distance of a swinging pendulum bob from the center as a function of time.

For numbers 13 to 15, please refer to the given function y = $\left(\frac{1}{3}\right)^x - 2$.

13. Which of the following is the y-intercept?
  a. -1
  b. -2
  c. 1
  d. 2

14. What can you say about the trend of the graph?
  a. decreasing
  b. either increasing or decreasing
  c. increasing
  d. no conclusion can be made

15. Which of the following is the horizontal asymptote?
  a. $y = -1$
  b. $y = -2$
  c. $y = 1$
  d. $y = 2$

Lesson 1 Intercepts, Zeroes, and Asymptotes of Exponential Functions

In the previous lessons, you learned how to determine domain and range of an exponential function. You were to only consider cases of exponential functions where *P(x)* is linear, in which case, $b^{P(x)}$ will always be defined for any value of *x*. Thus, the domain of an exponential function is the set of real numbers or R. For the range, note that $b^{P(x)} > 0$ for any value of *x*. hence, the range of an exponential function will depend on a and h.

What's In

To fully understand the topic, you need to recall on the laws of exponents, pay attention to the properties of exponential function, know the application of those laws and properties and be able to distinguish one from the other. Be patient enough to practice more in enhancing your skills. Keep in mind that an exponential function is different from other functions as its exponent is a variable.

Let us review the laws of exponents and the properties of equality for exponential equation.

Laws of Exponents

For any real numbers, a and b and any positive real numbers m and n,

a. $a^m a^n = a^{m+n}$
b. $(a^m)^n = a^{mn}$
c. $(ab)^n = a^n b^n$
d. $\frac{a^m}{a^n} = a^{m-n}, a \neq 0$
e. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$
f. $a^0 = 1$

Simplify each expression and express the answers with positive exponents.

Illustrations:

1. $x^3x^5 = x^{3+5}$

$= x^8$

2. $(x^{-2})^3 = x^{(-2)(3)}$

$= x^{-6}$

$= \frac{1}{x^6}$

3. $(2x^{-5})^{-3} = 2^{-3}x^{15}$

$= \frac{x^{15}}{2^3}$

$= \frac{x^{15}}{8}$

4. $\frac{16x^5}{12x^7} = \frac{16}{12} \bullet \frac{x^5}{x^7}$

$= \frac{4}{3} \bullet x^{5-7}$

$= \frac{4}{3} \bullet x^{-2}$

$= \frac{4}{3} \bullet \frac{1}{x^2}$

$= \frac{4}{3x^2}$

5. $\left[\frac{64x^{\frac{1}{3}}}{125x^{-\frac{2}{3}}}\right]^{\frac{2}{3}}$

$\left[\frac{64x^{\frac{1}{3}}}{125x^{-\frac{2}{3}}}\right]^{\frac{2}{3}} = \left[\frac{64}{125}\right]^{\frac{2}{3}} \bullet \left[\frac{x^{\frac{1}{3}}}{x^{-\frac{2}{3}}}\right]^{\frac{2}{3}}$

$= \left[\sqrt[3]{\frac{64}{125}}\right]^2 \bullet \left[x^{\frac{1}{3}\bullet\frac{2}{3}}\right]^{\frac{2}{3}}$ Express rational exponents in radical form and simplify

$= \left[\frac{4}{5}\right]^2 \bullet \left[x^{\frac{3}{3}}\right]^{\frac{2}{3}}$

$= \left[\frac{16}{25}\right] \bullet [x^1]^{\frac{2}{3}}$

$= \frac{16x^{\frac{2}{3}}}{25}$

Suppose you were asked to solve for the value of the variable that would make the equation true, how are you going to begin the task? So, to help you with this matter, let us recall what you have learned previously.

The Property of Equality for Exponential Equation

An exponential equation in one variable is an equation where the variable is an exponent.

In solving exponential equations, the property of equality for exponential equation also known as equating-exponents property implies that, if the bases are equal, the exponents must also be equal.

This could also be stated as follows,

"If a, b and c are real numbers and a ≠ 0, then $a^b = a^c$ if and only if b = c."

Examples: Solve for the value of the variable that would make the equation true.

1. $2^x = 2^4$ Since the bases are equal,
   $x = 4$ the exponents must be equal too.
   Since the $x = 4$, then $2^x = 2^4$.

2. $3^{4y} = 3^{16}$ Since the bases are equal,
   $4y = 16$ the exponents must be equal too.
   $y = 4$
   Since $y = 4$, then $3^{4y} = 3^{16} \rightarrow 3^{4(4)} = 3^{16} \rightarrow 3^{16} = 3^{16}$

3. $5^6 = 5^{x-2}$ The bases are equal,
   $6 = x - 2$ the exponents must be equal too.
   $x = 8$
   Since $x = 8$, then $5^6 = 5^{x-2} \rightarrow 5^6 = 5^{8-2} \rightarrow 5^6 = 5^6$

Use laws of exponents to solve to make the bases equal. Then apply the Equating-Exponents Property.

Solve the equation $2^{x-1} = 8$.
Solution: Write both sides with 2 as the base.

$2^{x-1} = 8$
$2^{x-1} = 2^3$
$x - 1 = 3$ By the additive inverse property
$x = 4$

Finding the Roots of Exponential Equation

1. Solve the exponential equation $2^{4x-1} = 8^{x-2}$.
   Solution: Use laws of exponents to make the bases equal. Then apply the Equating-Exponents.
   $2^{4x-1} = 8^{x-2}$
   $2^{4x-1} = 2^{3(x-2)}$
   $4x - 1 = 3(x - 2)$
   $4x - 1 = 3x - 6$
   $x = -5$
2. Solve the exponential equation $2^{x^2-5x} = \frac{1}{16}$.
   $2^{x^2-5x} = \frac{1}{16}$
   $2^{x^2-5x} = 16^{-1}$
   $2^{x^2-5x} = (2^4)^{-1}$
   $2^{x^2-5x} = 2^{-4}$
   $x^2 - 5x = -4$
   $x^2 - 5x + 4 = 0$
   $(x - 1)(x - 4) = 0$
   $x = 1$ or $x = 4$

Notes to the Teacher

Provide solutions for problems which will not be correctly solved by the learners. Students should have a mastery of the simplifying exponential expressions, laws of exponents, properties of equality and finding the roots of exponential equations.

What's New

Who Says Who?

Maria Corazon C. Tolentino

What could go wrong if my mind explodes?
The absence of my "x" that left my side,
to completely heal my heart.
There could have been us but if not "asymptote" decides,
numbers and variables collide
and my mind might collapse.

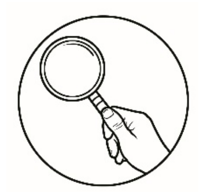
Who says who?
Exponents could be bossy too.
While base awaits, raise to power too.
It's just that my heart wants to subside,
from this pool of miseries of confusion.
Even inspiration is a piece of cake,
to cater to my mind's undecided state.

Who says who?
Nothing is yet to decide.
My "*x*" or your "*y*", who could be it now?
Absolute affection is indeed my direction,
To value the things in my perception.

Hey, you, who brings my heart,
be positive enough to my delight.
Can I conquer my fear without you at my sight?
Be brave, be brave, My heart!

Take note. not everyone shares the same idea as you have. We all have different ways of dealing with our problems. What might be easy for me might be difficult for you? You might be afraid to try learning this topic. Be not afraid.

Little by little, things that do not matter to us are the most essential for all you know. This poem reaches out to your inner self. The same with this module. It is talking to you as if it is your friend. Try to test your limit and appreciate that no matter what happens, you can always go back to basic. As there are new lessons that will be introduced to you in this module, try to think and learn this poem by heart, for it will lead you to the right path.



What is It

The following lesson will unlock the concepts on the properties of the exponential function. If you could notice, the exponential function has a great connection in Algebra, in Trigonometry, Calculus, all Sciences and Mathematics, and so on. There may be things that are still unclear to you, but the idea is for you to stay focused on what are the properties of an exponential function. What should you know before taking this module? How will you be able to find the intercepts, zeroes, and asymptote of an exponential function? How should you apply knowledge of these topics in real-life situation? How should you react to each of the examples given and discuss what you have learned to a partner?

The focus of this module is on determining intercepts, zeroes and asymptotes of an exponential function.

Determining the Zeroes of Exponential Equation

The zero of an exponential function refers to the value of the independent variable x that makes the function 0. Graphically it is the abscissa of the point of intersection of the graph of the exponential function and the x-axis. To find the zero of an exponential function $f(x)$, equate $f(x)$ to 0 and solve for x.

Examples:

Determine the zero of the given exponential function.

1. $f(x) = 3^x$

   Solution:

   To find the zero of the function, equate it to 0 and solve for x.

   $$f(x) = 3^x = 0$$
   $$3^x = 0$$

   The resulting equation suggests that $f(x)$ has no zero since no real value of x will make $3^x = 0$ a true statement.

2. $g(x) = 5^{3x-12} - 1$

   Solution:

   To find the zero of the function, equate it to 0 and solve for x.

   $$g(x) = 5^{3x-12} - 1 = 0$$
   $$5^{3x-12} - 1 = 0$$
   $$5^{3x-12} = 1$$
   $$5^{3x-12} = 5^0$$
   $$3x - 12 = 0$$
   $$3x = 12$$
   $$x = 4$$

   The zero of $g(x)$ is 4.

3. $h(x) = \left(\frac{1}{2}\right)^{3x+5} - 8$

Solution:

To find the zero of the function, equate it to 0 and solve for x.

$$h(x) = \left(\frac{1}{2}\right)^{3x+5} - 8 = 0$$
$$\left(\frac{1}{2}\right)^{3x+5} - 8 = 0$$
$$\left(\frac{1}{2}\right)^{3x+5} = 8$$
$$\left(\frac{1}{2}\right)^{3x+5} = 2^3$$
$$(2^{-1})^{3x+5} = 2^3$$
$$-3x - 5 = 3$$
$$-3x = 8$$
$$x = -\frac{8}{3}$$

The zero of $h(x)$ is $-\frac{8}{3}$.

4. $y = 4^{3x+2} - \left(\frac{1}{256}\right)^{2x-1}$

Solution:

To find the zero of the function, equate it to 0 and solve for x.

$$y = 4^{3x+2} - \left(\frac{1}{256}\right)^{2x-1} = 0$$
$$4^{3x+2} - \left(\frac{1}{256}\right)^{2x-1} = 0$$
$$(2^2)^{3x+2} = (256^{-1})^{2x-1}$$
$$2^{6x+4} = [(2^8)^{-1}]^{2x-1}$$
$$2^{6x+4} = (2^{-8})^{2x-1}$$
$$2^{6x+4} = (2)^{-16x+8}$$
$$6x + 4 = -16x + 8$$
$$22x = 4$$
$$x = \frac{2}{11}$$

The zero of y is $\frac{2}{11}$.

Intercepts of an Exponential Function

The y-intercept is a point at which the graph crosses the y-axis. The x-value is always at zero. When you want to find the intercepts from an equation, let the y-value equal to zero, then solve for x.

Examples:

1. Find the x-intercept and y-intercept of $y = 4^{x+1} - 2$.

Solution:

To find the y-intercept, let x = 0, then by substitution, we have

$y = 4^{x+1} - 2$
$y = 4^{0+1} - 2$
$y = 4^{1} - 2$
$y = 2.$

Then, the y-intercept is at (0, 2).

To find the x-intercept, let y = 0, then by substitution, we have

$y = 4^{x+1} - 2$
$0 = 4^{x+1} - 2$
$2 = 4^{x+1}$
$2^1 = (2^2)^{x+1}$
$2^1 = (2)^{2(x+1)}$
$1 = 2(x + 1)$
$1 = 2x + 2$
$1 - 2 = 2x$
$-1 = 2x$
$\frac{-1}{2} = x$

Thus, the x-intercept is at $(\frac{-1}{2}, 0)$.

2. Find the x-intercept and y-intercept of $y = 2^x - 64$.

Solution:

To find the y-intercept, let x = 0, then by substitution, we have

$y = 2^x - 64$
$y = 2^0 - 64$
$y = 1 - 64$
$y = -63.$

Then, the y-intercept is at (0, -63).

To find the x-intercept, let y = 0, then by substitution, we have

$y = 2^x - 64$
$0 = 2^x - 64$
$64 = 2^x$
$2^6 = 2^x$
$6 = x$

Thus, the x-intercept is at (6, 0).

3. Find the x-intercept and y-intercept of $y = 3.2^x + 8$.

Solution:

To find the y-intercept, let x = 0, then by substitution, we have

$y = (3.2)^x + 8$
$y = (3.2)^0 + 8$
$y = 1 + 8$
$y = 9$

Then, the y-intercept is at (0, 9).

To find the x-intercept, let y = 0, then by substitution, we have

$$y = 3.2^x + 8$$
$$0 = 3.2^x + 8$$
$$-8 = 3.2^x$$
$$-2^3 = 3.2^x$$

Since no way could make their bases equal, there is no x-intercept.

4. Find the x-intercept and y-intercept of $f(x) = -2(0.3^{2x+1}) + 4$.

Solution:

To find the y-intercept, let x = 0, then by substitution, we have

$$f(x) = -2(0.3^{2x+1}) + 4$$
$$f(0) = -2(0.3^{2(0)+1}) + 4$$
$$f(0) = -2[\left(\left(\frac{3}{10}\right)^{-1}\right)^1] + 4$$
$$f(0) = -2\left(\frac{3}{10}\right)^{-1} + 4$$
$$f(0) = -2\left(\frac{10}{3}\right) + 4$$
$$f(0) = \left(\frac{-20}{3}\right) + 4$$
$$f(0) = -\frac{8}{3}$$

Then, the y-intercept is at $(0, -\frac{8}{3})$.

To find the x-intercept, let y = 0, then by substitution, we have

$$f(x) = -2(0.3^{2x+1}) + 4$$
$$y = -2(0.3^{2x+1}) + 4$$
$$0 = -2(0.3^{2x+1}) + 4$$
$$4 = -2(0.3^{2x+1})$$
$$2^2 = -2(0.3^{2x+1})$$

Since no way could make their bases equal, there is no x-intercept.

Asymptotes of an Exponential Function Given by a Graph

A line that a curve approaches arbitrarily closely is an asymptote. An asymptote may be vertical, oblique or horizontal. Horizontal asymptotes correspond to the value the curve approaches as x gets very large or very small.

With the help of a table of values and a graph you can determine the asymptote of an exponential function. Now, take a look at the properties of the function $f(x) = 2^x$. In this case, a = 1, $P(x) = x$, and h = 0.

Assign integer values to x and find the corresponding values of $f(x)$.

For $x \geq 0$:

x	0	1	2	3	4	5	6	7	8	9
$f(x)$	1	2	4	8	16	32	64	128	256	512

Please take note, that as x increases, the value of $f(x)$ keeps on increasing rapidly.

For x < 0:

x	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10
$f(x)$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{128}$	$\frac{1}{256}$	$\frac{1}{512}$	$\frac{1}{1,024}$

Observe that as the value of x decreases, the value of $f(x)$ decreases as well. Notice that when x is negative and decreasing, the value of the function approaches zero. Thus, the graph has $y = 0$ as a horizontal asymptote. (Note: You will learn more about graphing an exponential function on another module.)

One property of the graph is that it passes the point (0, 1) or the graph has its y-intercept = 1.

Let us take this next example. Suppose $f(x) = 2^x + 2$. Our table of values in this case is as shown below.

x	-3	-2	-1	0	1	2	3
$f(x)$	$2\frac{1}{8}$	$2\frac{1}{4}$	$2\frac{1}{2}$	3	4	6	10

Comparing the two, we can see that the graph of $f(x) = 2^x + 2$ is shifted up by two units that of the $f(x) = 2^x$ on the graph.

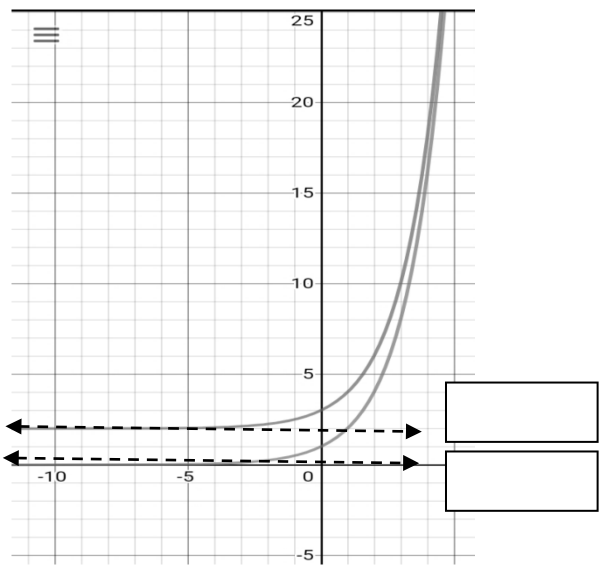


Figure 1. Asymptotes of $f(x) = 2^x$ and $f(x) = 2^x + 2$

From the graph, you can see that the horizontal asymptote of $f(x) = 2^x$ is $y = 0$, while the horizontal asymptote of $f(x) = 2^x + 2$ is $y = 2$

To help you understand more on this topic, here are some more samples for you to try. (Hint: Observe the value of d in the exponential function $f(x) = a \cdot b^x + d$)

Determine the asymptote of the following:

a. $f(x) = 5^x$

Solution: The asymptote is at $y = 0$.

b. $f(x) = 3^x + 2$.

Solution: The asymptote is at $y = 2$.

c. $y = 3^{x+2} - 5$.

Solution: The asymptote is at $y = -5$.

d. $y = -2 + 3^x$.

Solution: The asymptote is at $y = -2$.

e. $f(x) = 4^{x-3}$.

Solution: The asymptote is at $y = 0$.

Did you get the technique? That is, if you are given an exponential function it has a horizontal asymptote always. A function of the form f(x) = a(b^x)+c always has a horizontal asymptote at y = c. Now, if there are still confusing processes to you, do not hesitate to go back once again and verify the answers. Don't be hesitant to ask for help from your teacher. Have a happy attitude to get you where you want to be.

What's More

Activity 1.1

Solve for the zero of each exponential function below if it exists. Write your solutions and answers on a separate sheet of paper.

1. $f(x) = 14^x - 1$
2. $g(a) = -3^a + 27$
3. $h(x) = \left(\frac{1}{2}\right)^x - \frac{1}{8}$
4. $f(x) = 4^x$
5. $h(b) = -2(2^{b+3}) + 8$

Activity 1.2

Solve for the y-intercept of each exponential function below. Write your solutions and answers on a separate sheet of paper.

1. $f(c) = 3^c$
2. $g(x) = -\left(\frac{1}{3}\right)^x$
3. $h(x) = 5(2^x)$
4. $f(d) = 7^{d+2} - 1$
5. $g(x) = -6(2^{2x+3}) + 4$

Activity 1.3

Identify the asymptote of each exponential function below. Write your answers on a separate sheet of paper.

1. $g(x) = -7^x$
2. $h(x) = \left(\frac{3}{4}\right)^x$
3. $f(x) = 2^x + 5$
4. $g(z) = -4^{z-1} + 1$
5. $h(k) = \left(\frac{1}{2}\right)^k - 3$

Activity 1.4

Complete the table below.

exponential function	y-intercept	zero	asymptote
1. $f(x) = 3^x$			
2. $g(x) = -3^x$			
3. $g(x) = \left(\frac{1}{3}\right)^x - 1$			
4. $h(x) = 2(3^x) - 18$			
5. $h(x) = 81 - 3^{x+1}$			
6. $g(x) = 3^x + 1$			
7. $f(x) = -2\left(\frac{1}{3}^x\right)$			
8. $g(x) = 2\left(\frac{1}{3}^x\right) + 1$			
9. $f(x) = \left(\frac{1}{4}\right)^{x+2} + 3$			
10. $f(x) = -3^{2x+1} + 2$			

What I Have Learned

This time complete the statements below.

1. To solve for the y-intercept of $f(x) = a(b^{x-c}) + d$, replace _____ with 0, and solve for _____.
2. To solve for the zero of $f(x) = a(b^{x-c}) + d$, replace _____ with 0, and solve for _____.
3. If the range of $f(x) = a(b^{x-c}) + d$ is $(d, +\infty)\ or\ y > d$, the equation of the asymptote is y=_____.
4. If the range of $f(x) = a(b^{x-c}) + d$ is $(-\infty, d)\ or\ y < d$, the equation of the asymptote is y=_____.
5. Regardless of the value of _____ in $f(x) = b^x$, there is _____ zero of a function. Meanwhile, the y-intercept is _____ and the asymptote's equation is _____.

What I Can Do

The population growth model in a certain city is defined by the exponential function $f(x) = 20(1.5)^x$ where x is the number of years and f(x) is the population. Determine the intercept, zero and asymptote of the function if there is any. Interpret each. Use a separate sheet of paper for your answer.

	3	**2**	**1**
Proficiency **(score x 2)**	all 3 properties of the function are identified including both properties with or without values	2 properties are identified	0 or 1 property is identified
Interpretation **(score x 3)**	interprets properties of an exponential function in relation to the problem	explains the properties apart from their application to the problem	No attempt to explain or interpret the properties

Assessment

Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. What will you find if zero is substituted to x-variable of an exponential function?
    a. asymptote
    b. domain
    c. y-intercept
    d. zero

2. What will you find if zero is substituted to y-variable of an exponential function?
    a. asymptote
    b. domain
    c. y-intercept
    d. zero

3. Which determines the equation of the asymptote in $f(x) = a(b^{x-c}) + d$?
    a. a
    b. b
    c. c
    d. d

4. Which of the following is not true about $f(x) = -5^x$?
    a. There is no zero.
    b. The y-intercept is 1.
    c. The x-intercept is zero.
    d. The asymptote is $y = 0$.

5. What is the zero of $f(x) = 2^x - 8$?
    a. 0
    b. 1
    c. 2
    d. 3

6. What is the y-intercept of $f(x) = -\left(\frac{5}{12}\right)^x$?
    a. -1
    b. 0
    c. $\frac{5}{12}$
    d. 1

7. What is the y-intercept of $g(x) = -6^{x+1} + 1$?
    a. -5
    b. -1
    c. 1
    d. 7

8. What is the asymptote of $g(x) = 2^x + 7$?
    a. x = 2
    b. x = 7
    c. $y = 2$
    d. $y = 7$

9. Which of the following is true about $h(x) = 3^x - 9$?
    a. Its zero is -2.
    b. Its y-intercept is -8.
    c. Its asymptote is $y = 9$.
    d. It is a decreasing function.

10. Which is/are similar among $f(x) = 2^x, g(x) = 4^x \; and \; h(x) = 7^x$?
    a. asymptotes
    b. y-intercepts
    c. both a and b
    d. none

11. Which property is not the same for all the following functions:
    $f(x) = \quad 2^x, g(x) = -2^x, h(x) = \left(\frac{1}{2}\right)^x \; and \; j(x) = -\left(\frac{1}{2}\right)^x$?
    a. asymptotes
    b. range
    c. y-intercepts
    d. zeroes

12. Which is not the same for all the functions: $f(x) = 5^x, g(x) = 5^x + 1, h(x) = 5^x - 2$?
    a. asymptotes
    b. x-intercepts
    c. y-intercepts
    d. zeroes

13. Which is not true for $f(x) = 7^{x+1} \; to \; g(x) = -7^{x+1} \; and \; h(x) = 2(7^{x+1})$?
    a. Each exponential function has no zero.
    b. The y-intercepts are 7, -7 and 14, respectively.
    c. The range of each exponential function is $y > 7$.
    d. The asymptote of each exponential function is $y = 0$.

14. Which is true about $f(c) = 4(2^c) - 8$?
    a. The zero is the same as the zeroes of $g(c) = 4(2^c)$.
    b. The asymptote is the same as the asymptote of $g(c) = 4(2^c)$.
    c. The asymptote is the same as the asymptote of $h(c) = (2^c) - 8$.
    d. The y-intercept is the same as the y-intercept of $h(c) = (2^c) - 8$.

15. Which is not true about $f(x) = -4(2^{x+3}) - 16$
    a. The y-intercept is 48.
    b. Its asymptote is $y = -16$.
    c. Its domain is the set of real numbers.
    d. The zero of the exponential function is -1.

Additional Activities

Supply each set of exponential functions in the table below with correct data. Write also your observations about the similarities and differences in the features of each set of exponential functions, if there is any.

	y-intercept	**zero**	**asymptote**	**observations**
Set A				
$f(x) = 2^x$				
$g(x) = -2^x$				
Set B				
$f(x) = 3^x$				
$g(x) = \left(\frac{1}{3}\right)^x$				
Set C				
$f(x) = 4^x$				

$g(x) = 4^x + 1$				
$h(x) = 4^x - 1$				
Set D				
$f(x) = 2^x$				
$g(x) = 2^{x+1}$				
$h(x) = 2^{x-1}$				

Answer Key

What I Know

1. a
2. a
3. d
4. d
5. b
6. d
7. a
8. b
9. c
10. d
11. c
12. b
13. a
14. a
15. b

What's More

Activity 1.1

1. 0
2. 3
3. 3
4. None
5. -1

Activity 1.2

1. 1
2. -1
3. 5
4. 48
5. -44

Activity 1.3

1. y=0
2. y=0
3. y=5
4. y=1
5. y=-3

Activity 1.4

	y-intercept	Zero	asymptote
1	1	None	y=0
2	-1	None	y=0
3	2	0	y=-1
4	1	2	y=-18
5	-5	3	y=81

Assessment

1. c
2. d
3. d
4. c
5. d
6. a
7. a
8. d
9. b
10. c
11. c
12. a
13. c
14. c
15. a

References

Verzosa, Debbie Marie, et.al. *General Mathematics: Learner's Material, First Edition.* Philippines: Lexicon Press Inc. 2016.

Cox, Janelle (2020). Sample EssayRubric for Elementary Teachers. https://www.thoughtco.com/essay-rubric-2081367

General Mathematics Learner's Material. First Edition. 2016. pp. 88-96

*DepED Material: General Mathematics Learner's Material

For inquiries or feedback, please write or call:

Department of Education - Bureau of Learning Resources (DepEd-BLR)

Ground Floor, Bonifacio Bldg., DepEd Complex
Meralco Avenue, Pasig City, Philippines 1600

Telefax: (632) 8634-1072; 8634-1054; 8631-4985

Email Address: blr.lrqad@deped.gov.ph * blr.lrpd@deped.gov.ph