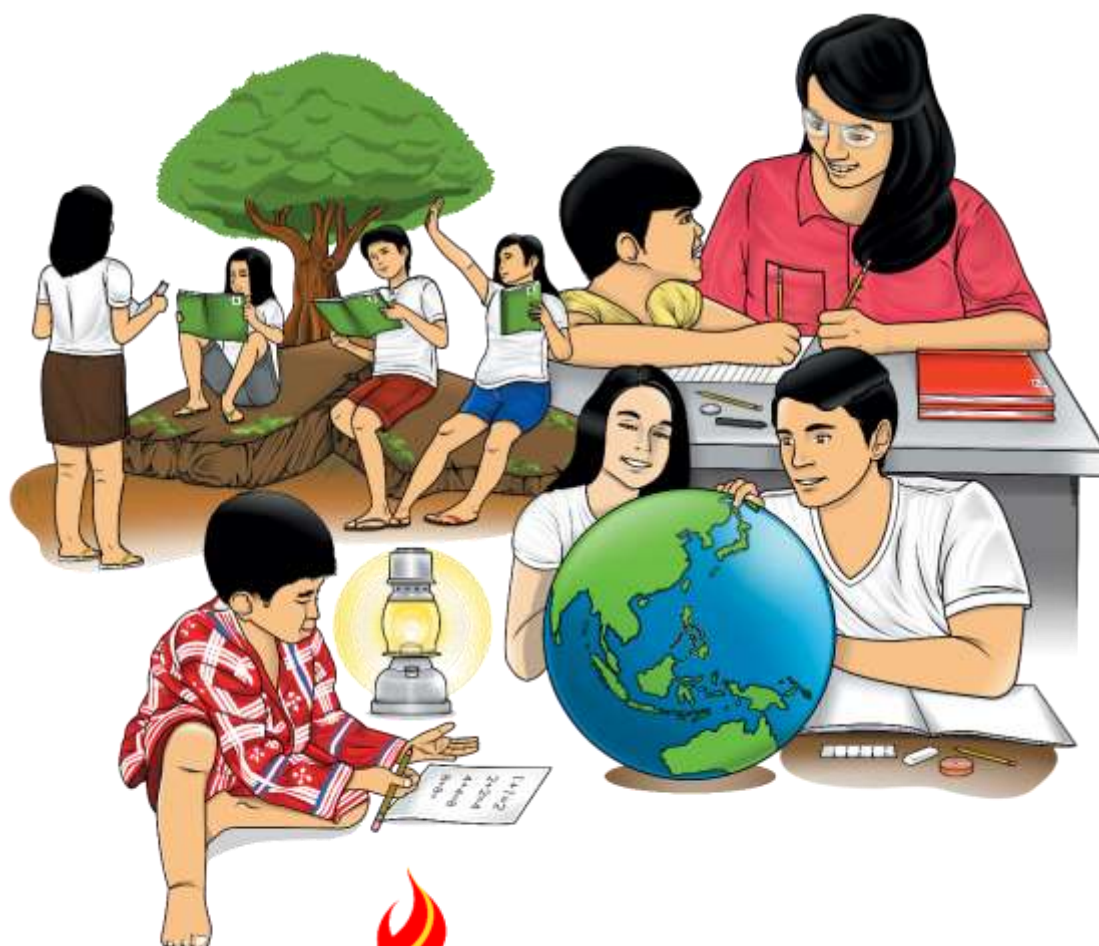


Senior High School

General Mathematics

Quarter 1 – Module 18:

Solving Exponential Equations and Inequalities



General Mathematics
Alternative Delivery Mode
Quarter 1 – Module 18: Solving Exponential Equations and Inequalities
First Edition, 2021

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Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.



What I Need to Know

This module was designed and written with you in mind. It is here to help you master how to solve exponential equation and inequality. The scope of this module permits it to be used in many different learning situations. The language used recognizes the diverse vocabulary level of students. The lessons are arranged to follow the standard sequence of the course. But the order in which you read them can be changed to correspond with the textbook you are now using.

After going through this module, you are expected to:

1. explain how to apply the properties in solving exponential equations and inequalities; and
2. solve exponential equations and inequalities.



What I Know

Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. What should be considered in solving an exponential equation?
 - a. Bases on both sides of the exponential equation must be the same.
 - b. Bases on both sides of the exponential equation must be simplified.
 - c. Exponents on both sides of the exponential equation must be the same.
 - d. Exponents on both sides of the exponential equation must be simplified.
2. Which of the following describe the statement: If $x_1 \neq x_2$, then $b^{x_1} \neq b^{x_2}$. Conversely, if $x_1 = x_2$, then $b^{x_1} = b^{x_2}$?
 - a. Addition Property of Equality
 - b. Distributive Property of Equality
 - c. Multiplication Property of Equality
 - d. One-to-one Property of Exponential Function
3. In solving for the value of the unknown variable in $4^{x+1} = 16$, what is the best thing to do first?
 - a. Divide 16 by 4.
 - b. Express 16 as 4^2 .
 - c. Multiply 4 by $x+1$.
 - d. Write $x+1$ as the exponent for both 4 and 16.

4. Which of the following best leads to the value of the unknown variable in $4^{x+1} = 16$?
- $4^{x+1} = 4^2$
 - $4^{x+1} = 2^4$
 - $4^{x+1} = 16^x$
 - $(4)^{2(x+1)} = 16$
5. What is the value of x in $4^{x+1} = 16$?
- 0
 - 1
 - 2
 - 4
6. Which of the following best leads to the value of the unknown variable in $27^x = 9$?
- $9^{3x} = 9$
 - $3^{9x} = 9$
 - $9^{3x} = 3^2$
 - $3^{3x} = 3^2$
7. What is the value of x in $27^x = 9$?
- $1/3$
 - $2/3$
 - 3
 - $3/2$
8. What is the first step in solving for x in the exponential inequality $2(5)^x > 10$?
- Multiply 2 by 5.
 - Divide both sides by 2.
 - Divide both sides by 5.
 - Divide both sides by 10.
9. What is the value of x in the given exponential inequality in item 8?
- $x > 5$
 - $x > 2$
 - $x > 1$
 - $x > 0$
10. Which of the following is equivalent to $10^{x-5} > 100^{x-10}$?
- $10(10)^{x-5} > 100^{x-10}$
 - $10^{x-5} > 10^{2x-20}$
 - $1^{x-5} > 10^{2x-20}$
 - $1^{x-5} < 10^{2x-20}$

11. Which best leads to the correct answer in solving for the unknown variable in the given exponential inequality in number 10?

- a. $x - 5 < x - 10$
- b. $x - 5 > x - 10$
- c. $x - 5 < 2x - 20$
- d. $x - 5 > 2x - 20$

12. What is the value of x for the given in number 10?

- a. $x < 10$
- b. $x > 10$
- c. $x < 15$
- d. $x > 15$

13. Which best describes base b of $\left(\frac{1}{3}\right)^{x+4} \geq \left(\frac{1}{9}\right)^x$?

- a. $0 < b < 1$
- b. $b < 1$
- c. $b > 1$
- d. $b > 0$

14. Which of the following best leads to the value of x in $\left(\frac{1}{3}\right)^{x+4} \geq \left(\frac{1}{9}\right)^x$?

- a. $\left(\frac{1}{3}\right)^{x+4} \leq \left(\frac{1}{3}\right)^{2x}$
- b. $\left(\frac{1}{3}\right)^{x+4} \geq \left(\frac{1}{3}\right)^x$
- c. $\left(\frac{1}{3}\right)^{x+4} \geq \left(\frac{1}{3}\right)^{-2x}$
- d. $\left(\frac{1}{3}\right)^{x+4} \leq \left(\frac{1}{3}\right)^{-2x}$

15. What is the solution to $\left(\frac{1}{3}\right)^{x+4} \geq \left(\frac{1}{9}\right)^x$?

- a. $(4, +\infty)$
- b. $[4, +\infty)$
- c. $(-\infty, 4)$
- d. $(-\infty, 4]$

Lesson**1****Solving Equations and Inequalities**

We are living in a world where diversity exists. Things vary from each other. Various decisions lead to several differing results. These differences, nevertheless, are always present. In the same manner, in the previous module, we have learned that statements containing exponential expressions may take various forms. And we have understood that despite their differences, what is important is to know how to classify each accordingly and how to deal with them.

***What's In***

Listed below are statements with exponential expressions. Which are exponential functions? exponential equations? exponential inequalities?

- | | | | |
|----------------------|----------------------|----------------------------|-------------------|
| (a) $4^x = 2^{x+1}$ | (d) $81 = 9^{3x}$ | (g) $y = e^x$ | (j) $g(x) = 6x^3$ |
| (b) $100 > 10^{x-2}$ | (e) $f(x) = 5^{x-4}$ | (h) $(0.81)^{2x} \geq 0.9$ | |
| (c) $16x = x^2$ | (f) $27 < 3x$ | (i) $25^{x-2} = 5^{3x}$ | |

Recall that an exponential function is not intended to be solved as it simply shows relationship between two variables. In the above list, (e) and (g) are both exponential functions. Why is (j) not considered as one? Though it has two variables, yet it is not an exponential expression since the exponent is not a variable.

Both exponential equation and inequality, on the other hand, are the ones whose x values satisfying the given statements, are meant to be solved. They both involve only one variable. Among the given above, (a), (d), and (i) are exponential equations. They are equations involving exponential expressions and are consisted of only one variable. How about (c)? It does not even have an exponential expression since its exponent is a constant.

The statements in (b) and (h) are the only exponential inequalities in the list. They both have exponential expressions consisting of one variable only. Also, they both use inequality symbols. (f) has inequality symbol, but why can't you consider it as exponential inequality? It is since its exponent is not a variable, hence it is not an exponential expression.



Notes to the Teacher

Stress once again that not all functions, equations, or inequalities with exponents, contain exponential expressions. Instead, only those involving variable exponents are considered as so.



What's New

Raise It!

This time let us see how fast you can compute mentally and how smart your reasoning power is. Just answer as fast as you can without looking at the solutions below. Ready? Let us begin!

1. How many times do you have to multiply 4 by itself to obtain 64?
2. How will you write it in symbols?
3. What kind of mathematical expression is it?
4. How many times do you have to multiply 4 by itself so that the result will be greater than 64?
5. How will you write the fourth question in symbols?
6. What kind of mathematical expression is being depicted by it?

Solutions:

1. Sounds like you find it easy. Correct, three times!
2. Expressing it in symbols, it is $4^x = 64$. Then, it becomes $4^3 = 64$, based on your first answer.
3. In the previous module, you have learned that $4^x = 64$ is an exponential equation since it is an exponential expression with only one variable and involving an equation.
4. You might answer 5, 6, 7, 8, and so on. They are all correct. But actually, even non-integers may be solutions as long as they are greater than 4. Hence the solution is $x > 4$.
5. When written in symbols, it is $4^x > 64$.
6. It is an exponential inequality since it involves an exponential expression with only one variable and an inequality.



What is It

In the previous activity you have been able to solve for the values of the unknown in both exponential equation and inequality. Most probably you have successfully solved them mentally. How do your solutions really work? Let us try to understand it further.

Solving Exponential Equation

One-to-one Property of Exponential Functions states that in $f(x) = b^x$, if $x_1 \neq x_2$, then $b^{x_1} \neq b^{x_2}$. Conversely, if $b^{x_1} = b^{x_2}$, then $x_1 = x_2$. This property paves the way in understanding how to solve exponential equation.

Example 1: Solve for the value of x in $4^{x+1} = 64$.

Solution:

$4^{x+1} = 4^3$	Express 64 as 4^3 , in order for both sides of the equation to have same bases.
$x + 1 = 3$	One-to-one Property of Exponential Functions states that if $b^{x_1} = b^{x_2}$, then $x_1 = x_2$.
$x + 1 - 1 = 3 - 1$	Use Addition Property of Equality in order to solve for the value of x .
$x = 2$	Combine like terms.

Example 2: Solve for the value of x in $3^{4x} = 9^{x+1}$.

Solution:

$3^{4x} = (3)^{2(x+1)}$	Express 9 as 3^2 , in order for both sides of the equation to have same bases.
$4x = 2x + 2$	One-to-one Property of Exponential Functions states that if $b^{x_1} = b^{x_2}$, then $x_1 = x_2$.
$4x - 2x = 2x + 2 - 2x$	Use Addition Property of Equality in order to solve for the value of x .
$2x = 2$	Combine like terms.
$x = 1$	Use Multiplication Property of Equality by multiplying both sides of the equation by $\frac{1}{2}$.

Solving Exponential Inequality

Recall that in an exponential function $f(x) = b^x$, $b > 0$ but $b \neq 1$. Now, the key to solving exponential inequality is the fact that if $b > 1$ and $x_1 > x_2$, then $b^{x_1} > b^{x_2}$. Otherwise, if $0 < b < 1$, then $b^{x_1} < b^{x_2}$. Let us further make this clearer by considering the next examples.

Example 3: Solve for the values of x in $5^x > 125^{x+8}$

Solution:

$5^x > (5)^{3(x+8)}$	Express 125 as 5^3 , for both sides of the inequality to have same bases.
$x > 3x + 24$	$b = 5$. It is a fact that if $b > 1$ and $b^{x_1} > b^{x_2}$, then $x_1 > x_2$.
$x - x - 24 > 3x + 24 - x - 24$	Use Addition Property of Equality in order to solve for the value of x .
$-24 > 2x$	Combine like terms.
$x < -12$	Use Multiplication Property of Equality by multiplying both sides of the equation by $\frac{1}{2}$.

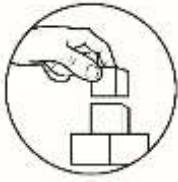
Hence, the solution to the exponential inequality $5^x > 125^{x+8}$ is the set of all real numbers less than -12. In symbols, that is, $x < -12$ or $(-\infty, -12)$.

Example 4: Solve for the values of x in $\left(\frac{1}{7}\right)^{2x+9} \leq \left(\frac{1}{343}\right)^{x-5}$.

Solution:

$\left(\frac{1}{7}\right)^{2x+9} \leq \left(\frac{1}{7}\right)^{3(x-5)}$	Express $\frac{1}{343}$ as $\left(\frac{1}{7}\right)^3$, in order for both sides of the inequality to have same bases.
$2x + 9 \geq 3x - 15$	$b = \frac{1}{7}$. It is a fact that if $0 < b < 1$ and $b^{x_1} < b^{x_2}$, then $x_1 > x_2$.
$2x + 9 - 2x + 15 \geq 3x - 15 - 2x + 15$	Use Addition Property of Equality in order to solve for the value of x .
$24 \geq x$ or $x \leq 24$	Combine like terms.

Thus, the solution to the exponential inequality $\left(\frac{1}{7}\right)^{2x+9} \leq \left(\frac{1}{343}\right)^{x-5}$ is the set of all real numbers less than or equal to 24. In symbols, that is, $x \leq 24$ or $(-\infty, 24]$.



What's More

Activity 1.1

Who Has a Point?

Observe each of the following pairs of solutions. Decide whether anyone of them got the correct answer. Answer the sets of guide questions.

Becca

$$16x^2 = 4^{x+3}$$

$$(2)^{4x^2} = (2)^{2(x+3)}$$

$$4x^2 = 2x + 6$$

$$\left(\frac{1}{2}\right)(4x^2 - 2x - 6 = 0)$$

$$2x^2 - x - 3 = 0$$

$$(2x - 3)(x + 1) = 0$$

$$2x - 3 = 0 \text{ and } x + 1 = 0$$

$$x = \frac{3}{2}, x = -1$$

Celia

$$16x^2 = 4^{x+3}$$

$$(4)^{2x^2} = 4^{x+3}$$

$$2x^2 = x + 3$$

$$2x^2 - x - 3 = 0$$

$$(2x - 3)(x + 1) = 0$$

$$2x - 3 = 0 \text{ and } x + 1 = 0$$

$$x = \frac{3}{2}, x = -1$$

1. How did you check whether $\frac{3}{2}$ and -1 are really solutions of the given exponential equation?
2. Were Becca and Celia both correct or both wrong?
3. What is the difference between their solutions?
4. Did the difference affect the solutions? Why?
5. Did Becca and Celia both use the One-to-one Property of Exponential Function? How?

Hector

$$(0.5)^{x-1} > (0.25)^{-x-2}$$

$$(0.5)^{x-1} > (0.5)^{2(-x-2)}$$

$$x - 1 > -2x - 4$$

$$3x > -3$$

$$x > -1 \text{ or } (-1, +\infty)$$

Dindo

$$(0.5)^{x-1} > (0.25)^{-x-2}$$

$$(0.5)^{x-1} > (0.5)^{2(-x-2)}$$

$$x - 1 < -2x - 4$$

$$3x < -3$$

$$x < -1 \text{ or } (-\infty, -1)$$

6. Have you noticed any difference in the solutions? What is it?
7. Who used the property for exponential inequality? How did he use it?
8. Taking Hector's solution, can 0 be a value of x ?
9. Considering Dindo's solution, will -2 make the inequality correct?
10. Were Hector and Dindo both correct? If not, whose work is right?

Activity 1.2

Find the Missing x!

Solve for the values of x for each of the following exponential equations and inequalities.

1. $8^{2-x} = 2$
2. $\left(\frac{1}{2}\right)^x < \frac{1}{8}$
3. $5^x = 25^{x-2}$
4. $3^{x+2} \geq 27$
5. $4^{3x} = 8^{x-1}$



What I Have Learned

Complete the following statements by filling in the blanks with the correct words or phrases.

1. _____ states that in $f(x) = b^x$, if $b^{x_1} = b^{x_2}$, then $x_1 = x_2$.
2. This property, as stated in the previous statement, applies in solving _____.
3. Given an exponential equation, first thing we see to it is that the bases of both sides of the equation are _____.
4. In solving an exponential inequality, if base b is greater than 1 and $b^{x_1} > b^{x_2}$, then _____.
5. Given that _____, and $b^{x_1} > b^{x_2}$, then $x_1 < x_2$.



What I Have Learned

Write a mathematical journal about the importance of understanding the concepts behind solving exponential equation and inequality. Include your own examples of exponential equation and inequality, with one example for each. Then explain how the properties apply in solving each example.

	4	3	2	1
Content (score x 2)	Very relative to the topic and well-organized	Somewhat relative to the topic and organized	Less relative to the topic and poorly organized	Not relative to the topic and very poorly organized
Proficiency (score x 2)	Both examples are correct and explained properly	Both examples are correct but properties are not explained properly	One example is correct and explained properly	One example is correct but not explained properly
Spelling, Grammar, and Punctuations (score x 1)	No spelling, punctuation or grammatical errors	Very few spelling, punctuation, and grammatical errors	Several spelling, punctuation, and grammatical errors	Many spelling, punctuation, and grammatical errors



Assessment

Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

- What should be considered in solving an exponential equation with base b and exponents x_1 and x_2 ?
 - If $b^{x_1} = b^{x_2}$, then $x_1 \neq x_2$.
 - If $b^{x_1} \neq b^{x_2}$, then $x_1 = x_2$.
 - If $b^{x_1} = b^{x_2}$, then $x_1 = x_2$.
 - If $b^{x_1} = b^{x_2}$, then $x_1 > x_2$.

- Which of the following is true in solving an exponential inequality with base b and exponents x_1 and x_2 ?
 - If $b > 1$ and $b^{x_1} > b^{x_2}$, then $x_1 > x_2$.
 - If $b > 1$ and $b^{x_1} > b^{x_2}$, then $x_1 < x_2$.
 - If $0 < b < 1$ and $b^{x_1} > b^{x_2}$, then $x_1 > x_2$.
 - If $0 < b < 1$ and $b^{x_1} < b^{x_2}$, then $x_1 < x_2$.

- In solving for the value of the unknown variable in $2^{5x} = 64$, what is the best thing to do first?
 - Simplify 64 into 8^2 .
 - Divide 64 by 2.
 - Express 64 as 2^6 .
 - Multiply $5x$ by 2.

4. Which of the following best leads to the value of the unknown in $2^{5x} = 64$?
- $2^{5x} = 64/2$
 - $2^{5x} = 2^6$
 - $2^{5x} = 8^2$
 - $2^{5x} = 64/2^{5x}$
5. What is the value of x in $2^{5x} = 64$?
- $6/5$
 - 6
 - 8
 - 32
6. Which of the following best leads to the value of the unknown in $8^{x^2} = 2^{2x+1}$?
- $8^{3x^2} = 2^{2x+1}$
 - $8^{x^2} = 8^{2x+1}$
 - $(2)^3 = 2^{2x+1}$
 - $(2)^{3x^2} = 2^{2x+1}$
7. What are the values of x in $8^{x^2} = 2^{2x+1}$?
- $-\frac{1}{3}, -1$
 - $-\frac{1}{3}, 1$
 - $\frac{1}{3}, -1$
 - $\frac{1}{3}, 1$
8. What is an important step in solving for x in any exponential inequality?
- Consider if $b > 1$ or if $0 < b < 1$.
 - Assume that $b < 0$.
 - See to it that the exponents are equal.
 - Always divide both sides by the common exponent.
9. Which among the following is a significant observation when solving for x value of $25^x < 125^{x-3}$?
- The exponents are almost the same.
 - The exponents both use x variable.
 - The bases are greater than 1.
 - The bases are both multiples of 5.
10. Which of the following best leads to the solution for the given in item 9?
- $25^x < (100 + 25)^{x-3}$
 - $25^x > (100 + 25)^{x-3}$
 - $(5)^{2x} < (5)^{3(x-3)}$
 - $(5)^{2x} > (5)^{3(x-3)}$

11. What is the value of x in the given exponential inequality in item 9?
- $x < 6$
 - $x > 7$
 - $x < 8$
 - $x > 9$
12. Which among the following is a significant observation when solving for x value of $0.49^x > 0.7^{x+1}$?
- The exponents both used variable x .
 - The exponent on the right side is 1 greater than the other.
 - The bases are multiples of 0.7.
 - The bases are greater than 0 but less than 1.
13. Which best leads to the correct answer in solving for the unknown variable in the given exponential inequality in number 12?
- $0.49^x < (0.7)^{2(x+1)}$
 - $(0.7)^{2x} < 0.7^{x+1}$
 - $0.49^x > (0.7)^{2(x+1)}$
 - $(0.7)^{2x} > (0.7)^{2(x+1)}$
14. Which of the following is a correct part of the solution for the given in item 12?
- $x > x + 1$
 - $x < x + 1$
 - $2x > x + 1$
 - $2x < x + 1$
15. What is the value of x for the given in number 12?
- $(-\infty, 1)$
 - $(-\infty, 1]$
 - $(1, +\infty)$
 - $[1, +\infty)$



Additional Activities

Solve for the values of x in $32^{3x^2} = 4^{11x+24}$ and in $\left(\frac{1}{16}\right)^{2x+5} = \left(\frac{1}{64}\right)^{x-1}$.



Answer Key

Assessment

1. c
2. a
3. c
4. b
5. a
6. d
7. b
8. a
9. c
10. c
11. d
12. d
13. b
14. d
15. a

What's More

Activity 1.1

By substituting each to x of the given. They are both correct.

Becca used $b=2$, while Celia used $b=4$. It did not, since both were correctly used as exponential form of the given.

Yes, they both did. They both used same bases for both sides of the equation, before equating their respective exponents.

Dindo interchanged the inequality symbol, while Hector used the same symbol all throughout.

Dindo used the property. Since $0 < b(0.5) < 1$, he interchanged the inequality symbol.

0 cannot be.

-2 can be.

Only Dindo got it right.

Activity 1.2

1. $x = \frac{3}{4}$
2. $x > 3$
3. $x = 4$
4. $x \geq 1$
5. $x = -1$

What I Know

1. a
2. d
3. b
4. a
5. b
6. d
7. b
8. b
9. c
10. b
11. d
12. c
13. a
14. a
15. b

References

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