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DepED
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Mathematics

Quarter 1-Module 10

Solving Word Problems on Rational Algebraic Expressions Transformable into Quadratic Equations

Week 4

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ADM

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MODULE 10

SOLVING WORD PROBLEMS ON RATIONAL ALGEBRAIC EXPRESSIONS TRANSFORMABLE INTO QUADRATIC EQUATIONS

In module 8, you have learned solving rational algebraic expressions reducible into quadratic equations. Mastery to this topic is a valuable tool to answer many real-life problems which you will learn from this module.

WHAT I NEED TO KNOW

LEARNING COMPETENCY

The learners will be able to:

- solve problems involving quadratic equations and rational algebraic equations. **M9AL-Ie-1**

WHAT I KNOW

Find out how much you already know about solving word problems on rational algebraic expression transformable into quadratic equation. Write the letter that you think is the best answer to each question on your answer sheet. Answer all items. After taking and checking this short test, take note of the items that you were not able to answer correctly and look for the right answer as you go through this module.

1. Mary and Carol are doing a Math project. Carol can do the work 2 hours longer as Mary. If they work together, they can finish the project in 4 hours. How long does it take Mary working alone to do the same project?
   A. 6 h
   B. 4 h
   C. 8 h
   D. 10 h

2. Two pipes working together can fill a tank in 35 minutes. If the large pipe alone can fill the tank in 24 minutes less than the time taken by the smaller pipe then find the time taken by larger pipe working alone to fill the tank.
   A. 30 mins.
   B. 60 mins.
   C. 40 mins.
   D. 80 mins.

3. It takes Mary 3 hours more to do a job than it takes Jane. If they work together, they can finish the same job in 2 hours. How long would it take Mary to finish the job alone?
   A. 3 hours
   B. 5 hours
   C. 6 hours
   D. 8 hours

4. Two conveyor belts can unload a shipment of tomatoes in 6 hours. Working alone, the slower belt would take 9 hours longer than the faster belt to do the job. How many hours would it take the faster conveyor belt working alone to do the job?
   A. 9 hours
   B. 10 hours
   C. 8 hours
   D. 6 hours

5. A motorcyclist traveled at a constant speed for 60 km. If he had gone 10 km/h faster, he would have shortened his traveling time by one hour. Find the speed of the motorcyclist.
   A. 19 km/h
   B. 18 km/h
   C. 20 km/h
   D. 17 km/h

6. A motorboat heads upstream a distance of 48 km on a river whose current is running at 3 km per hour (km/h). Then the motorboat returns. The trip upstream and back takes 12 hours. Assuming that the motorboat maintained a constant speed relative to the water, what was its speed?
   A. 8 km/h
   B. 9 km/h
   C. 7 km/h
   D. 10 km/h

7. A swimming pool can be filled by two pipes together in six hours. If the larger pipe alone takes five hours less than the smaller pipe alone to fill the pool, find the time in which smaller pipes alone would fill the pool.
   A. 15 hours
   B. 12 hours
   C. 10 hours
   D. 8 hours

8. A car drove from a city A to a city B. The distance between the two cities is 350km. After 200 km, the speed of the car decreased by 20 km/h. The total trip took 5 hours. Find the speed of the car in the first part of the journey.
   A. 60 km/h
   B. 70 km/h
   C. 80 km/h
   D. 90 km/h

9. Alex can do a job in one hour less than Jane. If Alex and Jane work together the job takes $\frac{6}{5}$ hours. How long would it take Alex working alone?
   A. 2 hours
   B. 4 hours
   C. 3 hours
   D. 6 hours

10. Two painters working together can paint the front of a house in 16 hours. One of the painters alone can finish this job in 24 hours less time than the other painter alone. How much time does the faster painter to do the job alone?
   A. 24 hours
   B. 32 hours
   C. 20 hours
   D. 18 hours

WHAT'S IN

Let us recall: ADD and SUBTRACT!

Here are a few steps to follow when you add or subtract rational expressions with like/unlike denominators.

1. To add or subtract rational expressions with unlike denominators, first find the LCM of the denominator.
2. Write each expression using the LCD.
3. Add or subtract the numerators.
4. Simplify as needed.

Study the examples below.

a.) $\frac{1}{x}+\frac{2x}{5}$ LCD = $5x$ $= \frac{1(5)+2x(x)}{5x} = \frac{5+x^2}{5x}$

b.) $\frac{x}{x+1}-\frac{2}{x+2}$ LCD = $(x+1)(x+2)$ $= \frac{x(x+2)-2(x+1)}{(x+1)(x+2)} = \frac{x^2+2x-2x-2}{x^2+3x+2} = \frac{x^2-2}{x^2+3x+2}$

Try This! You can trade your solutions and answers for cooperative review.

Add or subtract rational algebraic expressions and express the results in simplest forms.

1. $\frac{3}{x}+\frac{x}{6}$
2. $\frac{x}{2}+\frac{2x+1}{x}$
3. $\frac{5}{y}-\frac{3y+1}{2}$
4. $\frac{a+1}{a}-\frac{3}{2a+1}$
5. $\frac{m+5}{2m}+\frac{m-2}{m+1}$

WHAT'S NEW

Communication, character building, critical thinking and collaboration

Read and understand the conversation below and answer the given questions

GOODS FOR ALL

To address the needs of the people in surviving the present pandemic, the government utilized the reserved funds in providing relief goods (mostly food consisting of rice and canned goods) to households and mobilized disinfection efforts.

Gabriel and Wynter came from two different barangays. They talked about the distribution of relief goods in their area during the quarantine period through phone.

Gabriel: Hello Wynter. How are you? I just heard the news that ECQ is extended till the end of the month.

Wynter: I'm fine. I just finished my paper works at home. How's the distribution of relief goods in Barangay Masipag?

Gabriel: According to our barangay officials, they will distribute the goods this afternoon. How about in Barangay Matipid?

Wynter: They are still packing the goods. The chairman said that our barangay will take 7 hours longer to pack all the goods compared to your barangay.

Gabriel: Just have more patience Wynter. Last week, Barangay Masipag and Barangay Matipid worked together in packing the goods and it took 12 hours to finish it.

Wynter: Oh that's a very long and tiring day for them.

Gabriel: Yes I agree. Now I am just wondering that since our barangays decided to work separately, how long it will take to pack all the relief goods in each barangay?

Wynter: I am also curious about that!

a. *How long would it take Barangay Matipid to pack the relief goods by itself?*
b. *What equation would represent the problem?*
c. *How will you use the equation to solve what is asked?*

*The ancient greek mathematician Pythagoras believed that all **numbers** were **rational**, but one of his students Hippasus proved (using geometry, it is thought) that you could not write the square root of 2 as a fraction, and so it was irrational.*

WHAT IS IT

Communication, Critical Thinking, and Collaboration

It is important to be able to relate the language of mathematics to your everyday language. The problems you encounter at work, for example, will likely be expressed orally or in written form. It is important to practice expressing your ideas in words, or what we call *word equations*, and practice writing your answers in complete sentences.

Example 1: Two factories can produce a shipment of facemasks in 6 hours. Working alone, the slower factory would take 9 hours longer than the faster factory to do the job. How many hours would it take each factory working alone to do the job?

Solution:

Let t = time in hours for the faster factory

$\frac{1}{t}$ = rate of the faster factory

$t + 9$ = time in hours for slower factory

$\frac{1}{t+9}$ = rate of the slower factory

	Rate	**Time**	**Work**
Faster factory	$\frac{1}{t}$	6	$\frac{6}{t}$
Slower factory	$\frac{1}{t+9}$	6	$\frac{6}{t+9}$

Work done by faster factory + Work done by slower factory = One whole job

$\frac{6}{t} + \frac{6}{t+9} = 1$ *Substitute the values from the table into the word equation.*

$t(t+9)\left(\frac{6}{t} + \frac{6}{t+9}\right) = t(t+9)(1)$ *Multiply both members by the LCD t (t+9).*

$6(t+9) + 6t = t(t+9)$

$6t + 54 + 6t = t^2 + 9t$ *Simplify and write the quadratic equation so that one member is 0.*

$t^2 - 3t - 54 = 0$

$(t-9)(t+6) = 0$ *Factor and solve for t.*

$t - 9 = 0$ *or* $t + 6 = 0$

$t = 9$ *or* $t = -6$ *Since the time can't be negative, t=-6 is not an answer.*

Solving for the number of hours the slower factory can do the job, $t + 9 = 18$.

Answer: The faster factory could do the job in 9 h, while the slower factory could do the job in 18 h.

Example 2: Two rescue boats that have the same speed in still water travel in opposite directions in a river with a current of 5 km/h. The rescue boat going upstream departed 1h before the boat going downstream. A period after the boat going downstream has departed, a radio conversation between the boats indicated that one boat is 44 km upstream and the other boat is 75 km downstream. Approximate, to the nearest tenths of kilometer per hour, the speed of each boat in still water.

Solution:

Let r = rate of each rescue boat in still water in km/h
r + 5 = rate of the rescue boat going downstream in km/h
r – 5 = rate of the rescue boat going upstream in km/h

Since the distance D is the product of the rate r and time t, or D = rt, then $\boldsymbol{t = \frac{D}{r}}$. Use this equation and a table to organize the given information.

	Distance	**rate**	**time**
Boat going upstream	44	(r – 5)	$\frac{44}{r-5}$
Boat going downstream	75	(r + 5)	$\frac{75}{r+5}$

If the boat traveling upstream departed 1hr before the boat going downstream, then we have

(time for boat going upstream) = (time for boat going downstream) + 1

$$\frac{44}{r-5} = \frac{75}{r+5} + 1$$

$$(r-5)(r+5)(\frac{44}{r-5}) = (r-5)(r+5)\left[\frac{75}{r+5} + 1\right]$$

$$44(r+5) = 75(r-5) + (r-5)(r+5)$$

$$44r + 220 = 75r - 375 + r^2 - 25$$

$$0 = r^2 + 31r - 620$$

$$r = \frac{-31 \pm \sqrt{(31)^2 - 4(1)(-620)}}{2(1)}$$

$$r \approx \frac{-31 \pm \sqrt{3441}}{2} \quad \rightarrow \quad \boldsymbol{r \approx \frac{-31 \pm 58.660}{2}}$$

$\boldsymbol{r \approx 13.8}$ *or* $r \approx -44.8$ — *Not an appropriate solution*

Answer: The speed of each boat in still water is approximately 13.8 km/h.

WHAT'S

Critical Thinking

Solve the following problems:

1. A positive integer is six less than the other integer. The sum of the reciprocal of the two numbers is $\frac{14}{40}$.What are the two integers?

2. Two groups of volunteers can repack relief goods in 4 hours. One group would take 2 hours longer than the faster factory to the repacking. How many hours would it take each group working alone?

WHAT I HAVE LEARNED

To solve problems involving rational algebraic equation transformable to quadratic equations you may use the following procedures: (this is just almost the same with the previous module)
In solving problems involving the quadratic equation, you may use the following procedures:

1. Read and analyze the problem carefully.
2. Collect the necessary data.
3. Let the missing data be any variables and use algebraic expression to represent some data.
4. Convert the remaining data into mathematical sentence. This will serve as the "working rational algebraic equation".
5. Solve the rational algebraic equations.
6. Upon solving, observe if the resulted equation is quadratic.
7. If yes, find the solution set or roots of the quadratic equation using any method discussed on the previous lessons.
8. For assurance, just substitute the obtained roots to the "working rational algebraic equation". [you may obtain *extraneous root]

NOTE: Solving problems involving equations transformable to quadratic equations does not end in finding and checking the roots. Since, it is a real-life problem you must decide whether both or one or none of the solutions is reasonable.

WHAT I CAN DO

Critical Thinking

MORE PRACTICE MORE FUN!

Solve the following problems. Give your answers in 2 decimal places if necessary.

1. A boat can travel at a speed of 12km/h in still water. If it takes three hours to go 15 km upstream and back, find the speed of the current, giving your answer to 2 decimal places.
2. Gabo can do a job in one hour less than Wynter. If Gabo and Wynter work together the job takes $\frac{6}{5}$ hours. How long would it take each person working alone?
3. The time for the bus trip between two towns is 45 minutes shorter than the time for the train trip between the same two towns. If the towns are 350 km apart and the train speed is 15 km/h slower than the bus, find the speed of the bus in km/h.
4. A cistern can be filled by two pipes. The small pipe alone will take 24 minutes longer than the larger pipe to fill the cistern alone. The small pipe alone will take 32 minutes longer to fill the cistern alone than when the two pipes are operating together. How long will it take the larger pipe to fill the cistern alone.

ASSESSMENT

Write the letter of the correct answer on your answer sheet. If your answer is not among the choices, write E together with your final answer.

1. A worker can clean a pool in four hours less time than it takes another worker. If the men work together the job takes $\frac{8}{3}$ hours. How long would it take the slower man working alone?
   A. 6 hours
   B. 8 hours
   C. 7 hours
   D. 4 hours
2. Jessie and Mark are planning to paint a house together. Jessie thinks that if he works alone, it will take him 5 hours more than the time Mark takes to paint the entire house. Working together, they can complete the job in 6 hours. How long will it take Jessie to finish the job?
   A. 5 hours
   B. 15 hours
   C. 10 hours
   D. 20 hours
3. The distance between two cities A and B is 140 km. A car driving from A to B left at the same time as a car driving from B to A. The cars met after one hour, then the first car reached city B 35 minutes later than the second car reached city A. Find the speed of the car from city A to B.
   A. 60 km/h
   B. 50 km/h
   C. 80 km/h
   D. 70 km/h

4. A river is flowing at a rate of 5 km/h. A boat travels 12 km upstream and 10 km downstream in a total of 5 hours. What is the speed of the boat in still water?
   A. 17.6 km/h
   B. 27.6 km/h
   C. 43.5 km/h
   D. 54.6 km/h

5. Gabriel drove 25 km and then biked 10 km to the campsite. The trip took 2 hours. If he drove 20 km/h faster than he biked, how fast did he drive?
   A. 12.6 km/h
   B. 20.4 km/h
   C. 27.8 km/h
   D. 45.8 km/h
6. Two taps A and B fill a swimming pool together in two hours. Alone, it takes tap A three hours less than B to fill the same pool. How many hours does it take tap A to fill the pool separately?
   A. 3 hrs.
   B. 6 hrs.
   C. 9 hrs.
   D. 12 hrs.
7. Two faucets can fill a tank in 1 hour and 20 minutes. The first faucet takes more than two hours longer to fill the same tank when functioning without the second tap. How long does it take to fill the tank at first?
   A. 2 hours
   B. 6 hours
   C. 7 hours
   D. 4 hours
8. A motorboat makes a round trip on a river of 45 miles upstream and 45 miles downstream, maintaining the constant speed 12 miles per hour relative to the water. The entire round trip takes 8 hours. What is the speed of the current?
   A. 3 mi/h
   B. 2 mi/h
   C. 9 mi/h
   D. 6 mi/h
9. An airplane makes a trip of 2400 miles long into the wind and 2400 miles back with the same tail wind, maintaining the constant speed of 440 miles per hour relative to the air. The entire round trip takes 11hour. Find the speed of the wind.
   A. 30 mi/h
   B. 40 mi/h
   C. 20 mi/h
   D. 60 mi/h
10. A boat can cover 10 km up the stream and 5 km down the stream in 6 hours. If the speed of the stream is 1.5 km/h, find the speed of the boat in still water.
    A. 3.5 km/h
    B. 2.5 km/h
    C. 2 km/h
    D. 3 km/h

ADDITIONAL ACTIVITIES

Communication, Creativity and Collaboration

A REALITY OF RATIONAL ALGEBRAIC EXPRESSIONS:

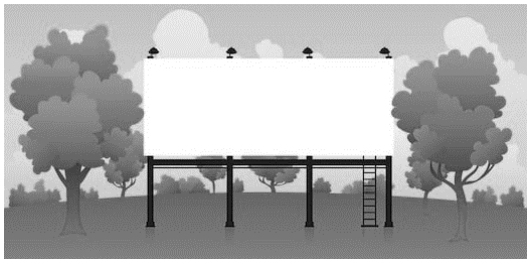
Cite and create a real-life situation where the concept of a rational algebraic equation transformable into a quadratic equation is being applied. Use the situation to answer the following questions.

1. How are the concept of a rational algebraic equations transformable into a quadratic equation applied in the situation?
2. What quantities are involved in the situation?
3. Formulate then solve a problem out of the given situation.
4. What do solutions obtained represent? Explain your answer.

PROBLEM – BASED WORKSHEET

A. Advertising

An amusement park wants to place a new rectangular billboard to inform visitors of their new attractions. Suppose the length of the billboard to be placed is 4 meters longer than its width and the area is 96 square meters. What will the length and width of the billboard?"



1. How will you represent the width and the length of the billboard?
2. What equation describes the area of the billboard?
3. What is the length and the width of the billboard?

B. Flag Ceremony

In one of the Flag raising ceremony, the school counsellor noticed that 780 students stand in rows and columns. Each row has equal number of students and each column has equal number of students. If the number of students in each row is 4 more than the number of rows, find the number of students in each row.

E-Search

You may also check the following links for your reference and further learnings on solving problems involving quadratic equations and rational algebraic equations.

- https://www.youtube.com/watch?v=ylgYZkzp4CA
- https://www.analyzemath.com/Algebra2/solutions-to-equations.html
- https://study.com/academy/topic/rational-expressions-quadratic-equations.html
- http://mathvids.com/lesson/mathhelp/1437-solving-rational-equation-to-quadratic-1

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Sinom L. Chua, Benson Tan, et. al. Mastering Intermediate Algebra II, SIBS Publishing House, Inc.

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https://www.freepik.com/free-vector/kids-having-online-lessons_7560046.htm
https://www.freepik.com/free-vector/illustration-with-kids-taking-lessons-online-design_7574030.htm

ANSWER KEY

WHAT'S IN

1. $\frac{x^2+18}{6x}$
2. $\frac{x^2+4x+2}{2x}$
3. $\frac{-3y^2-y+10}{2y}$
4. $\frac{2a^2+1}{a(2a+1)}$
5. $\frac{3m^2+2m+5}{2m(m+1)}$

WHAT I CAN DO

1. 4.90 km/h
2. 3hours, 2 hours
3. 91.5 km/h
4. 24 minutes

WHAT'S MORE

1. 4,10
2. $3+\sqrt{17}\ hours \approx 7.12\ hours$, $5+\sqrt{17}\ hours \approx 9.12\ hours$

WHAT I KNOW

1. C
2. B
3. C
4. A
5. C
6. B
7. A
8. C
9. A
10. A

ASSESSMENT

1. B
2. B
3. A
4. B
5. C
6. A
7. D
8. A
9. B
10. A

PROBLEM – BASED WORKSHEET

A.

1. If we represent the width, in meters, of the billboard by x, then its length is $x + 4$.
2. Since the area of the billboard is 96 m², then $\boldsymbol{x\,(x+4) = 96}$.
3. Solving the equation, we have,

$$\begin{aligned} x\,(x+4) &= 96 \\ x^2 + 4x - 96 &= 0 \\ (x+12)\,(x-8) &= 0 \\ x+12 = 0 \quad &\text{or} \quad x-8 = 0 \\ x = -12 \quad &\text{or} \quad \boldsymbol{x = 8} \end{aligned}$$

We only consider the positive value of x since the situation involves measures of length. Thus, the dimensions of the billboard are **8 meters** (*width*) and **12 meters** (***length***).

B. Let r be the number of rows, and $r + 4$ be the number of students in each row.

The sum of the product of the number of rows and the number of students in each row add up to 780. In symbols, $(r)\ (r+4) = 780$

Solving the resulting quadratic equation,

$$\begin{aligned} (r)\ (r+4) &= 780 \\ r^2 + 4r &= 780 \\ r^2 + 4r - 780 &= 0 \\ (r+30)\,(r-26) &= 0 \\ r+30 = 0 \quad &\text{or} \quad r-26 = 0 \\ r = -30 \quad &\text{or} \quad \boldsymbol{r = 26} \end{aligned}$$

Since the number of must be a positive number, we conclude that r = **26**. And the number of students in each row $\boldsymbol{r + 4 = 26 + 4 = 30}$ ***students***.