9

Mathematics

Quarter 1-Module 6

Nature of Roots of a Quadratic Equation

Week 2

Learning Code - M9AL-Ib-3

ALTERNATIVE DELIVERY MODE

ADM

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MODULE 6

NATURE OF ROOTS OF A QUADRATIC EQUATION

From your previous modules, you learned how to get the roots of a quadratic equation. At this point, you will explore on describing the characteristics of the roots of a quadratic equation without solving for the roots. This knowledge would come in handy on some real-life scenarios especially in decision making.

WHAT I NEED TO KNOW

LEARNING COMPETENCY

The learners will be able to:

- characterize the roots of a quadratic equation using the discriminant. **M9AL-Ib-3**

WHAT I KNOW

Find out how much you already know about the nature of roots of quadratic equation. Write the letter that you think is the best answer to each question on your answer sheet. Answer all items. After taking and checking this short test, take note of the items that you were not able to answer correctly and look for the right answer as you go through this module.

1. Which of the following is the discriminant of the quadratic equation: $ax^2 - bx + c = 0$, where a, b, c are real number and $a \neq 0$?
   A. b^2
   B. $b^2 - 4ac$
   C. $\sqrt{b^2 - 4ac}$
   D. $-b \pm \sqrt{b^2 - 4ac}$

2. How many real roots does the quadratic equation $x^2 + 8x + 12 = 0$ have?
   A. 0
   B. 1
   C. 2
   D. 3

3. Which of the following quadratic equations has no real roots?
   A. $2x^2 + 4x = 3$
   B. $z^2 - 8z - 4 = 0$
   C. $3m^2 - 2m + 5 = 0$
   D. $-2f^2 + f = -4$

4. Which of the following is the nature of the roots of the quadratic equation if the value of its discriminant is zero?
   A. The roots are not real.
   B. The roots are irrational and not equal.
   C. The roots are rational and not equal.
   D. The roots are rational and equal.

5. Which of the following is the nature of the roots of the quadratic equation if the value of its discriminant is negative?
   A. The roots are not real.
   B. The roots are rational and equal.
   C. The roots are rational and not equal.
   D. The roots are irrational and not equal.

6. Which of the following is the nature of the roots of the quadratic equation if the value of its discriminant is positive and a perfect square?
   A. The roots are not real.
   B. The roots are rational and equal.
   C. The roots are irrational and not equal.
   D. The roots are rational and not equal.

7. Which of the following is the nature of the roots of the quadratic equation if the value of its discriminant is positive and not a perfect square?
   A. The roots are not real.
   B. The roots are rational and equal.
   C. The roots are rational and not equal.
   D. The roots are irrational and not equal.

8. What is the value of r in the quadratic equation $x^2 - 10x - r = 0$ whose roots are equal rational and real?
   A. -25
   B. -10
   C. 10
   D. 25

9. Which of the following quadratic equations has equal roots?
   A. $x^2 + 6x + 9 = 0$
   B. $x^2 + 5x + 10 = 0$
   C. $2x^2 - 10x + 8 = 0$
   D. $3x^2 - 2x - 5 = 0$

10. What are the values of k in the quadratic equation $6x^2 - 5x - k = 0$ whose roots are imaginary?
   A. $k < \frac{-25}{24}$
   B. $k < \frac{25}{24}$.
   C. $k > \frac{-25}{24}$
   D. $k > \frac{25}{24}$.

WHAT'S IN

Communication, Collaboration

Before we begin, let us unlock the following vocabularies. Get your dictionary or any math book and find the meaning of the following words. Give at least two examples of each.

1. Real numbers
2. Imaginary numbers
3. Rational Numbers
4. Irrational Numbers

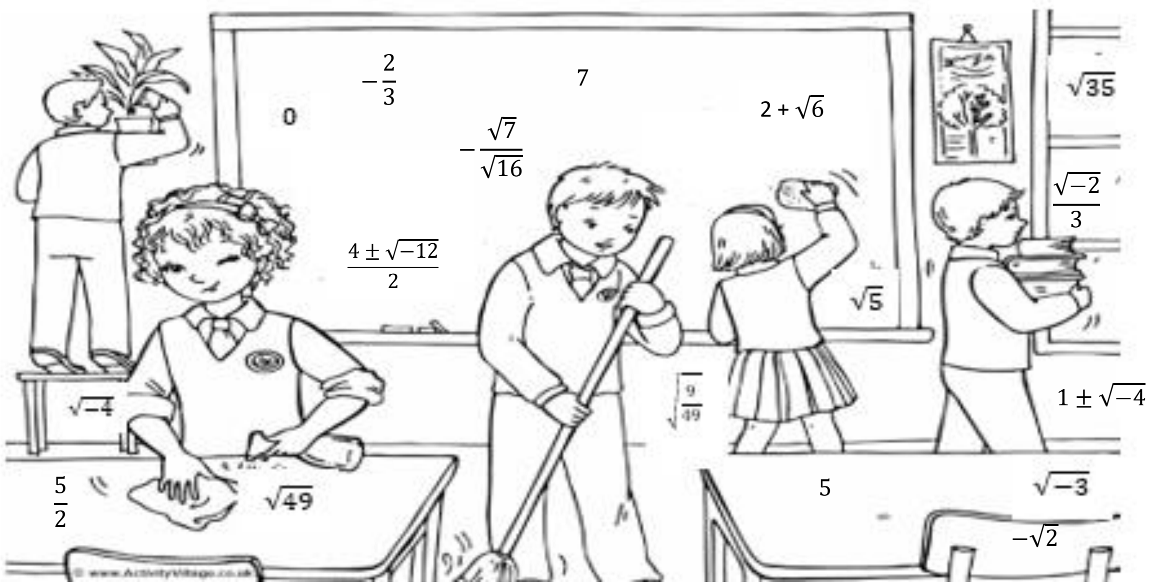
Did you find the meaning of the words above? Now apply these words in the next activity.

Let us FIND and INVESTIGATE!

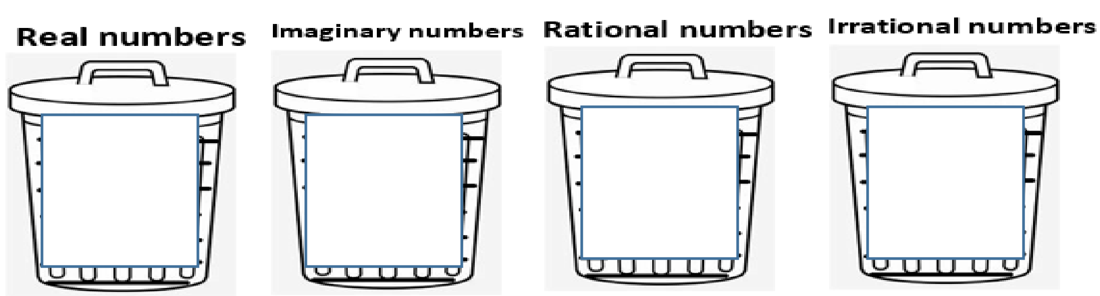
Real or Imaginary? Rational or Irrational?

This is a classroom hidden number game. In this game you must find out the hidden numbers and place it in the appropriate trash bin.

Reminder: a number can be written in two trash bins.



1Source: http://clipart-library.com/clean-classroom-cliparts.html



WHAT'S NEW

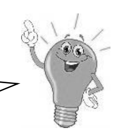
The quadratic equations listed can be solved by using quadratic formula as discussed from the previous topic. Using this method, find and examine the roots obtained to complete the table below. In this case, let r_1 and r_2 be the roots.
Put a check in the cell that corresponds to the characteristics of the roots attained in each equation.

EQUATIONS	**ROOTS**		**Real or Imaginary?**		**Rational or irrational?**		**Equal or unequal?**		**The value of b²-4ac**
	r_1	r_2	**Real**	**Imaginary**	**Rational**	**Irrational**	**Equal**	**Unequal**	
(a) $x^2 - 7x + 10 = 0$									9
(b) $x^2 - 4x + 7 = 0$									-12
(c) $x^2 - 10x + 25 = 0$									0
(d) $x^2 - 4x - 2 = 0$									24

If the quadratic formula is used to get the roots, the values of $b^2 - 4ac$ *(quantity under the radical sign) for those equations are* $9, -12, 0$ *and* 24 *respectively.*

WHAT IS IT

Communication, Critical Thinking, and Collaboration



When describing the natures or characters of the roots of a quadratic equation, it can be one of each of the following:

(a) Real or Imaginary
(b) Rational or irrational
(c) Equal or unequal

Given the form of the equation, $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$, then using the quadratic formula, the roots are

$$\frac{-b+\sqrt{b^2-4ac}}{2a} \quad \text{and} \quad \frac{-b-\sqrt{b^2-4ac}}{2a}$$

Looking at the two roots, we can say that they are real or imaginary by using the quantity under the radical sign. It depends whether it is positive, zero or negative.

Similarly, if the value inside the radical sign is a perfect square, then the roots are rational; if not, they are irrational.

In order to have equal roots, the quantity under the radical sign must be zero.

Therefore, the nature of the roots can be decided by using the quantity $b^2 - 4ac$, which is called the **discriminant** of the equation $ax^2 + bx + c = 0$.

Thus:

(a) If $b^2 - 4ac = 0$; the roots are *real, equal* and *rational.*
(b) If $b^2 - 4ac < 0$; the roots are *unequal* and *imaginary*.
(c) If $b^2 - 4ac > 0$ and a perfect square; the roots are *real*, *unequal* and rational.
(d) If $b^2 - 4ac > 0$ but not a perfect square; the roots are *real*, *unequal,* and *irrational.*

Applying this conclusion, you may check your answer on the last activity in the "WHAT'S NEW" part.

Example 1: By inspection, determine the nature of the roots of the following equations:

(a) $x^2 - 7x - 8 = 0$
(b) $2x^2 - 4x + 1 = 0$
(c) $3x^2 + 9x + 11 = 0$
(d) $x^2 - 12x + 36 = 0$

Solutions:

(a) The equation $x^2 - 7x - 8 = 0$ gives $a = 1$, $b = -7$, and $c = -8$
Substituting the given values of a,b, and c in the expression we have

$$b^2 - 4ac = (-7)^2 - 4(1)(-8)$$
$$= 49 + 32$$
$$= 81$$

Since 81 is greater than 0 and a perfect square, then the equation $x^2 - 7x - 8 = 0$ has ***2 real, rational, and unequal roots.***

(b) The equation $2x^2 - 4x + 1 = 0$ gives $a = 2$, $b = -4, and\ c = 1$
Substituting the given values of a,b, and c in the expression we have

$$b^2 - 4ac = (-4)^2 - 4(2)(1)$$
$$= 16 - 8$$
$$= 8$$

Since 8 is greater than 0 and not a perfect square, then the equation $2x^2 - 4x + 1 = 0$ has ***real, irrational, and unequal roots.***

(c) The equation $3x^2 + 9x + 11 = 0$ gives $a = 3, b = 9,$ and $c = 11$

Substituting the given values of a,b, and c in the expression we have

$$b^2 - 4ac = (9)^2 - 4(3)(11)$$
$$= 81 - 132$$
$$= -44$$

Since -44 is less than 0, then the equation $3x^2 + 9x + 11 = 0$ has ***imaginary* and unequal roots.**

(d) The equation $x^2 - 12x + 36 = 0$ gives $a = 1$, $b = -12$, and $c = 36$
Substituting the given values of a,b, and c in the expression we have

$$b^2 - 4ac = (-12)^2 - 4(1)(36)$$
$$= 144 - 144$$
$$= 0$$

Since the discriminant is equal to zero, then the equation $x^2 - 12x + 36 = 0$ has ***real, rational and equal roots.***

Example 2: Answer the following

(a) Given the equation $(3k + 1)x^2 + 2(k + 1)x + k = 0$, find the values of k for which the roots are equal.
(b) For what values of k are the roots of $6x^2 - 5x - k = 0$ is: i.) imaginary? ii.) real and unequal?
(c) Find r so that the roots of the equation $(r - 3)x^2 - (r - 6)x - 4 = 0$ are equal?

Solutions:

(a) Based on the given equation, $(3k + 1)x^2 + 2(k + 1)x + k = 0$, the expressions for a, b, c are $a = 3k + 1, b = 2(k + 1)$, and $c = k$.

For equal roots, the discriminant must be equal to zero. Therefore,

$$b^2 - 4ac = 0$$
$$[2(k + 1)]^2 - 4(3k + 1)(k) = 0$$
$$(2k + 2)^2 - (12k + 4)(k) = 0$$
$$4k^2 + 8k + 4 - 12k^2 - 4k = 0$$
$$-8k^2 + 4k + 4 = 0$$
$$8k^2 - 4k - 4 = 0$$

Dividing both sides by 4, we have:

$$2k^2 - k - 1 = 0$$

To solve for k, we have:

$$(k - 1)(2k + 1) = 0$$

Equating both factors to zero and solving for k,

k – 1 = 0 or 2k + 1 = 0
k = 1   2k = -1
$k = -\frac{1}{2}$

Thus, the values of k for which the roots are equal are 1 and $-\frac{1}{2}$.

(b) Given the equation of $6x^2 - 5x - k = 0$, we have $a = 6, b = -5,$ and $c = -k$

(i) In order to have imaginary roots, $b^2 - 4ac$ must be less than zero which is denoted by:

$$b^2 - 4ac < 0$$

Hence,

$$(-5)^2 - 4(6)(-k) < 0$$
$$25 + 24k < 0$$
$$24k < -25$$

Therefore, $\boldsymbol{k < \frac{-25}{24}}$.

(ii) In order to have real and unequal roots, $b^2 - 4ac$ must greater than 0 which is denoted by:

$$b^2 - 4ac > 0$$

Therefore, based on (i), $\boldsymbol{k > \frac{-25}{24}}$. ***[complement of the answer in (i)]***

(c) Given the equation $(r - 3)x^2 - (r - 6)x - 4 = 0$, we have a = r – 3, b = -(r – 6), and c = -4.

For equal roots, the discriminant must be equal to zero. Therefore,

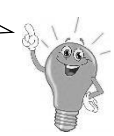
$$b^2 - 4ac = 0$$
$$[-(r - 6)]^2 - 4(r - 3)(-4) = 0$$
$$r^2 - 12r + 36 + 16r - 48 = 0$$
$$r^2 + 4r - 12 = 0$$

Solving for r, we have

$$(r - 2)(r + 6) = 0$$
$$r - 2 = 0 \quad \text{or} \quad r + 6 = 0$$
$$r = 2 \qquad\qquad r = -6$$

Therefore, the values of r for which the roots are equal are 2 and -6.

Critical Thinking,
Collaboration



WHAT'S MORE

I. Using the discriminant, characterize the roots of the following quadratic equations:
1. $x^2 - 3x - 5 = 0$
2. $x^2 = 5x - 11$
3. $5x^2 - 6x + 13 = 0$

II. Given $(3k + 1)x^2 + (11 + k)x + 9 = 0$. Find the values of k for which the roots are:
(a) equal;
(b) imaginary;
(c) real and unequal

III. Write a quadratic equation of each of the following types:
(a) two rational roots
(b) two imaginary roots
(c) two irrational roots
(d) one rational root

WHAT I HAVE LEARNED

The nature of the roots of a quadratic equation using the **discriminant** without solving the equation. To characterize the roots of the quadratic equation $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$, we have the following:

1. If $b^2 - 4ac = 0$, the roots are real, rational and equal.
2. If $b^2 - 4ac > 0$ and a perfect square, the roots are real, rational, and unequal.
3. If $b^2 - 4ac > 0$ and not a perfect square, the roots are real, irrational and unequal.
4. If $b^2 - 4ac < 0,$ the roots are imaginary and unequal.

WHAT I CAN DO

Critical Thinking

I. Characterize the roots of the following quadratic equations using the discriminant.

1. $6x^2 - x + 2 = 0$
2. $x^2 - 3x + 7 = 0$
3. $m^2 + 8m + 16 = 0$
4. $3x^2 + 8x - 8 = 0$
5. $3x^2 - 4x + 2 = 0$
6. $7x^2 - 10x + 1 = 0$
7. $t^2 - 3t + 2 = 0$
8. $8x^2 - 16x + 9 = 0$
9. $r^2 - r = 0$
10. $3t^2 + 4t = 8$

II. Solve the following:

1. Find the values of k that will make the roots of $4x^2 - kx + 4 = 0$ equal.
2. For what values of k are the roots of $kx^2 - 4x + 5 = 0$ are imaginary?
3. Determine the value of c if $4x^2 - 5x + c = 0$ has real and unequal roots.
4. The equation $7x^2 - 3x + m = 0$ has imaginary roots. Determine the value of m.
5. Find k so that the roots of the equation $w^2 + (k + 3)w + 3k = 0$ are equal.

ASSESSMENT

Write the letter of the correct answer on your answer sheet. If your answer is not among the choices, write E together with your final answer.

1. Which of the following is the discriminant of the quadratic equation: $x^2 + 2x + 5 = 0$?
   A. -16
   B. -4
   C. 4
   D. 16

2. How many real roots does the quadratic equation $5x^2 - 8x + 6 = 0$ have?
   A. 0
   B. 1
   C. 2
   D. 3

3. Which of the following quadratic equations has real roots?
   A. $x^2 + x + 1 = 0$
   B. $2x^2 + 5x + 3 = 0$
   C. $2x^2 - 6x + 6 = 0$
   D. $4x^2 + 3x + 3 = 0$

4. The roots of a quadratic equation are imaginary. Which of the following statement is true about the discriminant of equation?
   A. The discriminant is negative.
   B. The discriminant is equal to zero.
   C. The discriminant is positive and a perfect square.
   D. The discriminant is positive and not perfect square.

5. The roots of a quadratic equation are 2 and 5.Which of the following statement is true about the discriminant of equation?
   A. The discriminant is negative.
   B. The discriminant is equal to zero.
   C. The discriminant is positive and not perfect square.
   D. The discriminant is positive and a perfect square.

6. Given the equation, $4x^2 + 9x - 13 = 0$, which of the following is the nature of its roots?
   A. The roots are real, rational and equal.
   B. The roots are imaginary and unequal.
   C. The roots are real, irrational and unequal.
   D. The roots are real, rational, and unequal.

7. The discriminant of a quadratic equation is 34. Which of the following is true about its roots?
   A. The roots are real, rational and equal.
   B. The roots are imaginary and unequal.
   C. The roots are real, irrational and unequal.
   D. The roots are real, rational, and unequal.

8. What is the value of t that will make the roots of the equation $8x^2 - 4tx + 3 = 0$ equal?
   A. $t = \pm 6$
   B. $t = \pm 96$
   C. $t = \pm\sqrt{6}$
   D. $t = \pm 4\sqrt{6}$

9. Which of the following quadratic equations has rational, equal and real roots?
   A. $9 - 6x + x^2 = 0$
   B. $x^2 + 2x + 2 = 0$
   C. $x^2 + 4x - 21=0$
   D. $4x^2 + 12x + 10$

10. What is/are the value/s of k in the quadratic equation $2x^2 - 5x - k = 0$ whose roots are imaginary?
   A. $k < \frac{25}{8}$
   B. $k < -\frac{25}{8}$
   C. $k > -\frac{25}{8}$
   D. $k > \frac{25}{8}$

ADDITIONAL ACTIVITIES

Critical Thinking,
Collaboration, Creativity

The shortest way of describing the nature of the roots of a quadratic equation is not by solving for the roots but by just using the discriminant of the given quadratic equation.

Below is a specific scenario on how this lesson may help you in some decision making.

A. Apply the idea of the nature of roots of a quadratic equation to visualize, answer if it is possible or not, and then justify your answer.

SCENARIO 1
Is it possible to make a rectangular park of perimeter 80 m and area 400 m^2 ?

SCENARIO 2
The sum of the ages of two girls is 20 years. The product of their ages four years ago was 48.Is this possible?

SCENARIO 3
Is it possible to design a rectangular rice field whose length is twice its width and the area is 800 m^2?

PROBLEM – BASED WORKSHEET

SPOT THE ERROR

Maria and Jose wanted to determine the number of real solutions of the quadratic equation $5x^2 - 3x = 2$.

Maria

$5x^2 - 3x = 2$

$b^2 - 4ac = (-3)^2 - 4(5)(2)$

$= -31$

Since the discriminant is -31, there are no real roots.

Jose

$5x^2 - 3x = 2$

$5x^2 - 3x - 2 = 0$

$b^2 - 4ac = (-3)^2 - 4(5)(-2)$

$= 49$

Since the discriminant is 49, positive and perfect square, there are two real roots.

Who is correct? Justify your answer.

E-Search

You may also check the following link for your reference and further learnings on nature of the roots of quadratic equations:

- https://www.khanacademy.org/math/in-in-grade-10-ncert/x573d8ce20721c073:in-in-chapter-4-quadratic-equation/x573d8ce20721c073:in-in-10-quadratic-discriminant-and-number-of-solutions/v/discriminant-for-types-of-solutions-for-a-quadratic
- https://www.toppr.com/guides/maths/quadratic-equations/nature-of-roots/

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Oronce, O. A., Santos, G. C., & Ona, M. I. (n.d.). *Interactive Mathematics III (Concepts,Structures, adn Metods for High School.* pp 250-256Manila: Rex Book Store.

https://www.onlinemath4all.com/solving-word-problems-with-nature-of-roots-of-quadratic-equation.html

https://ya-webdesign.com/explore/trashcan-drawing-basura/
http://clipart-library.com/clean-classroom-cliparts.html
https://www.freepik.com/free-vector/woman-with-long-hair-teaching-online_7707557.htm
https://www.freepik.com/free-vector/kids-having-online-lessons_7560046.htm

https://www.freepik.com/free-vector/illustration-with-kids-taking-lessons-online-design_7574030.htm

WHAT I KNOW
1. B
2. C
3. C
4. D
5. A
6. D
7. D
8. A
9. A
10. A

ASSESSMENT
1. A
2. A
3. B
4. A
5. D
6. D
7. C
8. C
9. A
10. B

PROBLEM – BASED WORKSHEET

Solutions:

Maria made a mistake by getting the values of a, b and c without re-writing the equation in the form $ax^2 + bx + c = 0$.

Jose did the correct solution. He first re-write the equation in the form $ax^2 + bx + c = 0$ and came up with $a = 5$, $b = -3$ and $c = 2$.

Thus, the discriminant is 49. It is a positive number and a perfect square, therefore there are two real roots.

WHAT'S MORE

I.

1. The roots are real, irrational and unequal.
2. The roots are imaginary and unequal.
3. The roots are imaginary and unequal.

II.

a. $k = 1 \; or \; k = 85$
b. $1 < k < 85$
c. $k > 85 \; or \; k < 1$

ADDITIONAL ACTIVITIES

1. Yes, the dimension that will lead to an area of $400m^2$ from a perimeter of 80m is a 20m by 20m square. Since a square is a rectangle, thus the scenario is possible.
2. No, if the product of the girl's ages 4 years ago is 48, then the sum of their ages is 20 less 8.

   Let x and y be the girls' ages
   The equation will be,

   $x + y = 12$
   $xy = 48$

   Solving for x and y,

   If $x + y = 12$, then $y = 12 - x$.

   Substitute the value to the second equation, we have

   $x (12 - x) = 48$
   $-x^2 + 12x - 48 = 0$

   Solving for x using quadratic formula,

   $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
   $x = \frac{-12 \pm \sqrt{12^2 - 4(-1)(-48)}}{2(-1)}$
   $x = \frac{-12 \pm \sqrt{12^2 - 4(-1)(-48)}}{2(-1)}$
   $x = \frac{-12 \pm \sqrt{-48}}{-2}$

   Since the radicand is negative, the roots are imaginary. Thus, the product of the ages four years ago is impossible.
3. Yes, if we let x = width, then 2x = length. The area is:

   $x (2x) = 800$

   $2x^2 = 800$

   $x^2 = 400$

Hence x = 20. This gives a width of 20 m, and a length of 40 m.

ANSWER KEY

WHAT'S IN

Real Numbers	Imaginary Numbers	Rational Numbers	Irrational Numbers
$0, -\frac{2}{3}, 7, \frac{-\sqrt{7}}{\sqrt{16}}$, $2+\sqrt{6}, \sqrt{5}, \sqrt{35}, \sqrt{\frac{9}{49}}$, $\frac{5}{2}, \sqrt{49}$, $-\sqrt{2}$	$\frac{4\pm\sqrt{-12}}{2}, \frac{\sqrt{-2}}{3}, \sqrt{-3}$, $1 \pm \sqrt{-4}$	$0, -\frac{2}{3}, 7, \sqrt{\frac{9}{49}}, \frac{5}{2}, \sqrt{49}$	$2+\sqrt{6}, \frac{-\sqrt{7}}{\sqrt{16}}, \sqrt{5}$, $\sqrt{35}, \sqrt{2}$

WHAT'S NEW

EQUATIONS	ROOTS		Real or Imaginary?		Rational or irrational?		Equal or unequal?	
	r_1	r_2	Real	Imaginary	Rational	Irrational	Equal	Unequal
(a) $x^2 - 7x + 10 = 0$	5	2	√		√			√
(b) $x^2 - 4x + 7 = 0$	$\frac{4 + \sqrt{-12}}{2}$	$\frac{4 - \sqrt{-12}}{2}$		√				√
(c) $x^2 - 10x + 25 = 0$	5	5	√		√		√	
(d) $x^2 - 4x - 2 = 0$	$2 + \sqrt{6}$	$2 - \sqrt{6}$	√			√		√

WHAT I CAN DO

I.

1. imaginary and unequal
2. imaginary and unequal
3. real, rational and equal
4. real, irrational and unequal
5. imaginary and unequal
6. real, irrational and unequal
7. real, rational and unequal
8. imaginary and unequal
9. real, rational and unequal
10. real, irrational and unequal

II.

1. $k = \pm 8$
2. $k > \frac{4}{5}$
3. $c < \frac{25}{16}$
4. $m > \frac{9}{28}$
5. $k = 3$