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DepED
DEPARTMENT OF EDUCATION

Mathematics

Quarter 1 - Module 17

General Form and Standard Form of Quadratic Function

Week 7

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ADM

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MODULE 17

GENERAL FORM AND STANDARD FORM OF QUADRATIC FUNCTION

In the previous module, you learned the general form **$y = ax^2 + bx + c$** of a quadratic function and represented it in various ways. In this module, the standard form or vertex form **$y = a(x - h)^2 + k$** will be introduced. There will be instances that the standard form of quadratic function will be more appropriate to use when working on problems involving the vertex of the graph of a quadratic function.

WHAT I NEED TO KNOW

LEARNING COMPETENCY

The learners will be able to:

- transform the quadratic function in general form $y = ax^2 + bx + c$ into standard form (vertex form) $y = a(x – h)^2 + k$ and vice versa. **M9AL-Ig-12**

WHAT I KNOW

Find out how much you already know about general form and standard form of quadratic equation. Write the letter that you think is the correct answer to each question on your answer sheet. Answer all items. After taking and checking this short test, take note of the items that you were not able to answer correctly and look for the right answer as you go through this module.

1. Which of the following is standard form of a quadratic function?
   A) $y = a(x - h)^2 - k$ C) $y = a(x - h)^2 + k$
   B) $y = a(x - k)^2 - h$ D) $y = a(x - k)^2 + h$
2. The quadratic function $y = x^2 + 2x - 1$, is expressed in standard form as
   A) $y = (x + 1)^2 + 1$ C) $y = (x + 1)^2 + 2$
   B) $y = (x + 1)^2 - 2$ D) $y = (x + 1)^2 - 1$
3. Which must be the value of (h, k) in the quadratic function $y = 2(x - 3)^2 + 4$?
   A) (–3, –4) B) (–3, 4) C) (3, –4) D) (3, 4)
4. If $y = 2x^2 + 4x - 5$ is written in the form $y = a(x - h)^2 + k$, then what is the value of $a + h - k$?
   A) – 6 B) – 7 C) 6 D) 8
5. Which of the following is standard form of a quadratic function $y = ax^2 + bx + c$?
   A) $y = a\left(x - \frac{b}{2a}\right)^2 + \frac{4ac-b^2}{4a}$ C) $y = a\left(x - \frac{b}{2a}\right)^2 + \frac{4ac+b^2}{4a}$
   B) $y = a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac-b^2}{4a}$ D) $y = a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac+b^2}{4a}$

6. Which of the following is general form of a quadratic function?
   A) $y = ax^2 - bx - c$ C) $y = ax^2 + bx - c$
   B) $y = ax^2 - bx + c$ D) $y = ax^2 + bx + c$
7. The quadratic function $y = (x-1)^2 + 1$, is expressed in general form as
   A) $y = x^2 - 2x - 1$ C) $y = x^2 - 2x + 1$
   B) $y = x^2 - 2x$ D) $y = x^2 - 2x + 2$
8. Which must be the value of c if $y = (x+5)^2 + 7$ is transformed into general form?
   A) 12 B) 32 C) 34 D) 74
9. What is the value of b if $y = 2(x-5)^2 + 4$, is written in general form?
   A) – 20 B) –25 C) –10 D) – 5
10. If $y = (x+1)^2 - 10$ is written in general form, what is the value of $a + b - c$?
   A) – 9 B) – 7 C) 9 D) 12

WHAT'S IN

Today's lesson can be easily understood by recalling first the procedure on how to make a trinomial, a perfect square.

If $x^2 + bx + c$ is a trinomial expression, with b and c as constant, then to make the expression a perfect square trinomial, the value of c must be; $c = \left(\frac{b}{2}\right)^2$

Determine the number that must be added to make each of the following a perfect square trinomial. Then give the factors as a square of a binomial.

1) $x^2 + 4x +$ ______
2) $x^2 - 10x +$ ______
3) $x^2 + 3x +$ ______
4) $x^2 - 11x +$ ______
5) $x^2 + \frac{5}{2}x +$ ______

How did you find the activity? Were you able to recall how to find the constant term of a perfect square trinomial? Were you able to factor them as square of binomials?

How will this concept help you understand the topic in this module?

Are you ready to explore?

WHAT'S NEW

Communication, and Collaboration

THE MAXIMUM PRODUCT

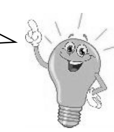
Vin and Rose are playing a number game called the maximum product. The rule says:

> The sum of two numbers is 24. What could be the maximum product of the two numbers, and what are those numbers?

What do you think are the numbers that satisfy the rule? How did you find out? Do you think there is an easier way to find the numbers that would give the maximum product?

WHAT IS IT

Communication, Critical Thinking, and Collaboration



Transforming Quadratic Function from General Form to Standard Form

One way of solving this is to list all 2 addends of 24 with respective products. 1 & 23 = 23 , 2 & 22 = 44, 3 & 21 = 63 , 4 & 20 = 80 ,…, up to 12 & 12 = 144. Since there are no restriction with number, all real numbers are possible options, the likes of $\frac{1}{5}$ and $23\frac{4}{5} = \frac{119}{25}$ are also included, and we are just looking for 2 addends with a maximum product. Thus, the maximum product is 144 when the numbers are both 12.

An alternative way of solving this problem is to recall our lessons on how to solve word problem, that is,

Let x = one of the number; and
$24 - x$ = the other number

If you let y be the product of the numbers, then the working equation is:

$$y = x(24 - x) \quad \text{or} \quad y = -x^2 + 24x,$$

The resulting equation is a quadratic function, which can be written as;

$$y = -(x^2 - 24x)$$

by applying the rule on the grouping symbol (group of variable x).

Also, this is equivalent to:

$$y = -(x^2 - 24x + 144) + 144.$$

by completing the squares and adding the additive inverse of the resulting value of constant of the perfect square trinomial. Writing this in a more compact form, we have: $y = -(x - 12)^2 + 144.$

The last equation is called the standard form of the quadratic function, in the form:

$$y = a(x - h)^2 + k$$

This is also called the vertex form of quadratic function which is very useful in solving problems modeled by the quadratic function. It easily gives you the vertex of the parabola at (h, k).

In the equation, the negative sign ($a = -1$) is the value of a, which tells us that there is a maximum product or value of k which is 144, and 12 is the value of h, which is one of the two numbers, the other number is of course being 12.

To learn more on transforming the general form of quadratic function to its equivalent standard form, study the given examples:

Example 1. Rewrite $y = x^2 - 6x - 11$, in standard form. Identify the constants a, h and k.

Solution:

Group the terms containing **x**.	$y = (x^2 - 6x) - 11$
Complete the expression in parenthesis to make it a perfect square trinomial by adding the constant $\left(\frac{-6}{2}\right)^2$ and subtracting the same value to the constant term.	$y = (x^2 - 6x + 9) - 11 - 9$
Simplify and express the perfect square trinomial as square of a binomial	$y = (x - 3)^2 - 20$

Thus, the equivalent standard form of the given quadratic function is $y = (x - 3)^2 - 20$ with $a = 1$, $h = 3$, and $k = -20$.

Example 2. Express $2x^2 + y = 10x - 3$ in the form $y = a(x - h)^2 + k$ and give the values of a, h, and k.

Solution:

Apply the addition property of equality to isolate variable **y** on the left side of the equation.	$y = -2x^2 + 10x - 3$
Group the terms containing variable **x**.	$y = (-2x^2 + 10x) - 3$

Factor out **a**. Here **a = – 2.**	$y = -2(x^2 - 5x) - 3$
Complete the expression in parenthesis to make it a perfect square trinomial by the constant $\left(\frac{-5}{2}\right)^2 = \frac{25}{4}$ and subtracting the value $-2\left(\frac{25}{4}\right) = -\frac{25}{2}$ from the constant term.	$y = -2\left(x^2 - 5x + \frac{25}{4}\right) - 3 - \left(-\frac{25}{2}\right)$
Simplify and express the perfect square trinomial as a square of a binomial.	$y = -2\left(x - \frac{5}{2}\right)^2 + \frac{19}{2}$

Thus, the equivalent standard form of the given quadratic function is $y = -2\left(x - \frac{5}{2}\right)^2 + \frac{19}{2}$ with $a = -2$, $h = \frac{5}{2}$, and $k = \frac{19}{2}$.

Transforming Quadratic Function from Standard Form to General Form

What if you are asked to rewrite the standard form of quadratic function to its general form? How will squaring of binomials help you do it? Study the examples below

Example 3. Write $y = -(x - 3)^2 + 9$ in general form, then identify the values of a, b, and c.

Solution:

First, we expand $(x - 3)^2$	$y = -(x^2 - 6x + 9) + 9$
Distribute negative inside the parenthesis.	$y = -x^2 + 6x - 9 + 9$
Simplify the equation.	$y = -x^2 + 6x$

The equivalent general form of $y = -(x - 3)^2 + 9$ is $y = -x^2 + 6x$. Therefore, $a = -1$, $b = 6$, and $c = 0$.

Example 4. Express $y = 3\left(x + \frac{1}{2}\right)^2 - 2$ in the form $y = ax^2 + bx + c$. Determine the constants a, b, and c.

Solution:

First, we expand $\left(x + \frac{1}{2}\right)^2$	$y = 3(x^2 + x + \frac{1}{4}) - 2$
Distribute negative inside the parenthesis.	$y = 3x^2 + 3x + \frac{3}{4} - 2$
Simplify the equation.	$y = 3x^2 + 3x - \frac{5}{4}$

The equivalent general form is of $y = 3\left(x + \frac{1}{2}\right)^2 - 2$ is $y = 3x^2 + 3x - \frac{5}{4}$. Therefore, $a = 3$, $b = 3$, and $c = -\frac{5}{4}$.

Are you ready to work on your own? Apply the steps done in the examples given on the next exercise.

WHAT'S MORE

Critical Thinking and Collaboration

A. Transform each quadratic function into standard form. Identify the constants a, h and k.

1) $y = x^2 + 10x - 13$
2) $y = x^2 - 6x + 5$
3) $y = x^2 - 2x + 3$
4) $y = x^2 - 3x + 16$
5) $y = x^2 - 18x - 9$
6) $y = 2x^2 - 3x + 4$
7) $y = 3x^2 - 12x - 4$
8) $y = 2x^2 - 8x - 22$
9) $y = 3x^2 + 6x - 1$
10) $y = 9x^2 + 18x + 4$

B. Transform each vertex form of quadratic function into general form. Identify the constants a, b, and c.

1) $y = (x + 2)^2 - 3$
2) $y = (x - 4)^2 + 11$
3) $y = -(x + 7)^2 - 23$
4) $y = (x - 11)^2 + 15$
5) $y = -\left(x + \frac{2}{3}\right)^2 + 1$
6) $y = -2(x + 5)^2 + 8$
7) $y = 3(x + 1)^2 - 29$
8) $y = -6(x - 5)^2 + 73$
9) $y = -3\left(x - \frac{4}{5}\right)^2 - \frac{1}{5}$
10) $y = 4\left(x + \frac{2}{7}\right)^2 + \frac{3}{7}$

How did you find the activity? Were you able to transform all quadratic function in standard form? If not, in which part did you find challenging? How did you cope up with it?

WHAT I HAVE LEARNED

A. The standard form (vertex form) of the quadratic function defined by the general form $\mathbf{y = ax^2 + bx + c}$, is $\mathbf{y = a(x - h)^2 + k}$, where **a**, **h** and **k** are any real numbers and $\mathbf{a \neq 0}$.
B. To convert a quadratic function from general form to standard form, use the process of completing the squares.
C. To convert a quadratic function from standard form, expand the square of binomial and then simplify.

WHAT I CAN DO

Critical Thinking

A. Transform each general form of quadratic function into standard form. Then, identify the values of a, h, and k.

1) $y = x^2 - 4x + 1$	6) $4x^2 - y = 24x - 31$
2) $y = 2x^2 - 4x + 4$	7) $2 + x^2 = y + 3x$
3) $y = x^2 - x + \frac{11}{4}$	8) $y = -(x^2 - 5x) + 6$
4) $y = -(x^2 - 4x) + 1$	9) $3x^2 - x = 1 - y$
5) $2x^2 + 12x = -3 - y$	10) $y - 2x = 5x^2 - 3$

B. Write the indicated letter of the quadratic function in the form $y = ax^2 + bx + c$, into the space provided that corresponds to its equivalent standard form.

1) $y = \left(x - \frac{1}{2}\right)^2 + \frac{3}{2}$	**I**) $y = x^2 - 2x - 3$
2) $y = 3\,(x + 2)^2 - \frac{1}{2}$	**S**) $y = 2x^2 + 5x - 3$
3) $y = (x - 1)^2 - 16$	**E**) $y = 3x^2 + 6x + \frac{23}{2}$
4) $y = 2\,(x + 1)^2 - 2$	**A**) $y = 3x^2 + 12x + \frac{23}{2}$
5) $y = (x - 1)^2 - 4$	**N**) $y = x^2 - 36$
6) $y = 2\left(x + \frac{5}{4}\right)^2 - \frac{49}{8}$	**T**) $y = x^2 - 2x - 15$
7) $y = (x - 3)^2 + 5$	**F**) $y = x^2 - 6x + 14$
8) $y = -2\,(x - 3)^2 + 1$	**M**) $y = x^2 - x + \frac{7}{4}$
9) $y = (x - 0)^2 - 36$	**U**) $y = -2x^2 + 12x - 17$
	H) $y = 2x^2 + 4x$

____ ____ ____ ____   ____ ____   ____ ____ ____

1 2 3 4   5 6   7 8 9

ASSESSMENT

Write the letter of the correct answer on your answer sheet. If your answer is not found among the choices, then write the correct answer.

1. Which of the following is standard form of a quadratic function?
   A) $y = a(x - h)^2 - k$ C) $y = a(x - h)^2 + k$
   B) $y = a(x - k)^2 - h$ D) $y = a(x - k)^2 + h$
2. The quadratic function $y = x^2 + 2x - 5$, is expressed in standard form as
   A) $y = (x + 1)^2 - 6$ C) $y = (x + 1)^2 + 4$
   B) $y = (x + 1)^2 - 4$ D) $y = (x + 1)^2 + 4$
3. Which must be the value of k if $y = x^2 + 14x - 1$ is transformed into standard form?
   A) – 50 B) – 48 C) 49 D) 50
4. Which must be the value of (h, k) in the quadratic function $y = 2(x + 3)^2 + 8$?
   A) (–3, –8) B) (–3, 8) C) (3, –8) D) (3, 8)

5. What is the value of h, if $y = 2x^2 + 8x - 5$ is written in standard form?
   A) – 4 B) – 2 C) 2 D) 4
6. Which of the following is general form of a quadratic function?
   A) $y = ax^2 - bx - c$ C) $y = ax^2 + bx - c$
   B) $y = ax^2 - bx + c$ D) $y = ax^2 + bx + c$
7. The quadratic function $y = \left(x + \frac{3}{2}\right)^2 - 1$, is expressed in general form as
   A) $y = x^2 + x - \frac{5}{4}$ C) $y = x^2 + 3x + \frac{5}{4}$
   B) $y = x^2 + 3x - \frac{5}{4}$ D) $y = x^2 + 3x + \frac{13}{4}$
8. Which must be the value of c if $y = 2(x-1)^2 - 3$ is transformed into general form?
   A) – 5 B) – 1 C) 1 D) 5
9. What is the value of b if $y = 3\left(x - \frac{1}{3}\right)^2 + 1$, is written in general form?
   A) – 2 B) – 1 C) $-\frac{2}{3}$ D) $-\frac{1}{3}$
10. If $y = (x-2)^2 - 1$ is written in general form, what is the value of $a^2 + b^2 - c^2$?
   A) 8 B) 16 C) 17 D) 26

ADDITIONAL ACTIVITIES

Communication, Critical Thinking, Creativity and Character Building

Activity 1: MAXIMUM GAS MILEAGE FOR A CAR

One of the greatly affected of the COVID19 pandemic were the transportation sectors of our society. Every types of vehicles were forced to adjust in accordance with implemented guidelines of our government. But we cannot deny the fact some of our car owners were curious about stability or durability of their cars.

Most cars get their best mileage when traveling at a relatively modest speed. The gas mileage M for a certain new car is modeled by the function

$$M(s) = -\frac{1}{28}s^2 + 3s - 31$$

where s is the speed in miles per hour and M is measured in miles per gallon.

What is the car's best gas mileage, and what speed is it attained?

1) What is the standard form of the given quadratic function?
2) What is the car's best gas mileage, and
3) What speed is it attained?

Activity 2: Reflection

After the lifting of the different level of quarantine impose on our respective society, (COVID19 pandemic), we will be dealing with the new form of standard of living. How would you adjust your lifestyle on the so called new normal?

E-Search

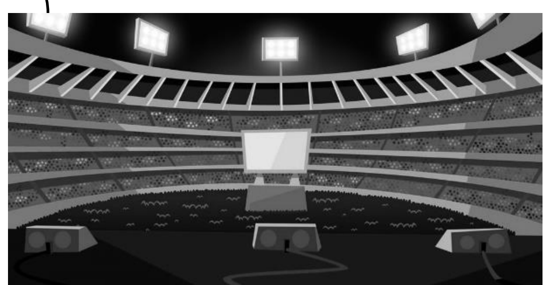
You may also check the following link for your reference and further learnings on general form and standard form of quadratic function.

https://www.youtube.com/watch?v=pwlTxyUghV0
https://www.youtube.com/watch?v=hXZ7Pf2RE-k
https://www.youtube.com/watch?v=pW5z2gCTdDw
https://www.youtube.com/watch?v=z3vmeBv-wtg
https://www.youtube.com/watch?v=JiP3-epY8Ko
https://www.youtube.com/watch?v=UbhFljl28Ts

PROBLEM – BASED WORKSHEET

THE CONCERT ARENA

A concert arena has a seating capacity of 15,000. With the ticket price set at P14, average attendance at recent concert has been 9,500. A market survey indicates that for each peso the ticket price is lowered, the average attendance increases by 1,000.



Let's Analyze!

1) Find a function that models the revenue in ticket price.
2) What is the general form of the quadratic function (#1)?
3) What ticket price is so high that no one attends, and hence no revenue is generated?
4) Find the price that maximizes revenue from the ticket sales.

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WHAT I CAN DO

A.

	Standard Form	a	h	k
1.	$y = (x-2)^2+5$	1	2	5
2.	$y = 2(x-1)^2+2$	2	1	2
3.	$y = \left(x-\frac{1}{2}\right)^2+\frac{5}{2}$	1	$\frac{1}{2}$	$\frac{5}{2}$
4.	$y = -(x-2)^2+5$	– 1	2	5
5.	$y = -2(x+3)^2+21$	– 2	– 3	21
6.	$y = 4(x-3)^2-5$	4	3	– 5
7.	$y = \left(x-\frac{3}{2}\right)^2-\frac{1}{4}$	1	$\frac{3}{2}$	$-\frac{1}{4}$
8.	$y = -\left(x-\frac{5}{2}\right)^2+\frac{49}{4}$	– 1	$\frac{5}{2}$	$\frac{49}{4}$
9.	$y = -3\left(x-\frac{1}{6}\right)^2+\frac{13}{12}$	– 3	$\frac{1}{6}$	$\frac{13}{12}$
10.	$y = 5\left(x+\frac{1}{5}\right)^2-\frac{16}{5}$	5	$-\frac{1}{5}$	$-\frac{16}{5}$

B.

M	A	T	H	I	S	F	U	N
1	2	3	4	5	6	7	8	9

ASSESSMENT

1. C
2. A
3. A
4. B
5. B
6. D
7. C
8. B
9. A
10. A

ADDITIONAL ACTIVITIES

A.

1. $M(s) = -\frac{1}{28}(s-42)^2+32$
2. 32 miles per gallon
3. 42 miles per hour

PROBLEM – BASED WORKSHEET

1. The model we want is a function that gives the revenue for any ticket price. Revenue (y) = ticket price x attendance

Ticket price	→	x
Amount ticket price is lowered	→	14 – x
Increase in attendance	→	100 (14 – x)
Attendance	→	9500 + 1000 (14 – x)

Thus, the function is y = x [9500 + 1000 (14 – x)]

2. Simplifying the function in #1, we get, $y = -1000x^2 + 23\,500x$
3. P23.50 or higher (of course, revenue is also zero if the ticket price is zero)
4. A ticket price of P11.75 yields the maximum revenue of P138, 062.50.

ANSWER KEY

WHAT I KNOW

1. C	3. D	5. B	7. D	9. A
2. B	4. D	6. D	8. B	10. D

WHAT'S IN

1.	4	$(x + 2)^2$
2.	25	$(x - 5)^2$
3.	$\frac{9}{4}$	$\left(x + \frac{3}{2}\right)^2$
4.	$\frac{121}{4}$	$\left(x - \frac{11}{2}\right)^2$
5.	$\frac{25}{16}$	$\left(x + \frac{5}{4}\right)^2$

WHAT'S MORE

A. Standard Form

Standard Form	a	h	k
1. $y = (x + 5)^2 - 38$	1	-5	-38
2. $y = (x - 3)^2 - 4$	1	3	-4
3. $y = (x - 1)^2 + 2$	1	1	2
4. $y = \left(x - \frac{3}{2}\right)^2 + \frac{55}{4}$	1	$\frac{3}{2}$	$\frac{55}{4}$
5. $y = (x - 9)^2 - 90$	1	9	-90
6. $y = 2(\left(x - \frac{3}{4}\right)^2 + \frac{23}{8}$	2	$\frac{3}{4}$	$\frac{23}{8}$
7. $y = 3(x - 2)^2 - 16$	3	2	-16
8. $y = 2(x - 2)^2 - 30$	2	2	-30
9. $y = 3(x + 1)^2 - 4$	3	-1	-4
10. $y = 9(x + 1)^2 - 5$	9	-1	-5

B. General Form

General Form	a	b	c
1. $y = x^2 + 4x + 1$	1	4	1
2. $y = x^2 - 8x + 27$	1	-8	27
3. $y = -x^2 - 14x - 72$	-1	-14	-72
4. $y = x^2 - 22x + 136$	1	-22	136
5. $y = -x^2 - \frac{4}{3}x + \frac{5}{9}$	-1	$-\frac{4}{3}$	$\frac{5}{9}$
6. $y = -2x^2 - 20x - 42$	-2	-20	-42
7. $y = 3x^2 + 6x - 26$	3	6	-26
8. $y = -6x^2 + 60x - 77$	-6	60	-77
9. $y = -3x^2 + \frac{24}{5}x - \frac{53}{25}$	-3	$\frac{24}{5}$	$-\frac{53}{25}$
10. $y = 4x^2 + \frac{16}{7}x + \frac{37}{49}$	4	$\frac{16}{7}$	$\frac{37}{49}$