9

Mathematics

Quarter 1-Module 5

Solving Quadratic Equation Using Quadratic Formula

Week 1

Learning Code - M9AL-Ib-2.3

ALTERNATIVE DELIVERY MODE

ADM

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MODULE 5

SOLVING QUADRATIC EQUATION USING QUADRATIC FORMULA

If you recall the previous lessons, the methods are just applicable for a specific quadratic equation. For example, the process of "factoring" is appropriate only if the quadratic expression is factorable. But, do not worry because there is one process that is applicable for all forms of quadratic equations that will be tackled in this module.

WHAT I NEED TO KNOW

LEARNING COMPETENCY

The learners will be able to:

- solve quadratic equations by:(d) using the quadratic formula. **M9AL-Ib-2.3**

WHAT I KNOW

Find out how much you already know about solving quadratic equations using quadratic formula. Write the letter that you think is the best answer to each question on your answer sheet. Answer all items. After taking and checking this short test, take note of the items that you were not able to answer correctly and look for the right answer as you go through this module.

1. Which of the following is the most appropriate method to solve for the roots of quadratic equation $2.3x^2 + 1.5x - 3.4 = 0$?
   A. Extracting the Square Root
   B. Factoring
   C. Completing the Squares
   D. Quadratic Formula

2. The solutions to the quadratic equation $ax^2 + bx + c = 0$ are ______________
   A. x = $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
   B. x = $\frac{b\pm\sqrt{b^2-4ac}}{a}$
   C. x = $\frac{-b\pm\sqrt{b^2+4ac}}{2a}$
   D. x = $\frac{-b\pm\sqrt{b^2-4ac}}{a}$

3. Which is the correct substitution of the values of a, b and c in the quadratic formula of the equation $2(x^2 - 2x) = 5$?
   A. $x = \frac{-2\pm\sqrt{2^2-4(1)(5)}}{2(1)}$
   B. $x = \frac{-(-2)\pm\sqrt{(-2)^2-4(1)(5)}}{2(1)}$
   C. $x = \frac{-(-4)\pm\sqrt{(-4)^2-4(2)(-5)}}{2(2)}$
   D. $x = \frac{-4\pm\sqrt{4^2-4(2)(-5)}}{2(2)}$

4. Solve $x^2 - 6x + 2 = 0$ using quadratic formula.
   A. $2 \pm \sqrt{5}$
   B. $3 \pm \sqrt{7}$
   C. $\pm\sqrt{5}$
   D. $\pm\sqrt{7}$

5. If the roots of $2x^2 = -3(x + 2)$ is solve using quadratic formula, what is the first thing to consider?
   A. Identify the value of a, b, and c.
   B. Rewrite the equation in general form.
   C. Substitute the values of a, b, and c in the quadratic formula
   D. Divide both sides by a common factor
6. What are the real roots of the quadratic equation in #5?
   A. $\frac{3\pm\sqrt{57}}{4}$ B. $\frac{-3\pm\sqrt{57}}{4}$ C. $\frac{3\pm\sqrt{39}}{4}$ D. None
7. Solve for the real roots of $x^2 - 9x = -19$ using quadratic formula.
   A. $\frac{9\pm\sqrt{7}}{4}$ B. $\frac{-9\pm\sqrt{5}}{2}$ C. $\frac{9\pm\sqrt{5}}{2}$ D. None
8. Find the positive real root of the quadratic equation $-3x^2 = 8x - 12$ using quadratic formula.
   A. $\frac{-4+2\sqrt{13}}{3}$ B. $\frac{-4-2\sqrt{13}}{3}$ C. $\frac{4+2\sqrt{13}}{3}$ D. $\frac{4-2\sqrt{13}}{3}$
9. Which of the following quadratic equations does not have real roots when solved using quadratic formula?
   A. $x^2 - 8x - 33 = 0$ C. $x^2 + 13x - 30 = 0$
   B. $x^2 - 5x + 3 = 0$ D. $x^2 - 6x + 13 = 0$
10. Erick throws the ball from the roof onto the ground 9m below. He tosses the ball with an initial downward velocity of 3m per second. With H is the height of an object after t seconds, v is the initial velocity, and h is the initial height, the condition above is given by the equation $H = -16t^2 + vt + h$, how long does it take for the ball to hit the ground?
   A. -0.66 sec B. 0.85 sec C. 1.2 sec D. 2.14 se

WHAT'S IN

Before answering the questions in "**WHAT'S NEW**" part, let us have a quick discussion on how to simplify radicals. Take note that a simplified radical has none of the factors of the radicand can be written as powers greater than or equal to the index, there are no fractions under the radical sign and no radicals in the denominator.

Study how the given radical expressions are simplified.

a. $4 + \sqrt{500} = 4 + \sqrt{(100)(5)}$ — *Factor out 500 such that one number is the greatest possible perfect square factor.*

$= 4 + 10\sqrt{5}$ — *Simplify the radical part and combine terms, whenever possible.*

b. $\frac{-6-\sqrt{12}}{4} = \frac{-6-\sqrt{(4)(3)}}{4}$ — *Factor out 12 such that one number is the possible perfect square factor.*

$= \frac{-6-2\sqrt{3}}{4}$ — *Simplify the radical part.*

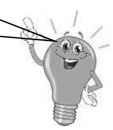
$= \frac{2(-3-\sqrt{3})}{4} = \frac{-3-\sqrt{3}}{2}$ — *Factor out the common monomial factor of the numerator, then simplify.*

Try This! Simplify the following radicals.

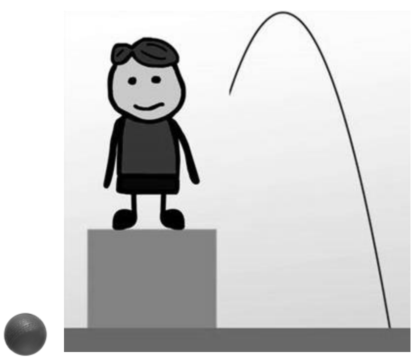
1. $\sqrt{125}$
2. $27 - \sqrt{49}$
3. $15 - \sqrt{112}$
4. $\frac{9+\sqrt{108}}{18}$
5. $\frac{-7+\sqrt{7^2-4(6)(-5)}}{12}$

WHAT'S NEW

Communication and Collaboration



Please Throw the Ball!



A certain group of boys play a ball outside Erick's house when one of them accidentally kicked it hard and went up to Erick's roof. They requested him to get the ball and throw it back to them. Without hesitation, Erick helped them and threw the ball from the roof onto the ground 9.14m below. He tossed the ball with an initial downward velocity of 3.05m per second.

With H as the height of an object after t seconds, v as the initial velocity, and h as the initial height, the condition above is given by the equation $H = -16t^2 + vt + h$.

What will be the equation to find how long it takes the ball to hit the ground? Do you think the methods of solving quadratic equation that you have learned in the previous modules will help you solve the equation you made?

WHAT IS IT

Communication, Critical Thinking, and Collaboration

The quadratic equation that can be formed from the story model earlier is $-16t^2 + 3.05t + 9.14 = 0$. Do you think the methods discussed previously can be used to solve for the roots of the obtained equation? Why?

There are quadratic equations that are difficult to solve by extracting the square roots, factoring or even completing the squares. These quadratic equations need a formula that will help you in solving for their roots.

By completing the square of the quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, a formula can be developed that gives the solutions to any quadratic equation in standard form. The formula is called Quadratic Formula.

Step	Illustrative Example
1. Place the constant term on the right side of the equation. All the terms with unknowns are on the left side.	$ax^2 + bx + c = 0$
2. The numerical coefficient of x^2 should be 1. Divide each term of the equation with the numerical coefficient of x^2 if necessary.	$ax^2 + bx = -c$ $x^2 + \frac{b}{a}x = -\frac{c}{a}$
3. To get the constant term needed to complete the square, get the numerical coefficient of x, divide it by 2 and square it. Add the result to both sides of the equation.	$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{c}{a} + \frac{b^2}{4a^2}$
4. Factor the perfect square trinomial.	$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2-4ac}{4a^2}$
5. Extract the square root from both sides. Two values will be obtained for the right side of the equation.	$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2-4ac}{4a^2}}$ $x + \frac{b}{2a} = \pm\frac{\sqrt{b^2-4ac}}{\sqrt{4a^2}}$
6. Equate the linear expressions to each of the two values.	$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2-4ac}}{2a}$
7. Solve each of the resulting linear equations.	$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}, for\ a \neq 0$ This is the quadratic formula.

Example 1: Solve for the real roots of each equation using quadratic formula.

a. $x^2 - 2x - 19 = 0$
b. $2x^2 + 6x = -3$
c. $2x^2 - 3x + 4 = 0$

Solutions

a. In $x^2 - 2x - 19 = 0$, $a = 1$, $b = -2$, and $c = -19$,

$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ *Quadratic formula*

$x = \frac{-(-2)\pm\sqrt{(-2)^2-4(1)(-19)}}{2(1)}$ *Substitute the values of a, b and c*

$x = \frac{2 \pm\sqrt{4+76}}{2}$ *Simplify*

$x = \frac{2 \pm\sqrt{80}}{2}$ *Evaluate the square root, if possible.*

$x = \frac{2 \pm 4\sqrt{5}}{2}$

$x = \frac{2(1 \pm 2\sqrt{5})}{2}$ *Simplify.*

$x = 1 \pm 2\sqrt{5}$

Therefore, the roots are $\{1 \pm 2\sqrt{5}\}$

b. When $2x^2 + 6x = -3$ is changed to standard form, we have $2x^2 + 6x + 3 = 0$. The value of a = 2, b = 6 and c = 3.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ *Quadratic formula*

$x = \frac{-6 \pm \sqrt{(6)^2 - 4(2)(3)}}{2(2)}$ *Substitute the values of a, b and c*

$x = \frac{-6 \pm \sqrt{36 - 24}}{4}$ *Simplify*

$x = \frac{-6 \pm \sqrt{12}}{4}$ *Evaluate the square root, if possible.*

$x = \frac{-6 \pm \sqrt{(4)(3)}}{4}$

$x = \frac{-6 \pm 2\sqrt{3}}{4}$ *Simplify*

$x = \frac{-3 \pm \sqrt{3}}{2}$

The roots are $\left\{\frac{-3 \pm \sqrt{3}}{2}\right\}$

c. In $2x^2 - 3x + 4 = 0$, $a = 2$, $b = -3$, and $c = 4$

x = $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Quadratic formula

x = $\frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(4)}}{2(2)}$ Substitute the values of a, b and c

x = $\frac{3 \pm \sqrt{-23}}{4}$ Simplify

Notice that the radicand is negative. It indicates that the roots are non-real.

Example 2: Erick throws the ball from the roof onto the ground 9.14m below. He tosses the ball with an initial downward velocity of 3.05 m per second. With H as the height of an object after t seconds, v as the initial velocity, and h as the initial height, the condition above is given by the equation $H = -16t^2 + vt + h$, how long does it take for the ball to hit the ground?

Solution:

This is the story model we had earlier. Given the equation $H = -16t^2 + vt + h$, if the ball hits the ground then the height, H, is zero. Thus, the quadratic equation is $-16t^2 + 3.05t + 9.14 = 0$. Solving for the time it takes the ball to hit the ground,

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ *Quadratic formula*

$x = \frac{-(3.05) \pm \sqrt{(3.05)^2 - 4(-16)(9.14)}}{2(-16)}$ *Substitute the values of a, b and c*

$x = \frac{-3.05 \pm \sqrt{9.3025 + 584.96}}{2}$ *Simplify*

$x = \frac{-3.05 \pm \sqrt{594.2625}}{-32}$ *Evaluate the square root, if possible.*

$x = \frac{-3.05 \pm 24.38}{-32}$

$x = -0.67$ or $x = 0.86$

Since -0.67 seconds is not possible, thus the ball will hit the ground in 0.86 seconds

WHAT'S MORE

Critical Thinking

TEST YOURSELF!

I. Use the quadratic formula to solve each of the following quadratic equations:
   a. $x^2 - 9x = -10$
   b. $2m^2 + 13m + 20 = 0$
   c. $2y^2 + 8y = 9$
   d. $9t - 5t^2 = -12$
   e. $3b^2 + 13b + 20 = 0$

II. Solve for the unknown in the problem using the quadratic formula.
   The area of a table cloth is 7 square ft. If the wifth is 3 feet shorter than the length,then what are the dimensions of the table cloth?

WHAT I HAVE LEARNED

Quadratic Formula

For any quadratic equation of the form $ax^2 + bx + c = 0$, where a, b, and c are real numbers and $a \neq 0$, the solutions are:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

WHAT I CAN DO

Critical Thinking

A. Solve for the real roots of each equation using quadratic formula.

1. $x^2 - 2x - 4 = 0$	6. $x^2 + 3x - 8 = 0$
2. $3x^2 + 2x - 21 = 0$	7. $4x^2 - 4x + 11 = 0$
3. $x^2 - 6x + 21 = 0$	8. $-2x^2 + 4x + 1 = 0$
4. $x^2 = 8x + 4$	9. $4x^2 = -7x + 15$
5. $x^2 + 4 = -12x$	10. $3x^2 + 1 = 5x$

B. Solve.

A rocket is launched straight up from the side of a cliff 43.89 m high. If the initial velocity (v) is 34.14 m/sec and the height (H) is given by the formula $H = -16t^2 + vt + h$, find the time at which the rocket will hit the ground.

ASSESSMENT

Write the letter of the correct answer on your answer sheet. If your answer is not found the choices, write E together with your final answer.

1. If the quadratic equation $ax^2 + bx + c = 0$ is solve using quadratic formula, the solutions are:

   A. $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ C. $x = \frac{-b\pm\sqrt{b^2+4ac}}{2a}$

   B. $x = \frac{b\pm\sqrt{b^2-4ac}}{a}$ D. $x = \frac{-b\pm\sqrt{b^2-4ac}}{a}$

2. Which is the correct substitution of the values of a, b and c in the quadratic formula of the equation $x^2 = -7x - 5$?

   A. $x = \frac{-(-7)\pm\sqrt{(-7)^2-4(1)(-5)}}{2(1)}$ C. $x = \frac{-7\pm\sqrt{7^2-4(1)(5)}}{2(1)}$

   B. $x = \frac{-(-7)\pm\sqrt{(-7)^2-4(-1)(-5)}}{2(1)}$ D. $x = \frac{-7\pm\sqrt{7^2-4(1)(-5)}}{2(1)}$

3. Solve for the roots of $2x^2 + 6x = -3$ using quadratic formula.

   A. $\frac{-3\pm\sqrt{3}}{2}$ B. $\frac{3\pm\sqrt{3}}{2}$ C. $\frac{2\pm\sqrt{3}}{2}$ D. $\frac{-2\pm\sqrt{3}}{2}$

4. What is/are the positive real root/s of the quadratic equation $x^2 - 5x + 3 = 0$?

   A. $\frac{5+\sqrt{13}}{2}$ B. $\frac{-5-\sqrt{13}}{2}$ C. $\frac{5\pm\sqrt{13}}{2}$ D. $\frac{-5\pm\sqrt{13}}{2}$

5. How will you describe the roots of the quadratic equation $x^2 + 3 = 5x$ when solved using quadratic formula?

   A. Equal B. Irrational C. Rational D. Non-real

6. Use quadratic formula in solving for the roots of $10x^2 = -7x + 6$.

   A. $\frac{1}{2}, -\frac{6}{5}$ B. $-\frac{1}{2}, -\frac{6}{5}$ C. $\frac{1}{2}, \frac{6}{5}$ D. $-\frac{1}{2}, \frac{6}{5}$

7. Find the roots of $x^2 = 8x + 4$ using quadratic formula.

   A. $-4 \pm \sqrt{5}$ B. $4\pm2\sqrt{5}$ C. $4\pm\sqrt{5}$ D. $-4\pm2\sqrt{5}$

8. Solve for the positive real roots of $5x^2 + 21x = -18$ using quadratic formula.

   A. 3 C. No positive roots

   B. $\frac{6}{5}$ D. No real roots

9. What are the roots of $x^2 + 6x = 10$?
   A. $-3 \pm \sqrt{19}$ B. $3 \pm \sqrt{19}$ C. $-3 \pm \sqrt{-4}$ D. $3 \pm \sqrt{-4}$

10. A rocket is launched straight up from the side of a cliff 40m high. If the initial velocity (v) is 30 m/sec and the height (H) is given by the formula $H = -16t^2 + vt + h$, find the time at which the rocket will hit the ground.
    A. 2.78sec B. -0.90sec C. 3.1sec D. -0.6sec

ADDITIONAL ACTIVITIES

Critical Thinking and Creativity

Think it up!

A. There are four ways of solving quadratic equations that we've learned. List down the advantages and disadvantages of using each method.

Method of solving quadratic equation	Advantages	Disadvantages
Extracting the square root		
Factoring		
Completing the Squares		
Quadratic Formula		

B. Solve the following quadratic equation using the most appropriate method. Give your reasoning on choosing that method.
   1. $x^2 + 8x - 4 = 0$
   2. $x^2 - 54 = 0$
   3. $6x^2 + 11x - 255 = 0$
   4. $x^2 + 7x + 12 = 0$

C. Reflect!

There are several options to choose in solving for the roots of quadratic equation. For a given equation, it may be more effective to use one method instead of another. In this situation we need be careful and decisive in choosing the most efficient method.

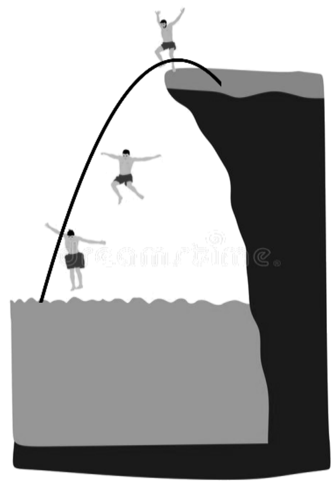
In real life, have you ever encountered situation where you need to make decisions? What options did you consider? Did you choose the best option?

PROBLEM – BASED WORKSHEET

Analyze the given situation to answer the problem.

The Cliff Diver

A cliff diver jumps from about 17m above the water. His height in meters from the water t seconds after he jumps is given by the formula $h = -4.9t^2 + 1.5t + 17$.



How long will it take for the diver to reach the water?

E-Search

You may also check the following link for your reference and further learnings on solving quadratic equation using quadratic formula.

- https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:quadratic-functions-equations/x2f8bb11595b61c86:quadratic-formula-a1/v/using-the-quadratic-formula
- https://www.math-only-math.com/word-problems-using-quadratic-formula.html
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https://www.freepik.com/free-vector/illustration-with-kids-taking-lessons-online-design_7574030.htm

ADDITIONAL ACTIVITIES

A.

Method of solving quadratic equation	Advantages	Disadvantages
Extracting the square root	Can be easily used for equation of the form $ax^2 = k$	Limited only to certain quadratic equation
Factoring	For equations that can be factored out easily	Not all equations are factorable
Completing the Squares	Can be easily used for equation with a =1 and b is even, factorable or not	May encounter complicated computations
Quadratic Formula	Applicable to all quadratic equation; Gives exact roots	Time consuming and may lead to errors in computation

B.

1. The equation is not factorable. Since a = 1 and b is even, completing the squares is the most appropriate. The roots are $\{-4 \pm 2\sqrt{5}\}$
2. The equation can be transformed into $ax^2 = k$, thus extracting the square root is the most appropriate. The roots are $\{\pm 3\sqrt{6}\}$
3. The equation is not easily factorable, and the numbers are large, thus the most appropriate method is using quadratic formula. The roots are $\left\{\frac{17}{3}, -\frac{15}{2}\right\}$.
4. The equation is easily factorable. The roots are {-3, -4}

WHAT I KNOW		ASSESSMENT	
1. D	6. A	1. A	6. A
2. A	7. C	2. C	7. B
3. C	8. A	3. A	8. C
4. B	9. D	4. C	9. A
5. B	10.B	5. B	10. A

PROBLEM – BASED WORKSHEET

Solutions:

The diver's height when he reaches the water is 0m. Since the numerical coefficients are decimals, quadratic formula is the most appropriate method to use:

$$0 = -4.9t^2 + 1.5t + 17$$

Here we have, a = -4.9, b = 1.5, and c = 17

Then,

$$t = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$t = \frac{-1.5 \pm \sqrt{(1.5)^2-4(-4.9)(17)}}{2(-4.9)}$$

$$t = \frac{-1.5 \pm \sqrt{335.45}}{-9.8}$$

$$t = 0.15 \pm (-1.87)$$

$t = 0.15 + (-1.87)$ or $t = 0.15 - (-1.87)$

$t = -1.72$ or $t = 2.02$

Since -1.72 seconds is not a valid solution, the diver will reach the water at 2.02 seconds.

ANSWER KEY

WHAT'S IN

1. $5\sqrt{5}$
2. 20
3. $15 - 4\sqrt{7}$
4. $\frac{3+2\sqrt{3}}{6}$
5. $\frac{1}{2}$

WHAT I CAN DO

A.

1. $1 \pm \sqrt{5}$
2. $\frac{7}{3}, -3$
3. no real roots
4. $4 \pm 2\sqrt{5}$
5. $-6 \pm 4\sqrt{2}$
6. $\frac{-3 \pm \sqrt{41}}{2}$
7. no real roots
8. $\frac{2 \pm \sqrt{6}}{2}$
9. $-3, \frac{5}{4}$
10. $\frac{5 \pm \sqrt{13}}{6}$

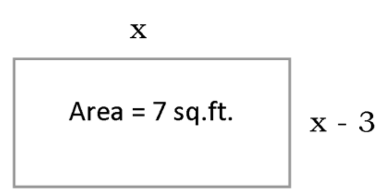
B. 3.04 seconds

WHAT'S MORE

I.

1. $\frac{9 \pm \sqrt{41}}{2}$
2. $-\frac{5}{2}, -4$
3. $\frac{-4 \pm \sqrt{34}}{2}$
4. *no real roots*
5. Non-real roots: $\frac{-13 \pm \sqrt{-71}}{6}$

II. Refer to the figure.



Let x = length
$x - 3$ = width

The equation will be:

$lw = A$

$x(x - 3) = 7$

Solving for the dimensions

$x^2 - 3x = 7$

$x^2 - 3x - 7 = 0$

By quadratic formula,

$x = \frac{3 \pm \sqrt{37}}{2}$

or $x = 4.54$ft and $x = 1.54$

Thus, the length is $x = 5.54$ ft and the width is 2.54ft.