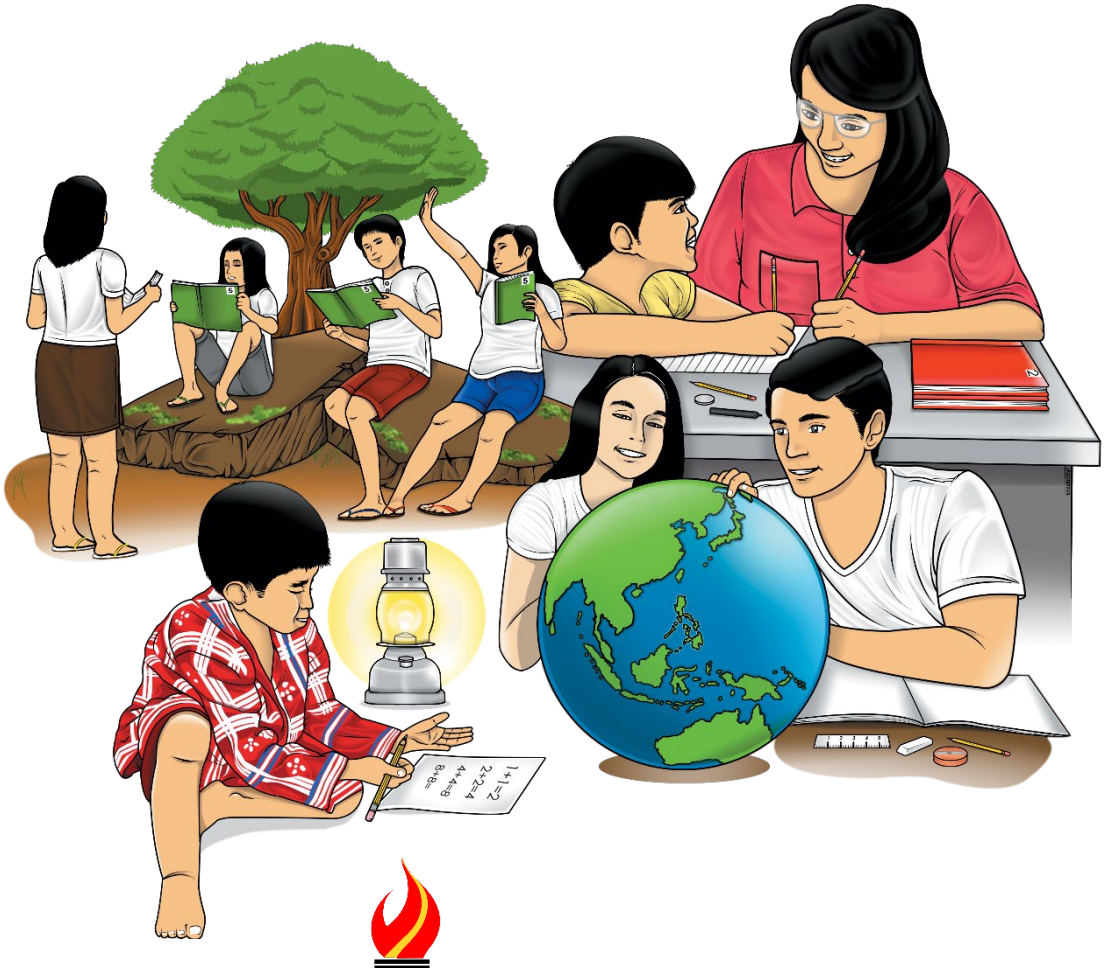
Senior High School

DepED
DEPARTMENT OF EDUCATION

General Mathematics

Quarter 1 – Module 19: Representations of Exponential Functions



ALTERNATIVE DELIVERY MODE
ADM

General Mathematics
Alternative Delivery Mode
Quarter 1 – Module 19: Representations of Exponential Functions
First Edition, 2021

Published by the Department of Education
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Undersecretary: Diosdado M. San Antonio

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Printed in the Philippines by _____

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Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

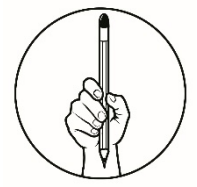
Thank you.

What I Need to Know

This module was designed and written with you in mind. It is here to help you master the representations of exponential functions through the table of values, graphs, and equations. The scope of this module permits it to be used in many different learning situations. The language used recognizes the diverse vocabulary level of students. But the order in which you read them can be changed to correspond with the textbook you are now using.

After going through this module, you are expected to:

1. define the exponential function; and
2. demonstrate how to represent exponential function using table of values, graphs, and equations.



What I Know

Read and analyze each item carefully. Circle the letter that corresponds to your answer for each statement.

1. In the function, $f(x) = b^x$, x is called _____.
   a. base
   b. coefficient
   c. exponent
   d. intercept

2. Which of the following is **NOT** an application of exponential function?
   a. bacterial growth
   b. compound interest
   c. monthly salary
   d. population decline

3. Which of the following is an example of exponential function?
   a. $f(x) = x^2 + 1$
   b. $f(x) = \frac{2}{x}$
   c. $f(x) = 4^{x+3}$
   d. $f(x) = 3x^2$

4. The following are examples of exponential function **except one**.
   a. $f((x) = 5^{-2x}$
   b. $f(x) = 10^{x+3}$
   c. $f(x) = 3^{3x}$
   d. $f(x) = (x^{-2})^{-2}$

5. In exponential function, the function has the value of 1 when the exponent is equal to _____.
   a. 0 c. -1
   b. 1 d. none of the choices

6. The function $f(x) = 2^x$ is an increasing function. The function $f(x) = 2^{-x}$ is _____ function.
   a. increasing
   b. decreasing
   c. either increasing or decreasing
   d. no conclusion can be made from the given

7. In the function $f(x) = 2^x$, $x < 0$, the x-axis becomes the _____ of the graph.
   a. intercept b. asymptote c. base d. slope

8. The domain of the function $f(x) = 2^x$ is the set of _____.
   a. negative real numbers c. zero
   b. positive real numbers d. set of real numbers

9. The range of the function $f(x) = 2^x$ is the set of _____.
   a. negative real numbers c. zero
   b. positive real numbers d. set of real numbers

10. The function $f(x) = 4^{-x}$ is the same as the function _____.
   a. $f((x) = \left(\frac{1}{4}\right)^x$ c. f(x) = -4x
   b. $f(x) = \left(\frac{1}{2}\right)^x$ d. $f(x) = \left(\frac{1}{4}\right)^{-x}$

11. In the function $f(x) = 2(1.25)^x$, find f(x) if x = 0, 1, 2.
   a. f(x) = 1.6, 2, 2.5 c. f(x) = 2, 2.5, 3.125
   b. f(x) = 1.28, 1.6, 2 d. f(x) = 2.5, 3.125, 3.906

12. In order to determine if the function $f(x) = 5(1.5)^x$ is exponential growth or decay, you have to look at _____.
   a. x b. 1.5 c. 5 d. 5^x

13. The exponential function in question no.12 is an example of _____.
   a. decay b. progress c. growth d. stagnant

14. The exponential function $f(x) = 0.65^x$ is an example of _____.
   a. decay b. progress c. growth d. stagnant

15. The annual sales at a company are ₱400,000.00 in the year 2019 and increasing at the rate of 5% per year. What will be the total amount after 20 years?
   a. ₱1,010,780.08 c. ₱1,114,385.04
   b. ₱962,647.69 d. ₱1,061319.08

Lesson 1 Representations of Exponential Functions

In your previous lesson, you learned about exponential equations and inequalities. In this module, you will extend your knowledge of exponents to growth and decays. Specifically, this module is about the exponential function. Several examples of exponential growth and decay can be observed in some real-life situations in the field of business and economics, health, demography, and sciences. But why is it called the exponential function? What are its characteristics that differentiate it from other functions?

The lessons and activities in this module will explain what you have to know about exponential functions.

After going through this module, you are expected to:

1. define the exponential function; and
2. represent an exponential function through its table of values, graphs, and equations.

What's In

In your previous lesson, you learned how to solve exponential equations and inequalities. As a review, prepare yourself in doing this first drill.

Let us have the equation $32 = 2^x$.

To find the value of x, first transform 32 into its exponential form. So, 32 becomes 2^5. Then, equate it with 2^x. This becomes $2^5 = 2^x$ since they both have the same base. Therefore, the result is x=5. Now, find the value of x on the following equations:

a) $3^{x+1} = 243$

b) $2^{4x+1} = 512$

c) $9^{2x} = 27$

What's New

Recently, a new corona virus has caused a world pandemic. In the Philippines, it spreads from five cases in March up to ten thousand by May. The contamination of cases is likened to what we call exponential spread.

For your **Activity 1**, construct a table of values that would represent the COVID-19 confirmed cases in the Philippines on the first 15 days of March starting on March 5. You may do your research online, then plot the values that you get on a Cartesian plane. You may use paper and pencil or any applicable graphing apps such as MS Excel, GeoGebra, or Desmos. You may use the table below as your reference.

Table 1

t, Days	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
x, cases															

Questions to ponder:

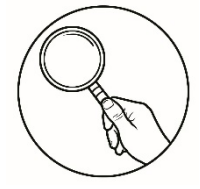
1. What can you observe about the table and graph? Is it linear?

   _____

2. Does it curve slowly or rapidly? _____

3. In order to slow down the growth, what can you do to the curve? In real-life, what must be done to flatten the curve? _____

From the foregoing activity, you have constructed an exponential function that depicts actual cases and are represented in a form of table of values, a graph, and equation.



What is It

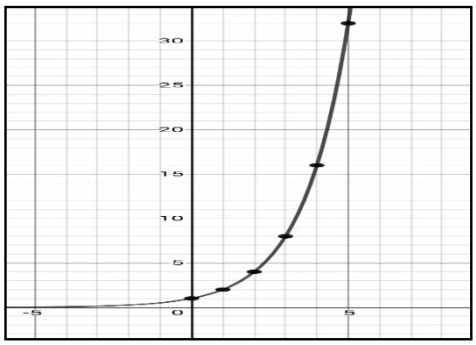
Now, observe that there is a set of values that can be found in x (t, as used in the preceding problem) which corresponds to a certain value in $f(x)$. In the case of the exponential function, the values of $f(x)$ curves rapidly on a given value of x.

This is the characteristic of an exponential function that differentiates it from other functions. Exponential function can be described using the form $f(x) = b^x$, where b is a constant called the base while x is a variable power or simply the exponent.

Let us study the different behaviors of the graphs of exponential functions relative to its independent and dependent variables. Take a look at the function $f(x) = 2^x$. Here x is the input and $f(x)$ is the output. Consider the values in the following table. Substitute the values of x to the function to get $f(x)$.

Table 2

x	0	1	2	3	4	5
f(x)	1	2	4	8	16	32



Looking at Table 2, you can observe that as x increases by 1 unit, $f(x)$ doubles its value from its previous value. To show how rapidly $f(x)$ changes a graph of the function is shown in Fig.1 below:

Fig.1. You can see how steep the curve moves upward from its initial value. This nature of the function has x values that are real numbers, i.e., $x \in \mathbb{R}$. Here, the y-intercept is 1 (x=0). At $x < 0$, the x-axis becomes the *asymptote* of the graph. On the other hand, y-values or f(x) contains only positive integers. Moreover, you can observe that the constant 2 is greater than 1 and is not equal to zero. If this is the case where b is greater than and not equal to 1, you will have what we call exponential growth. It is an increasing function.

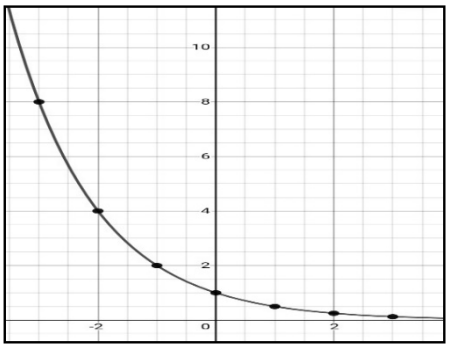


Fig.2. Now take a look at the function $f(x) = (½)^x$, here $0<b<1$ where $b=1/2$. You may notice that this function is also the same as $f(x) = 2^{-x}$. Observe the graph of the function on the left. The curve moves steeply downward going to the right but not touching x-axis. Still, the x inputs are real numbers. If this is the case, where $0<b<1$, you will have what we call exponential decay. It is a decreasing function.

Variations of graphs may be tried relative to functions $y=2^x$ and $y=2^{-x}$.

But first let us have an activity that will facilitate more understanding of these methods.

Activity 2 Move Me

Materials Needed: pen/pencil, graphing paper/bond paper, MS Excel, Desmos, Mechanics:

1. In a bond paper/graphing paper, plot the following functions.
   a. $f(x) = 3^x$ b. $f(x) = -2^x$ c. $f(x) = 2^x + 1$

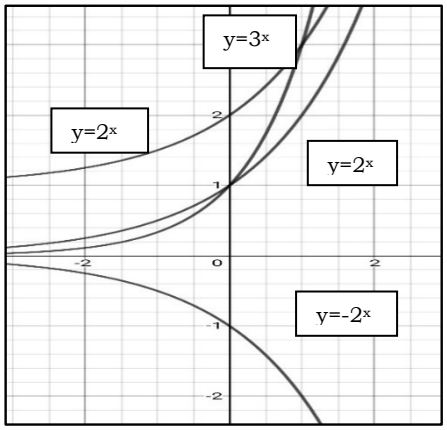
2. Create a table of values for question no.1 (altogether in one table) using the following x-values: -2, -1, 0, 1, 2.

3. Identify which among the functions are growth and decay.

Questions

1. Relative to $f(x) = 2^x$, what can you say about the movement of the curves?

2. Were you able to correctly identify which is an exponential growth and decay? Explain how you did that.



In the foregoing discussion, you were able to learn about the graph of the exponential function of $y=2^x$ relative to other functions. To cite some behaviors of these functions relative to $y=2^x$, let us briefly summarize the phenomena: Fig.3. In $y=3^x$, both functions have y=1 when x=0. The difference is that $y=3^x$ has moved 1 unit up at x=1. By looking at the graph $y=3^x$ moves more rapidly and steeply towards the y-axis. In $y=-2^x$ each y of $y=2^x$ is multiplied by -1 so that they become opposites. $y=-2^x$ is the reflection of $y=2^x$ about the x-axis. In $y=2^x+1$ the graph shifts 1 unit up relative to $y=2^x$. Let us try some real-life examples:

Example 1:

Jose is planning to buy a gift worth ₱500 for his mom's birthday. So, he planned to save money from what remains on his daily allowance. On the first day, he was able to save ₱5.00. Each day, he decided to double his previous savings. At what day can he be able to buy the gift?

Table 3 shows the pattern how Jose saves his money:

Table 3

day	savings
1	5
2	10
3	20
4	40
5	80
6	160
7	320
8	640

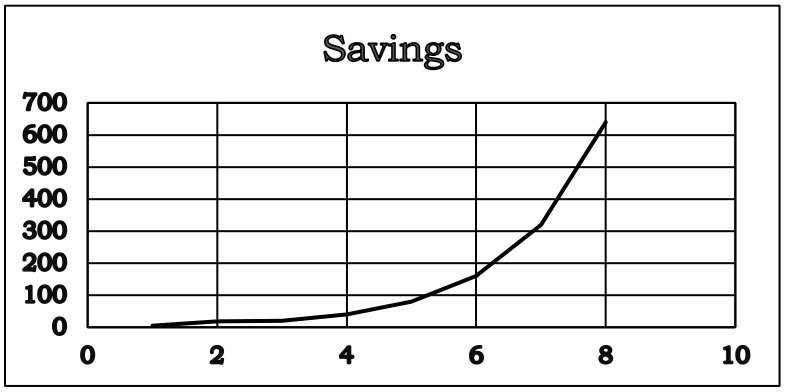


Figure 4

Conclusion: Jose can buy the gift for her mother's birthday on the 8th day.

Example 2:

Mang Leonardo bought his son a motorcycle worth ₱125,000.00. But father and son planned to sell the same motorcycle after 3 years. The value of the motorcycle depreciates at 5% per annum. How much would be the value of the motorcycle after 3 years?

To solve this problem, use the function $A(t) = P(1 - r)^t$, take note that instead of addition, we use subtraction in (1-r). Again, P is the initial amount, r is the rate of interest, and t is the time. Substituting values, we get:

$$A(t) = 125000(1 - .05)^3$$

The table of values for 3 years is shown below:

Table 4

t, time	0	1	2	3
A(t)	125,000	118,750	112,812.5	107171.88

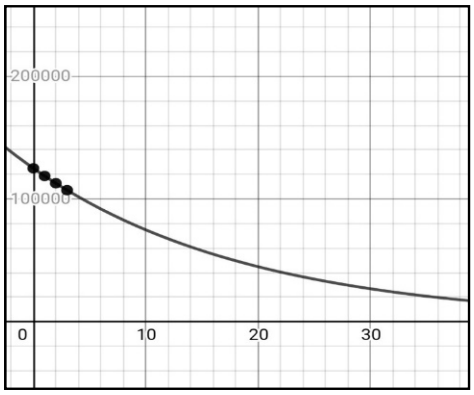
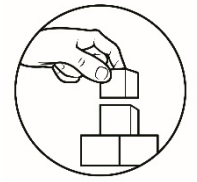


Figure 5

Conclusion: The motorcycle has depreciated to an amount of ₱107,171.88.



What's More

Activity 1.1

Find the values of the exponential function f(x) when x=1, 2, 3, 4, 5. Identify whether it is growth or decay.

1. $f(x) = \left(\frac{1}{3}\right)^{x-1}$
2. $f(x) = -\left(\frac{1}{2}\right)^{x} + 1$
3. $f(x) = 5(2)^x$
4. $f(x) = 2(2)^{\left(\frac{1}{2}\right)x}$
5. $f(x) = \frac{2}{5}(2)^x - 1$

Activity 1.2

Use MS Excel or any graphing apps to plot the graph of the following function. Identify if it is exponential growth or decay.

1. $f(x) = -2^{x-2}$
2. $f(x) = 3^{-x}$
3. $f(x) = 4^x$
4. $f(x) = -\left(\frac{1}{2}\right)^{-2x}$
5. $f(x) = -4^{-x}$

Activity 1.3

Analyze and solve.
A new car costs ₱450.000.00. This value subsides by 10% each year.

1. Write an exponential model that represents this situation after t years.

   Exponential function f(x) = _____

2. How much will the car be worth after 5 years? _____

   Given: A_0 = _____ r = _____ t = _____

3. Create a table of values for t = 1-4 years

Activity 1.4

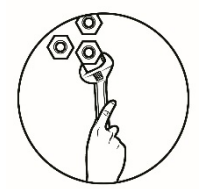
Analyze and solve.
Mr. Morales invests ₱10,000.00 in a company stock. This stock value depreciates by 1.5% each year.

1. Write an exponential model that represents this situation after t years.
2. How much will be the value of the stock after 5 years? _____
3. Create a table of values for t = 1-4 years.

What I Have Learned

Now, try to summarize the behavior of this function at b>1 and 0<b<1 by filling in the blanks with correct word or words.

The function $f(x) = 2^x$ curves upward from its initial value. This nature of the function has x values that are _____ numbers. Here, the y-intercept is 1 (x=0). At x < 0, the x-axis becomes the _____ of the graph. On the other hand, y-values or f(x) contain only _____ integers. Moreover, you can observe that the constant 2 is greater than 1 and is not equal to zero. If this is the case where b is greater than and not equal to 1, you will have what we call an exponential _____. It is an increasing function. In the case, where 0<b<1, you will have what we call exponential _____. It is a decreasing function.



What I Can Do

There are many ways of applying exponential functions in our lives. Examples of these are population growth, bacterial growth, radioactive decay, medical dosage, and compound interest. An example of its application can be seen below.

Field of Application: Compound Interest
Situation analysis: Ms. Gomez decided to have an initial deposit to CDO Bank worth ₱5,000.00 to save for future use. This decision came after an agent of the bank told her that the bank is offering a 5% interest compounded annually. How much would be Ms. Morales' total money in the bank after 5 years?

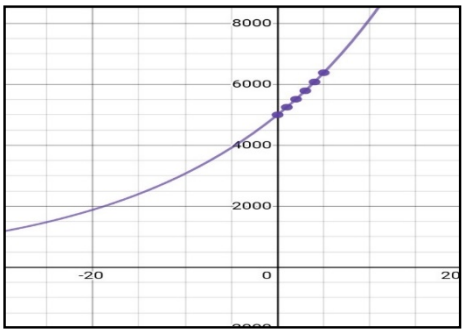
$$A(t) = P(1+r)^t$$

Use the given formula to find the total amount of money after 5 years.
Data manipulation: when t=0, $A(0) = P(1+.05)^0 = 5000$
Presentation: Using a table of values, we have:

Table 3

t	1	2	3	4	5
A(t)	5250.00	5512.50	5788.13	6077.53	6381.41

This is the graph of the function:



Interpretation: The table and the graph show that it has an increasing function. Thus, the values increase at certain period of time. It is an exponential growth. Ms. Gomez' money earned ₱1,181.14 after 5 years in the bank. What personal values worthy of emulation does Ms. Gomez show in this situation?

Now, It's Your Turn

1. Create your own or similar real-life situation where exponential function is applied.
2. In a bond paper, present the problem from **Field of application** up to **Presentation** as illustrated above.
3. You can use graphing paper, MS Excel, Desmos or any graphing app to graph the function. You can also use calculators to solve the table of values.
4. Your grade will be according to these criteria: Clarity of Presentation 60%, Organization and Accuracy, 30%, and Applicability to Current Situations, 10% with a total of 100%.

Assessment

Read and analyze each item carefully. Circle the letter that corresponds to your answer for each statement.

1. In the function, $f(x) = 2^x$, 2 is called _____.
   a. constant
   b. variable power
   c. coefficient
   d. base

2. Which of the following is an application of exponential function?
   a. monthly salary
   b. shooting a cannon
   c. radioactive decay
   d. distance travelled

3. The following are examples of exponential function ***except one.***
   a. $f(x) = 2^{-x}$
   b. $f(x) = 2^{x-3}$
   c. $f(x) = 5^{2x}$
   d. $f(x) = \left(\frac{3}{x-1}\right)^2$

4. One of the properties of the base in the exponential function is that it cannot be ___.
   a. equal to1
   b. greater than 0
   c. greater than 0 but less than 1
   d. greater than 1

5. In exponential function, when the exponent is equal to zero the function has a value equal to _____.
   a. 0
   b. 1
   c. -1
   d. imaginary number

6. The function $f(x) = 3^{-x}$ is a decreasing function. The function $f(x) = 3^x$ is _____ function.
   a. increasing
   b. decreasing
   c. either increasing or decreasing
   d. slanting

7. In the function $f(x) = 4^x$, the x-axis becomes the asymptote of the graph at _____.
   a. $x = 0$
   b. $x > 0$
   c. $x < 0$
   d. $x = 1$

8. The domain of the function $f(x) = 2^{x-1}$ is the set of _____.
   a. negative real numbers
   b. positive real numbers
   c. zero
   d. real numbers

9. The range of the function f(x) = $\left(\frac{1}{2}\right)^x$ is the set of _____.

a. positive numbers
b. zero
c. negative numbers
d. all of the choices

10. The exponential function f(x) = $\left(\frac{1}{20}\right)^x$ is an example of _____.

a. decay
b. progress
c. growth
d. stagnant

11. The exponential function f(x) = $2(10)^x$ is an example of _____.

a. decay
b. progress
c. growth
d. stagnant

12. In the function f(x) = $\frac{1}{2}(1.5)^x$, find f(x) if x = -1, 0, 1.

a. f(x) = 0.22, 0.33, 0.5
b. f(x) = 0.5, 0.75, 1.125
c. f(x) = 0.22, 0.5, 1.125
d. f(x) = 0.33, 0.5, 0.75

13. The annual sales at a company are ₱100,000.00 in the year 2020 and increasing at the rate of 4% per year. What is its total amount after 10 years?

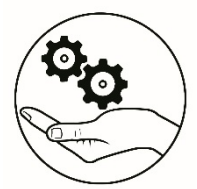
a. ₱136,856.91
b. ₱153,945.41
c. ₱148,024.43
d. ₱142,331.18

14. A new car costs ₱150.000.00. This value subsides by 5% each year. How much will the car be worth after 6 years?

a. ₱116,067.14
b. ₱110,263.78
c. ₱122,175.94
d. ₱104,750.59

15. At the start of the experiment in the laboratory, there are 1000 bacteria in a petri dish. The relationship between time t, in minutes, and the number of bacteria, N(t), can be represented by the function N(t) = $15(2)^{\frac{t}{10}}$. How many bacteria will there be after 150 minutes?

a. 491,520
b. 15,360`
c. 61,440
d. 983,040



Additional Activities

In doing this activity, you may need MS Excel or a mobile app such as Desmos to sketch the graph.

One of the regions in the Philippines is Region IV-A more popularly called CALABARZON region. In the last Philippine census, CALABARZON has around 14 million Filipinos living in the region with 2.58% population growth rate. Now using the formula: $N(t) = N_0 b^t$ where N(t) is the number of the population, N_o is the initial count of the number of populations, b is the growth factor, and t is the time period.

Construct a table of values and sketch the graph of its population for five years from 2021 – 2025.

Answer:

The following are given on this problem:

N_o = _____ b = (1+.0258) = _____
t = _____

1. The table of values for this function: (round off your answers to three decimal digits)

Table 4

The graph of this function: (use the space provided)

t	0	1	2	3	4	5
N(t) (in million)	14					

Questions:

1. What is the value of N(t) after 5 years? _____
2. After 5 years, how many people have been added from the initial number of the population? _____
3. Is this an increasing or decreasing function? _____

Answer Key

What I know	Assessment
1. B	1. D
2. D	2. C
3. C	3. D
4. D	4. A
5. A	5. B
6. C	6. A
7. B	7. C
8. D	8. D
9. B	9. A
10. A	10. A
11. C	11. C
12. B	12. D
13. C	13. C
14. A	14. B
15. D	15. A

Activity 1.1

1. if x = 1, $f(x) = (1/3)^{x-1} = (1/3)^{1-1} = (1/3)^0$

   Then f(x) = 1.

   f(x) = 0.33, 0.11 0.4, 0.01

2. f(x) = 0.5, 0.75, 0.88, 0.94, 0.97

3. f(x) = 10, 20, 30, 40, 80, 160

4. f(x) = 2.83, 4, 5.66, 8, 11.31

5. f(x) = -0.2, 0.6, 2.2, 5.4, 11.8

Activity 1.2

1. growth
2. decay
3. growth
4. decay
5. decay

What's More

Activity 1.3

a. $A(t) = A_0(1-r)^t$

b. Given: A_o = 450000

r = 10%

t = 5

$A(t) = 450000(1-0.10)^5$

$= 450000(0.90)^5$

= 265,720.50

c.

t	1	2	3	4
A(t)	405000	364500	328050	295245

Activity 1.4

a. $I(t) = I_0(1-r)^t$

b. $I(t) = 10000(1-.015)^5$

$= 10000(0.985)^5$

= 9272.17

c.

t	1	2	3	4
I(t)	9850	9702.25	9556.72	9413.37

References

AlgebraLAB. 2020. *Applications of Exponential Functions*. http://www.algebralab.org/lessons/lesson.aspx?file=Algebra_ExponentsApps.xml

CK-12. 2020. *Applications of Exponential Functions*. https://www.ck12.org/algebra/applications-of-exponential-functions/lesson/Applications-of-Exponential-Functions-BSC-ALG/?referrer=concept_details

Dorey, Kevin. 2013. *Exponential Decay Word Problems*. Uploaded on March 14, 2013. https://m.youtube.com/watch?v=Wt4KJfBwSml

**General Mathematics Learner's Material*. First Edition. 2016. pp. 77-81, 88-97

Hutchinson, E. (2014, Oct.8). *Exponential Growth Problem (Bacteria): Ex 2*. Uploaded on October 8, 2914. https://m.youtube.com/watch?v=bcBDqrd2wZ0

Khan Academy. 2020. *Exponential Model Word Problem: Bacteria Growth*. https://www.khanacademy.org/math/algebra2//x2ec2f6f830c9fb89:logs/x2ec2f6f830c9fb89:exp-models/v/solving-exponential-model-word-problem-2

Kuang, Yang et.al. 2020. *Understanding the Rules of Exponential Functions*. https://www.dummies.com/education/math/calculus/understanding-the-rules-of-exponential-functions/

MathBitsNotebook. 2020. *Exponential Functions*. https://mathbitsnotebook.com/Algebra1/FunctionGraphs/FNGTypeExponential.html

Math Insight. 2020. *The Exponential Function*. https://mathinsight.org/exponential_function

Monterey Institute. 2020. *Introduction to Exponential Function*. http://www.montereyinstitute.org/courses/DevelopmentalMath/COURSE_TEXT2_RESOURCE/U18_L1_T1_text_final.html

Orines et.al. 2009. *Advanced Algebra, Trigonometry, and Statistics*. Quezon City, SD Publications, Inc.

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