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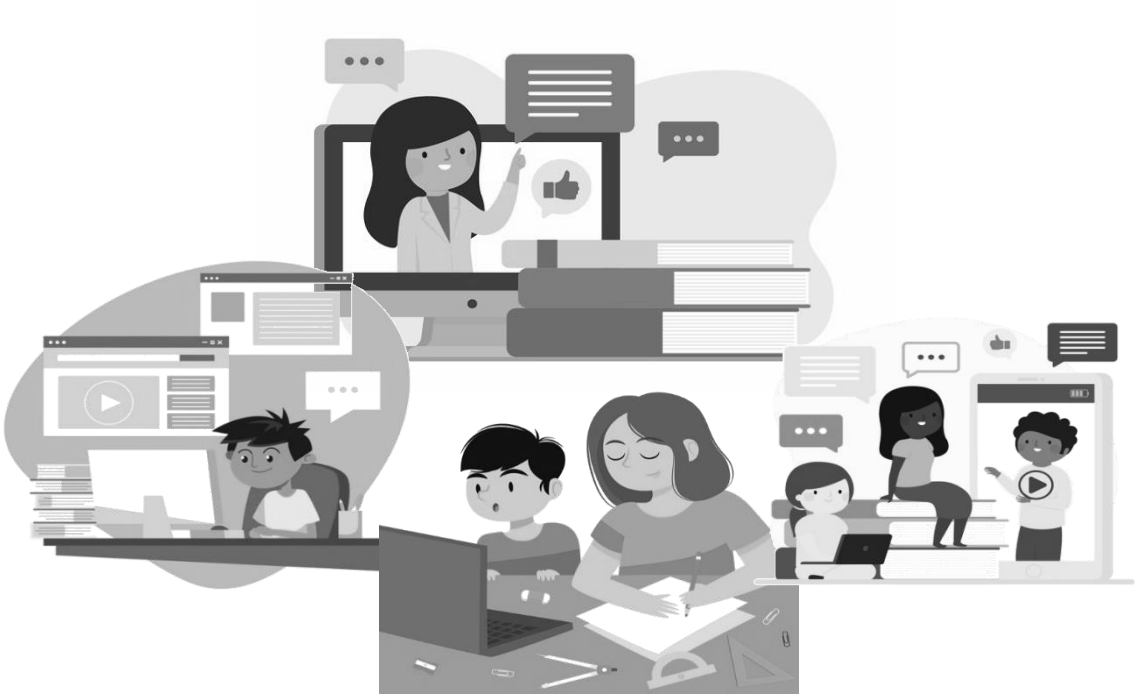
DEPARTMENT OF EDUCATION

Mathematics

Quarter 1-Module 4

Solving Quadratic Equation By Completing the Square

Week 1



ALTERNATIVE DELIVERY MODE
ADM

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MODULE 4

SOLVING QUADRATIC EQUATION BY COMPLETING THE SQUARE

You already learned how to solve quadratic equation by factoring. However, there are limitations in using this method especially if the trinomial is not factorable. In this module, you will learn another way of solving quadratic equation - by completing the square.

WHAT I NEED TO KNOW

LEARNING COMPETENCY
The learners will be able to:

- solve quadratic equation by completing the square. **M9AL – Ia-b- 1**

WHAT I KNOW

Find out how much you already know about solving quadratic equation by completing the square. Write the letter that you think is the best answer to each question on your answer sheet. Answer all items. After taking and checking this short test, take note of the items that you were not able to answer correctly and look for the right answers as you go through this module.

1. What should be the middle term to make this a perfect square trinomial $m^2 + \_\_ + 100$?
   A. 5m C. 20m
   B. 10m D. 50m
2. What must be added to $x^2 - 4x + \_\_\_\_\_$ to make it a perfect square trinomial?
   A. 2 C. 8
   B. 4 D. 16
3. In solving the equation $x^2 - 8x = -1$ by completing the square, what must be added to both sides to make the left side a perfect square trinomial.
   A. 1 C. 16
   B. 4 D. -2
4. Which of the following is a perfect square trinomial?
   A. $x^2 + 10x + 25$ C. $x^2 - 5x + 25$
   B. $x^2 + 10x - 25$ D. $x^2 + 10x + 5$
5. What is the factored form of $x^2 - 6x + 9$?
   A. $(x + 3)(x + 3)$ C. $(x - 3)(x + 3)$
   B. $(x - 3)(x - 3)$ D. $(x + 3)(x + 1)$
6. In the expression $x^2 - bx + \frac{49}{4}$, what is b to make the expression a perfect square.
   A. 49 B. 7 C.4 D. 1

7. What is the square of binomial in question number 6?
   A. $\left(x - \frac{7}{2}\right)^2$ C. $\left(x + \frac{7}{2}\right)^2$
   B. $\left(x - \frac{7}{4}\right)^2$ D. $\left(x + \frac{7}{4}\right)^2$
8. Find the solution set of quadratic equation $b^2 - 2b - 3 = 0$ by completing the square.
   A. 1, -3 C. 2, -4
   B. -2, -4 D. -1, 3
9. What are the roots of the quadratic equation $x^2 + 5x = -6$?
   A. 2, 3 C. -1, 6
   B. 3, -4 D. -2, -3
10. What must be added to $x^2 + 22x +$ _____ to make it a perfect square trinomial?
   A. 100 C. 112
   B. 110 D. 121

WHAT'S IN

Collaboration and Critical Thinking

The concept of "perfect square trinomial" is the basis in the process of "completing of the square". In this case, let us have a recall to illustrate this.

A trinomial is a perfect square if:

1. The first and third terms are perfect squares containing positive numerical coefficients or positive numbers.

2. The second term is twice the product of the square roots of the two perfect square terms.

$$4x^2 + 12x + 9$$

$$(2x)^2 \qquad (3)^2$$

$$2(2x)(3)$$

Try this!

Complete the table below.

Trinomial	**Is it a perfect square trinomial?**	**Factors or Reason**
$x^2 + 8x + 16$	Yes	$(x + 4)^2$
$y^2 + 10y - 25$	No	The sign of the third term (25) is negative.
$p^2 - 24p + 144$		
$r^2 + 6r + 36$		
$9z^2 + 24z - 16$		
$a^2 - \square + 121$	Yes	$(a - 11)^2$
$\square + 40b + 100$	Yes	$(2b + 10)^2$

To factor a perfect square trinomial:

1. Find the positive square roots of each of the two perfect square terms.
2. Connect the two squares roots with the sign of the numerical coefficient of the remaining term (the second term) of the given trinomial.
3. Indicate that this binomial is used twice as a factor.

Example:

$$x^2 - 6x + 9$$

$$\sqrt{x^2} = x \qquad \sqrt{9} = 3$$

$$(x - 3)^2$$

WHAT'S NEW

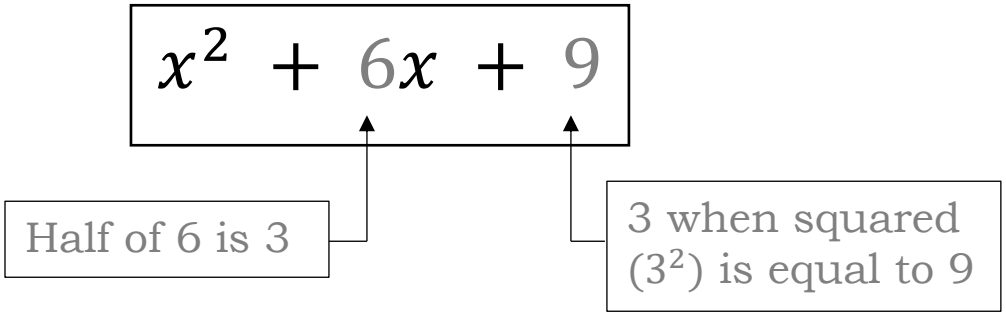
Communication, Critical Thinking, and Collaboration

An expression such as $x^2 + 8x$ is an "incomplete" square. But if you add 16 to $x^2 + 8x$, we will get $x^2 + 8x + 16$ which is a perfect square trinomial and equivalent to $(x + 4)^2$ in factored form. When we add 16 to $x^2 + 8x$, we are completing the square.

Look for a pattern in the following table.

Incomplete Square	Completed Square	Factored Form
$x^2 + 4x$	$x^2 + 4x + 4$	$(x + 2)^2$
$y^2 + 6y$	$y^2 + 6y + 9$	$(y + 3)^2$
$b^2 - 8b$	$b^2 - 8b + 16$	$(b - 4)^2$
$m^2 - 10m$	$m^2 - 10m + 25$	$(m - 5)^2$

Notice that the constant term in the completed square is the square of half the coefficient of x. For example:



This suggests the procedure of "completing the square"

Completing the Square

In an expression of the form $x^2 + bx$ or $x^2 - bx$, add the constant term $\left(\frac{b}{2}\right)^2$ to complete the square.

In $x^2 + 2bx$, if we take $\frac{1}{2}$(2b) or b, and square it, we will get the third term b^2.

Examples:

Complete the square. Then write each completed square in factored form.

1. $x^2 + 18x$

Solution:

To complete the square, add $\left(\frac{18}{2}\right)^2$ or 81.

Completed square: $x^2 + 18x + 81$

Factored form: $(x + 9)^2$

2. $d^2 - 7d$

Solution:

To complete the square, add $\left(\frac{-7}{2}\right)^2$ or $\frac{49}{4}$.

Completed square: $d^2 - 7d + \frac{49}{4}$

Factored form: $\left(d - \frac{7}{2}\right)^2$

3. $w^2 - \frac{2}{3}w$

Solution:

To complete the square, add $\left(\frac{\frac{-2}{3}}{2}\right)^2$ or $\frac{1}{9}$.

Completed square: $w^2 - \frac{2}{3}w + \frac{1}{9}$

Factored form: $\left(w - \frac{1}{3}\right)^2$

WHAT IS IT

Solving quadratic equations by factoring is simple and easy to do. However, there are some quadratic equation that are very difficult to factor. We can solve any quadratic equation using the method called completing the square.

Example 1: Solve $x^2 - 8x - 9 = 0$ by completing the square.

Solution:

$x^2 - 8x - 9 = 0$	*Original equation*
$x^2 - 8x = 9$	*Add 9 to both sides*
$x^2 - 8x + 16 = 9 + 16$	*Add 16 to both sides. The right side is a perfect square trinomial.*

***Note: To get the constant that will complete the square take the coefficient of x, divide it by 2, then square it. We have* -8 as the numerical coefficient of -8x. Thus, $\left(\frac{-8}{2}\right)^2 = 16$**

$x^2 - 8x + 16 = 25$	
$(x - 4)^2 = 25$	*Factor the trinomial square.*
$x - 4 = \sqrt{25}$	*Extract the square root of both sides.*
$x - 4 = \pm 5$	
$x - 4 = 5$ *or* $x - 4 = -5$	*Equate the linear expressions to each of the two values.*
$x = 9$ *or* $x = -1$	*Solve each of the resulting linear equations.*

The solution set is $\{-1, 9\}$**.**

Checking:

For $x = -1$

$x^2 - 8x - 9 = 0$

$(-1)^2 - 8(-1) - 9 = 0$

$1 + 8 - 9 = 0$

$0 = 0$

For $x = 9$

$x^2 - 8x - 9 = 0$

$(9)^2 - 8(9) - 9 = 0$

$81 - 72 - 9 = 0$

$0 = 0$

Example 2: Solve $3x^2 - 4x + 1 = 0$ by completing the square.

Solution:

$3x^2 - 4x + 1 = 0$	*Original equation*
$3x^2 - 4x = -1$	*Add -1 to both side of the equation.*
$\frac{3x^2}{3} - \frac{4x}{3} = -\frac{1}{3}$	*Divide by 3.*
$x^2 - \frac{4x}{3} = -\frac{1}{3}$	*The coefficient of x is* $-\frac{4}{3}$*.* $\left(-\frac{4}{3} \div 2\right)^2 = \left(-\frac{2}{3}\right)^2 = \frac{4}{9}$
$x^2 - \frac{4}{3}x + \frac{4}{9} = -\frac{1}{3} + \frac{4}{9}$	*Add* $\frac{4}{9}$ *to both sides.*
$x^2 - \frac{4}{3}x + \frac{4}{9} = \frac{1}{9}$	
$(x - \frac{2}{3})^2 = \frac{1}{9}$	*Factor the trinomial square.*
$\sqrt{\left(x - \frac{2}{3}\right)^2} = \sqrt{\frac{1}{9}}$	*Extract the root of both sides.*
$x - \frac{2}{3} = \pm\frac{1}{3}$	
$x - \frac{2}{3} = -\frac{1}{3}$ *or* $x - \frac{2}{3} = \frac{1}{3}$	
$x = \frac{1}{3}$	$x = 1$

The solution set is $\left\{\frac{1}{3}, 1\right\}$

**Checking the solution set will serve as your exercise.*

Example 3: Solve $x^2 - 4x - 8 = 0$ by completing the square.

Solution:

$x^2 - 4x - 8 = 0$	*Original equation*
$x^2 - 4x = 8$	*Adding 8 on both side of the equation.*
	The coefficient of x is -4.
	$(-\mathbf{4} \div \mathbf{2})^{\mathbf{2}} = (-\mathbf{2})^{\mathbf{2}} = \mathbf{4}$
$x^2 - 4x + 4 = 8 + 4$	*Add 4 on both sides.*
$x^2 - 4x + 4 = 12$	
$(x - 2)^2 = 12$	*Factor the trinomial square.*
$x - 2 = \pm\sqrt{12}$	*Extract the square root of both sides.*
$x - 2 = \pm\sqrt{4 \cdot 3}$	
$x - 2 = \pm 2\sqrt{3}$	
$x = 2 \pm 2\sqrt{3}$	

The solution set is $\{\mathbf{2 - 2\sqrt{3}}, \mathbf{2 + 2\sqrt{3}}\}$

Checking:

For x = 2 - $2\sqrt{3}$	***For x = 2 + $2\sqrt{3}$***
$x^2 - 4x - 8 = 0$	$x^2 - 4x - 8 = 0$
$(2 - 2\sqrt{3})^2 - 4(2 - 2\sqrt{3}) - 8 = 0$	$(2 + 2\sqrt{3})^2 - 4(2 + 2\sqrt{3}) - 8 = 0$
$4 - 8\sqrt{3} + 12 - 8 + 8\sqrt{3} - 8 = 0$	$4 + 8\sqrt{3} + 12 - 8 - 8\sqrt{3} - 8 = 0$
$0 = 0$	$0 = 0$

WHAT'S MORE

To master your skills in solving quadratic equation by completing the square, solve and check your solutions for the following:

1. $x^2 + 6x + 5 = 0$
2. $2x^2 + 3x - 2 = 0$
3. $x^2 - 4x + 12 = 0$
4. $5x^2 + 26x + 5 = 0$
5. $7x^2 - 2x = 1$

WHAT I HAVE LEARNED

To have a better understanding, below are the summarized steps in "completing the square".

1. Place the constant term on the right side of the equation. All the terms with unknowns are on the left side.
2. If the coefficient of the quadratic term, $a \neq 1$, divide each term of the equation by the numerical coefficient.
3. To get the constant term needed to complete the square, get the numerical coefficient of linear term, divide it by 2 and square it. Add the result to both sides of the equation.
4. Factor the perfect square trinomial.
5. Extract the square root from both sides. Two values will be obtained for the right side of the equation.
6. Equate the linear expressions to each of the two values.
7. Solve each of the resulting linear equations.
8. Check your answer by substituting to the original equation.

WHAT I CAN DO

Find the solution set of each of the following quadratic equations using the process of "completing the square". Show your solution.

1. $x^2 + 10x + 2 = 0$
2. $b^2 - 2b - 15 = 0$
3. $m^2 - 7m + 12 = 0$
4. $2w^2 + 2w - 4 = 0$
5. $3n^2 + 24n = -48$
6. $2y^2 - 3 = 5y$
7. $p^2 + 2p - 15 = 0$
8. $3y^2 + 12y + 12 = 0$
9. $2x^2 + 4x + 1 = 0$
10. $a = 2a^2 - 1$

ASSESSMENT

Write the letter of the correct answer on your answer sheet. If your answer is not among the choices, write E together with your final asnwer.

1. Which of the following will make the expression $x^2 + 6x + \_\_$ a perfect square trinomial?
   A. 3
   B. 6
   C. 9
   D. 12

2. What is the missing term to make the expression $p^2 + \_\_\_\_ + 49$ a perfect square trinomial?
   A. 14p
   B. 4p
   C. 16p
   D. 9p

3. Which of the following will have a perfect square trinomial on one side of equation if 36 is added to both sides of equation?
   A. $x^2 - 12x = -1$
   B. $x^2 - 6x = -1$
   C. $x^2 - 9x = -1$
   D. $x^2 + 24x = -1$

4. Which of the following is a perfect square trinomial?
   A. $x^2 - 2x + 1$
   B. $x^2 - 4x + 6$
   C. $x^2 - 4$
   D. $x^2 + 9$

5. In the expression $x^2 - bx + 100$, what is the value of b to make the expression a perfect square?
   A. 10
   B. 5
   C. 15
   D. 20

6. To find the value of b in question number 5, get the _____ of the product of the square roots of the x^2 and 100.
   A. thrice
   B. once
   C. twice
   D. triple

7. What is the squared binomial in question number 5?
   A. $(x - 5)^2$ C. $(x + 20)^2$
   B. $(x - 10)^2$ D. $(x + 10)^2$

8. Find the solution set of quadratic equation $6x^2 = 10 + 11x$ by completing the square.
   A. $\frac{5}{2}, -\frac{2}{3}$ C. $\frac{5}{2}, \frac{2}{3}$
   B. $-\frac{5}{2}, -\frac{2}{3}$ D. $\frac{2}{5}, -\frac{2}{3}$

9. What are the roots of quadratic equation $x^2 + \frac{5}{3}x + 1 = 0$?
   A. $\sqrt{11}$ C. $\frac{-5 \pm \sqrt{11}}{4}$
   B. $\frac{5 \pm \sqrt{11}}{8}$ D. no real roots

10. What constant should be added to both sides of $x^2 + 6x - 1 = 0$ a make the left-hand side a perfect square trinomial?

    A. 4 B. 6 C. 9 D. 12

ADDITIONAL ACTIVITIES

Critical Thinking and Creativity

A. The scenario below can be represented by a quadratic equation. Write a mathematical model showing the situation then solve for what is asked on the problem. If the process of "completing the square" is appropriate to solve for the unknown, then use it. Otherwise, use other methods that you know.

Led television screens are usually measured by the length of its diagonal. The television set has 60-inch diagonal. The screen is 12 inches wider than its height. Find the dimensions of the screen.

Exploration

In the next module, we will solve the value of x of a quadratic equation $ax^2 + bx + c = 0$,where a, b, c and $a \neq 0$ by using a specific formula. One way of deriving this formula is by completing the square. Try it now without looking ahead!

PROBLEM – BASED WORKSHEET

An Open Box

A rectangular board is 6 in longer than its width. An open box was made by cutting 3 in. squares at each corner and folding the sides.

1. Draw a diagram to illustrate the situation.

2. Write expressions that represent the width, length and height of the box.

   _____

3. Suppose the volume of the box is 336 cubic in., write the mathematical sentence that will represent the situation.

   _____

4. Use completing the squares to find the dimensions of the rectangular board.
5. What are the dimensions of the box? _____

E-Search

You may also check the following link for your reference and further learnings on solving quadratic equation using completing square.

- https://www.youtube.com/watch?v=0IFPixtQpYM
- https://www.youtube.com/watch?v=9UgvReIRsp8
- https://www.youtube.com/watch?v=prx_Bf2hakw

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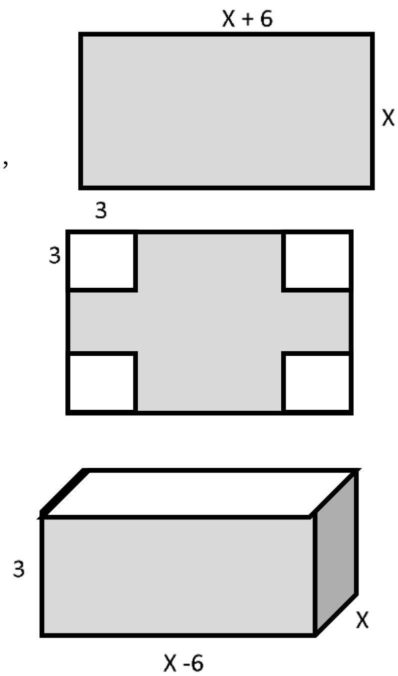
ADDITIONAL ACTIVITIES

A. Height=36 inches
   Width= 48 inches
B. *Please see module 5*

WHAT I KNOW	ASSESSMENT
1. C	1. C
2. B	2. A
3. C	3. A
4. A	4. A
5. B	5. D
6. D	6. C
7. A	7. B
8. D	8. A
9. D	9. D
10. D	10. C

PROBLEM – BASED WORKSHEET



1. Let x = width of the board
then x + 6 = length of the board

2. If squares with side 3 in. will be cut from the corners,
then the box will have,
width = x - 6
length = x
height = 3

3. With V = lwh, the volume of the box will be

$3x(x-6) = 336$

4. To find the dimensions on the board using

$3x^2 - 18x = 336$

$3(x^2 - 6x + 9) = 336 + 27$

$3(x - 3)^2 = 363$

$(x - 3)^2 = 121$

$x - 3 = \pm 11$

$x = 11 + 3$ and $x = -11 + 3$

$x = 14$ and $x = -8$

But a negative value of x does not fit the conditions of the problem, thus the value of x is 14. Thus, the board is 14 in. by 20 in.

5. The dimensions of the box are:
width = 8 in
length = 14 in
height = 3 in

ANSWER KEY

WHAT'S IN

Trinomial	Is it a perfect square trinomial?	Factors or Reason
$x^2 + 8x + 16$	Yes	$(x + 4)^2$
$y^2 + 10y - 25$	No	The sign of the third term (25) is negative.
$p^2 - 24p + 144$	YES	$(p - 12)^2$
$r^2 + 6r + 36$	NO	The square root of 36 is ±6 and if you multiply this by 2, the coefficient of the middle term must be 12 or -12
$9z^2 + 24z - 16$	YES	$(3z + 4)^2$
$a^2 - -\square + 121$	Yes	$(a - 11)^2$
$+ 40b + 100$	Yes	$(2b + 10)^2$

WHAT'S MORE

1. $\{-5, -1\}$
2. $\left\{-2, \frac{1}{2}\right\}$
3. no real solution
4. $\left\{-5, -\frac{1}{5}\right\}$
5. $\frac{1}{7} \pm \frac{2\sqrt{2}}{7}$

WHAT I CAN DO

1.) $\{-5 \pm \sqrt{23}\}$

2.) $\{-3, 5\}$

3.) $\{4, 3\}$

4.) $\{-2, 1\}$

5.) $\{-4\}$

6.) $\left\{-\frac{1}{2}, 3\right\}$

7.) $\{-5,3\}$

8.) $\{-2\}$

9.) $\left\{\frac{-2\pm\sqrt{2}}{2}\right\}$

10.) $\left\{-\frac{1}{2}, 1\right\}$