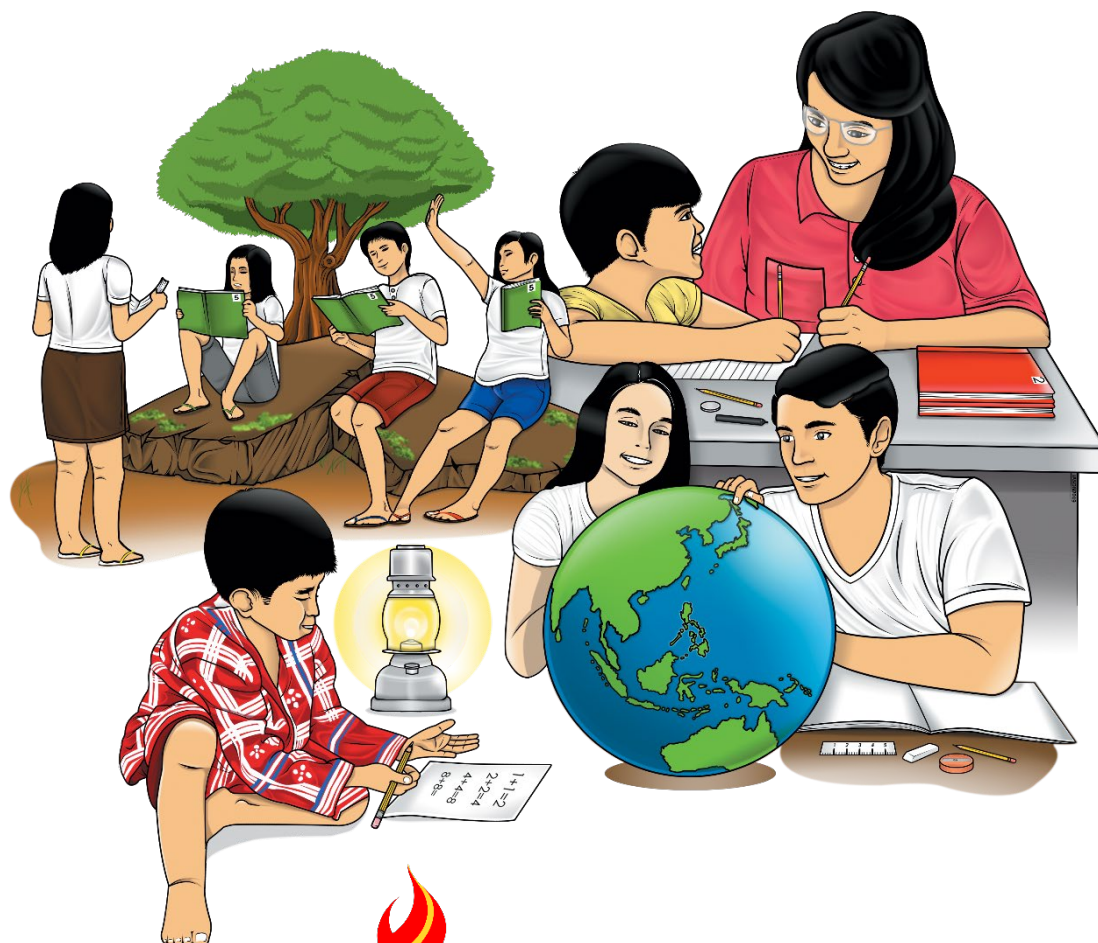


General Mathematics

Quarter 1 – Module 12:

The Inverse of One-to-one Functions



General Mathematics
Alternative Delivery Mode
Quarter 1 – Module 12: The Inverse of One-to-one Function
First Edition, 2020

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General Mathematics

Quarter 1 – Module 12:

The Inverse of One-to-one Functions

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

Thank you.



What I Need to Know

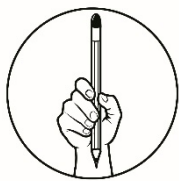
This module was designed and written with you in mind. It is here to help you understand the inverse function. Particularly, this will provide you guide on how to find the inverse of a one-to-one function. Enjoy as you immerse yourself in solving for the inverse function intuitively or using a set of more established steps.

The module is composed of one lesson, namely:

- Lesson 1 – The Inverse of a One-to-one Function

After going through this module, you are expected to:

1. determine the inverse of a one-to-one function.
2. write a letter to a family member or peer about making amends on regretful events which cannot be undone.



What I Know

Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

1. Which is related to the word “inverse”?
 - a. cancel
 - b. change
 - c. delete
 - d. interchange
2. What is the inverse of addition?
 - a. division
 - b. multiplication
 - c. subtraction
 - d. composition
3. What is the inverse of division?
 - a. addition
 - b. multiplication
 - c. subtraction
 - d. composition

4. What is the inverse of multiplication?
- addition
 - division
 - subtraction
 - composition
5. What is the inverse of subtraction?
- addition
 - division
 - multiplication
 - composition
6. Which of the following functions has/have inverse function?
- one-to-one
 - many-to-one
 - both
 - none
7. What is the mathematical symbol for inverse of $f(x)$?
- $\frac{1}{f(x)}$
 - $f(x)^{-1}$
 - $'f(x)$
 - $f^{-1}(x)$
8. What is the inverse of $f(x) = 3x - 4$?
- $f(x) = 3x + 4$
 - $f(x) = 4x - 3$
 - $f(x) = \frac{x+3}{4}$
 - $f(x) = \frac{x+4}{3}$
9. What is the inverse of $f(a) = a^3 + 5$?
- $f(a) = 3a - 5$
 - $f(a) = \sqrt[3]{a - 5}$
 - $f(a) = \sqrt[3]{a} - 5$
 - $f(a) = \frac{a}{3} + 5$

10. Which is a property of an inverse function?
- The inverse of $f^{-1}(x)$ is $-f^{-1}(x)$.
 - $f^{-1}(f(x)) = x$ for all positive x in the domain of f .
 - $f^{-1}(f^{-1}(x)) = x$ for all x in the domain of f^{-1} .
 - $f(f^{-1}(x)) = x$ for all x in the domain f^{-1} .
11. What is the inverse of $f(x) = -2x + 7$?
- $f(x) = -\frac{x-7}{2}$
 - $f(x) = -\frac{x}{2} + 7$
 - $f(x) = 7x - 2$
 - $f(x) = 2x - 7$
12. What is the inverse of $f(b) = b^3 + 4$?
- $f(b) = 3b - 4$
 - $f(b) = \frac{b}{3} + 4$
 - $f(b) = \sqrt[3]{b} + 4$
 - $f(b) = \sqrt[3]{b-4}$
13. What is the inverse of $g(x) = 2x - 8$?
- $g'(x) = \frac{x+8}{2}$
 - $g'(x) = -2x + 8$
 - $g^{-1}(x) = \frac{x+8}{2}$
 - $g^{-1}(x) = -2x + 8$
14. What is the inverse function of $f(c) = \frac{2c+1}{4c-5}$?
- $f^{-1}(c) = \frac{4c-1}{2c+5}$
 - $f^{-1}(c) = \frac{5c+1}{4c-2}$
 - $f'(c) = \frac{2c-1}{4c+5}$
 - $f'(c) = \frac{4c-5}{2c+1}$
15. Which is not involved in the process of finding the inverse of a function?
- Write the function in the form $y = f(x)$.
 - Interchange the x and y variables.
 - Write in the function in the simplest form.
 - Solve for y in terms of x .

Lesson**1****The Inverse of One-to-One Functions**

Among the functions, only a one-to-one function has an inverse which is a function also.

***What's In***

So far, you have known different faces of functions in the previous lessons. Likewise, you have categorized them already into groups of one-to-one and many-to-one functions. Let's have a quick review!

Directions: In the first column, identify each of the following as linear function (LF), quadratic function (QF) or rational function (RF). In the second column, decide whether each is one-to-one or many-to-one function.

Function	LF, QF or RF	One-to-one or many-to-one
1. $f(x) = 2x + 5$		
2. $g(x) = 2x^2 - 4x + 1$		
3. $h(x) = (x - 1)^2 - \frac{1}{2}$		
4. $f(x) = \frac{2x-1}{x+5}$		
5. $g(x) = x$		

Do you ever wonder if inverses of these functions are functions as well? Well, both the original equation and its inverse are both functions. In this lesson, you will delve into these functions with function inverses.

***Notes to the Teacher***

Solutions should be provided for exercises which will not be successfully answered by the learners especially for "What I Can Do" Part.



What's New

I Can See Your Mind

Let's have a mind game. Ready?

Think of a number and multiply it by 2. Then, subtract 1 from it. Now, add 4 to the difference. Lastly, give me your answer and I'll tell the number you are thinking of.

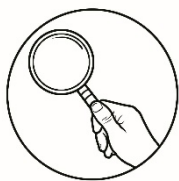
Can you tell me how I will know the original number you have chosen by giving me the final answer?

The key lies in the command “undo”. Familiar with it? Yes, this game follows the same principle as with the “undo” button we click when we are preparing documents using our laptops, cellphones or the likes. When you want to bring back how the document looks like a while ago, you keep clicking this button and the document gradually goes back to its previous layout. It keeps deleting the changes you do to the document one by one from the most recent to the earliest change you made.

Meanwhile, what you did with your chosen number is you multiplied it by 2 and then added 3 to it. Why 3? Because you subtracted 1 and then added 4 to the number which is the same as adding 3 to it. Going back to the principle of “undo”, this is how I guessed your original number by telling me your final answer.

Commands	Undo
Step 3. Add 3 to it. $(2x + 3)$	Step 1. Subtract 3 from your answer y . $(y - 3)$
Step 2. Multiply it by 2. $(2x)$	Step 2. Divide it by 2. $\frac{y-3}{2}$
Step 1. Think of a number. (x)	Answer will be the number you are thinking. (x)

By that way, I have seen your mind and have created a new set of commands. It's now your turn to try it with your family members or peer. Experience their oohs and aahs!



What is It

Inverse Function Defined

The inverse of a function is a function with domain B and range A given that the original function has domain A and range B.

This inverse function of function f is denoted by f^{-1} . It is defined by the equation $f^{-1}(y) = x$ if and only if $f(x) = y$ for any y in range B. Since both are functions, then a function has to be one-to-one for its inverse to be a function at the same time. If it is a many-to-one function, its inverse is one-to-many which is not a function.

How to Find the Inverse of One-to-one Function

Intuitively, the inverse of a function may be known by the principle of “undo”. That is, by considering the inverses of the operations performed, the inverse of a function may be computed easily.

Example 1

Given $f(x) = 3x - 8$, the inverse of a function may be solved intuitively.

Solution:

Steps	In symbols
Step 1. The last operation performed is subtraction, the inverse operation of which is addition. To x , add 8.	$x + 8$
Step 2. The second to the last operation performed is multiplication, the inverse operation of which is division. Divide $x + 8$ by 3.	$\frac{x + 8}{3}$
Step 3. Equate it to $f^{-1}(x)$ to denote that it is the inverse function of $f(x) = 3x - 8$.	$f^{-1}(x) = \frac{x + 8}{3}$

However, it is not that easy in some cases. In later examples, you will understand what I mean by saying that there is a more general method that may be followed.

To find the inverse of a one-to-one function, consider the following steps:

- Express the function in the form $y = f(x)$;
- Interchange the x and y variables in the equation;
- Solve for y in terms of x .

Example 2

If it exists, solve for the inverse of $g(x) = x^2 - 6x - 7$.

Solution:

Recognize that $g(x)$ is a quadratic function whose graph is a parabola opening upward. It fails the horizontal line test because it has x -values which correspond to the same y -value. And since it is not a one-to-one function, then its inverse is not a function. Simply put, it has no inverse function.

Alternate Solution:

$$y = x^2 - 6x - 7 \quad (\text{change } g(x) \text{ to } y)$$

$$x = y^2 - 6y - 7 \quad (\text{interchange } x \text{ and } y)$$

$$y^2 - 6y = x + 7 \quad (\text{solve for } y, \text{ APE})$$

$$y^2 - 6y + 9 = x + 7 + 9 \quad (\text{solve for } y, \text{ by completing the square, by APE})$$

$$(y - 3)^2 = x + 16 \quad (\text{solve for } y, \text{ by factoring})$$

$$y - 3 = \pm\sqrt{x + 16} \quad (\text{solve for } y, \text{ by getting the square root of both sides})$$

$$y = \pm\sqrt{x + 16} + 3 \quad (\text{solve for } y, \text{ by APE})$$

Notice that for some values of x , there are two values of y . For instance, if $x=1$, $y = \sqrt{17} + 3$ and $y = -\sqrt{17} + 3$. Therefore, the inverse function of $g(x)$ does not exist.

Example 3

Find the inverse of the rational function $h(x) = \frac{4x+8}{x-3}$.

Solution:

$$y = \frac{4x+8}{x-3} \quad (\text{change } g(x) \text{ to } y)$$

$$x = \frac{4y+8}{y-3} \quad (\text{interchange } x \text{ and } y)$$

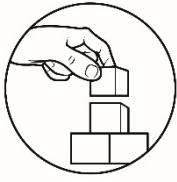
$$xy - 3x = 4y + 8 \quad (\text{solve for } y, \text{ MPE})$$

$$xy - 4y = 3x + 8 \quad (\text{solve for } y, \text{ by APE})$$

$$y(x - 4) = 3x + 8 \quad (\text{solve for } y, \text{ by factoring})$$

$$y = \frac{3x+8}{x-4} \quad (\text{solve for } y, \text{ by MPE})$$

$$h^{-1}(x) = \frac{3x+8}{x-4} \quad (\text{the inverse function})$$



What's More

Activity 1

Directions: By intuition, give the inverse function of each of the following. Write your answers on a separate sheet of paper.

1. $f(x) = x + 2$

2. $g(x) = 12x - 1$

3. $h(x) = -\frac{x}{4}$

4. $f(x) = x$

5. $g(x) = \frac{3x+5}{8}$

Activity 2

Directions: Solve for the inverse function of each of the following if it exists.

1. $f(x) = 25x - 18$

2. $g(x) = \frac{12x-1}{7}$

3. $h(x) = -\frac{9x}{4} - \frac{1}{3}$

4. $f(x) = x^9$

5. $f(a) = a^3 + 8$

6. $g(a) = a^2 + 8a - 7$

7. $f(b) = (b + 6)(b - 2)$

8. $h(x) = \frac{2x+17}{3x+1}$

9. $h(c) = \sqrt{2c + 2}$

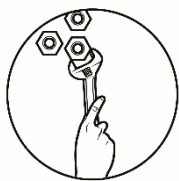
10. $f(x) = \frac{x+10}{9x-1}$



What I Have Learned

Answer the following questions.

1. What is an inverse function?
2. What is the symbol of an inverse function?
3. Do all kinds of functions have inverse function?
4. How do you solve for the inverse of a one-to-one function?

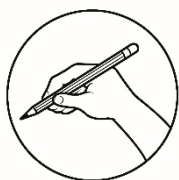


What I Can Do

Show that $f(x) = |5x|$ has no inverse function. Present your work through a comic strip. Use a separate sheet of paper for your work.

The following rubric will be used in rating your work:

Criteria	3	2	1
Solution (score x 4)	with correct solution	with minor computational errors	with major computational errors
Strategy (score x 3)	with insightful strategy that demonstrates conceptual understanding of mathematical rules and properties	with appropriate strategy that demonstrates procedural understanding, but not conceptual understanding	with absence of logic in strategy that will not lead to the correct answer
Presentation (score x 3)	images and captions show an exceptional degree of creativity; with clear identification of characters	images and captions show some degree of creativity; with identification of some characters	images and captions show a little degree of creativity; characters cannot be clearly identified



Assessment

Multiple Choice. Choose the letter of the best answer. Write the chosen letter on a separate sheet of paper.

- Which relates to “inverse”?
 - redo
 - opposite
 - delete
 - interchange
- What is the inverse of $f(x) = x + 36$?
 - $f^{-1}(x) = \frac{x}{36}$
 - $f^{-1}(x) = 36x$
 - $f^{-1}(x) = x - 36$
 - $f^{-1}(x) = -x - 36$

3. What is the inverse of $f(x) = \frac{x}{25}$?
 - a. $f^{-1}(x) = x + 25$
 - b. $f^{-1}(x) = 25x$
 - c. $f^{-1}(x) = x - 25$
 - d. $f^{-1}(x) = -25x$

4. What is the inverse of $f(x) = -3x$?
 - a. $f^{-1}(x) = x + 3$
 - b. $f^{-1}(x) = -\frac{x}{3}$
 - c. $f^{-1}(x) = x - 3$
 - d. $f^{-1}(x) = \frac{x}{3}$

5. What is the inverse of $f(x) = x - 10$?
 - a. $f^{-1}(x) = 10 + x$
 - b. $f^{-1}(x) = -\frac{x}{10}$
 - c. $f^{-1}(x) = -10x$
 - d. $f^{-1}(x) = -(x - 10)$

6. Which characterizes an inverse function?
 - a. Given that it is $f^{-1}(x)$, its domain and range are the same as the domain and range of $f(x)$.
 - b. It is denoted by $y = f^{-1}(x)$.
 - c. Its inverse is one-to-one or many-to-one.
 - d. It is one-to-one or many-to-one.

7. What is the mathematical symbol for inverse of $f(x)$?
 - a. $\frac{1}{f(x)}$
 - b. $f(x)^{-1}$
 - c. $'f(x)$
 - d. $f^{-1}(x)$

8. What is the inverse of $f(x) = -2x - 8$?
 - a. $f(x) = 8x + 2$
 - b. $f(x) = 2x + 8$
 - c. $f(x) = \frac{x+2}{8}$
 - d. $f(x) = \frac{x+8}{-2}$

9. What is the inverse of $f(x) = b^5 + 2$?
 - a. $f(x) = 2b - 5$
 - b. $f(x) = 5b + 2$
 - c. $f(x) = \sqrt[5]{b-2}$
 - d. $f(x) = \sqrt[5]{b} - 2$

10. Which is a property of an inverse function?

- a. The inverse of $f^{-1}(x)$ is $f(x)$.
- b. $f^{-1}(f(x)) = x$ for all negative x in the domain of f .
- c. $f^{-1}(f^{-1}(x)) = x$ for all x in the domain of f^{-1} .
- d. $f(f^{-1}(x)) = x$ for all positive x in the domain of f^{-1} .

11. What is the inverse of $f(x) = -6x - 5$?

- a. $f(x) = \frac{x+5}{-6}$
- b. $f(x) = -\frac{x}{6} + 5$
- c. $f(x) = 6x + 5$
- d. $f(x) = 5x + 6$

12. What is the inverse of $f(c) = (c + 1)^3 - 1$?

- a. $f(c) = \sqrt[3]{c}$
- b. $f(c) = \sqrt[3]{c+1} - 1$
- c. $f(c) = \sqrt[3]{c-1} + 1$
- d. $f(c) = 1 - \sqrt[3]{c+1}$

13. What is the inverse of $g(x) = 9x + 20$?

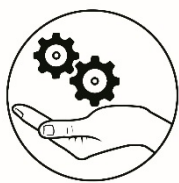
- a. $g'(x) = \frac{x+20}{-9}$
- b. $g'(x) = -20x + 9$
- c. $g^{-1}(x) = \frac{x-20}{9}$
- d. $g^{-1}(x) = -9x - 20$

14. What is the inverse function of $f(d) = \frac{d-12}{2d+1}$?

- a. $f'(d) = \frac{d+12}{-2d-1}$
- b. $f'(d) = \frac{-d+12}{-2d-1}$
- c. $f'(d) = \frac{d-12}{2d-1}$
- d. $f'(d) = \frac{-d-12}{2d-1}$

15. Which is involved in the process of finding the inverse of a function?

- a. Write the function in the form $x = f(x)$.
- b. Cancel the x variable.
- c. Write in the function in the simplest form.
- d. Solve for y in terms of x .



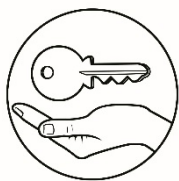
Additional Activities

In real life, can we undo events? Have you experienced any conflict with your family or peer on concerns like showing respect, being honest and trustworthy or being helpful and cooperative? What do you do to make amends? This time try writing a letter to a family member or peer expressing your regret over an event. Pour out your heart and feel light after then.

To make sure you'll put smile on their faces, try scoring your letter using the rubric below:

Criteria	4	3	2	1
Content (score x 3)	Focus on actions to take to resolve the situation; sincere and polite tone; admit one's fault; with follow up	Involves only three of the four characteristics cited at the left	Involves only two of the four characteristics cited	Involves only one of the four characteristics cited
Grammar and mechanics (score x 2)	Sentences are clear; use commas and other punctuations properly; no lengthy narration in every sentence; sentences are arranged properly	Involves only three of the four characteristics cited at the left	Involves only two of the four characteristics cited	Involves only one of the four characteristics cited

If you scored your letter and it ranged from 15 to 20, proceed giving your letter wholeheartedly. If the score you give is below 15, consider revising it before giving it to your loved one. This is a rare moment, make it count.



Answer Key

What I Know

1. D
2. C
3. B
4. B
5. A
6. A
7. D
8. D
9. B
10. D
11. A
12. D
13. C
14. B
15. C

What's More

Activity 12.1

$$1. f^{-1}(x) = x - 2$$

$$2. g^{-1}(x) = \frac{12}{x+1}$$

$$3. h^{-1}(x) = -4x$$

$$4. f^{-1}(x) = x$$

$$5. g^{-1}(x) = \frac{3}{8x-5}$$

Activity 12.2

$$1. f^{-1}(x) = \frac{x+18}{25}$$

$$2. g^{-1}(x) = \frac{12}{7x+1}$$

$$3. h^{-1}(x) = \frac{9}{-4(x+\frac{1}{3})}$$

$$\text{or } h^{-1}(x) = -\frac{9}{4x} + \frac{27}{4}$$

$$4. f^{-1}(x) = \sqrt[9]{x}$$

$$5. f^{-1}(a) = \sqrt[3]{a-8}$$

6. It has no inverse.

Initially, the given is

not a one-to-one

function. Or, by

solving for the inverse,

$$y = \pm\sqrt{x+23} - 4. \text{ There}$$

are y-values each of

which is paired to two

x-values.

7. It has no inverse.

Initially, the given is

not a one-to-one

function. Or, by

solving for the inverse,

$$y = \pm\sqrt{x+16} - 2. \text{ There}$$

are ordinates each of

which is paired to two

abscissas.

$$8. h^{-1}(x) = \frac{-x+17}{3x-2} \text{ or } -\frac{x-17}{3x-2}$$

$$9. h^{-1}(c) = \frac{c^2-2}{2}$$

$$10. f^{-1}(x) = \frac{x+10}{x-1}$$

Assessment

1. D
2. C
3. B
4. B
5. A
6. B
7. D
8. D
9. C
10. A
11. A
12. B
13. C
14. D
15. D

Reference

Verzosa, Debbie Marie, et.al. 2016. *General Mathematics: Learner's Material, First Edition*. Philippines: Lexicon Press Inc.

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